# General Relativity and Applications
# 4. Black Holes

**Edmund Bertschinger**

**MIT Department of Physics and**
**Kavli Institute for Astrophysics and Space Research**

# What is a black hole?

A massive spacetime curvature singularity,

(a point or ring of infinite density and tidal acceleration)

Surrounded by an event horizon

(a spacetime boundary between causally disconnected regions of the universe)

# Outline

The Astrophysics of Black Holes

The Physics of Black Holes

The Future of Black Hole Studies

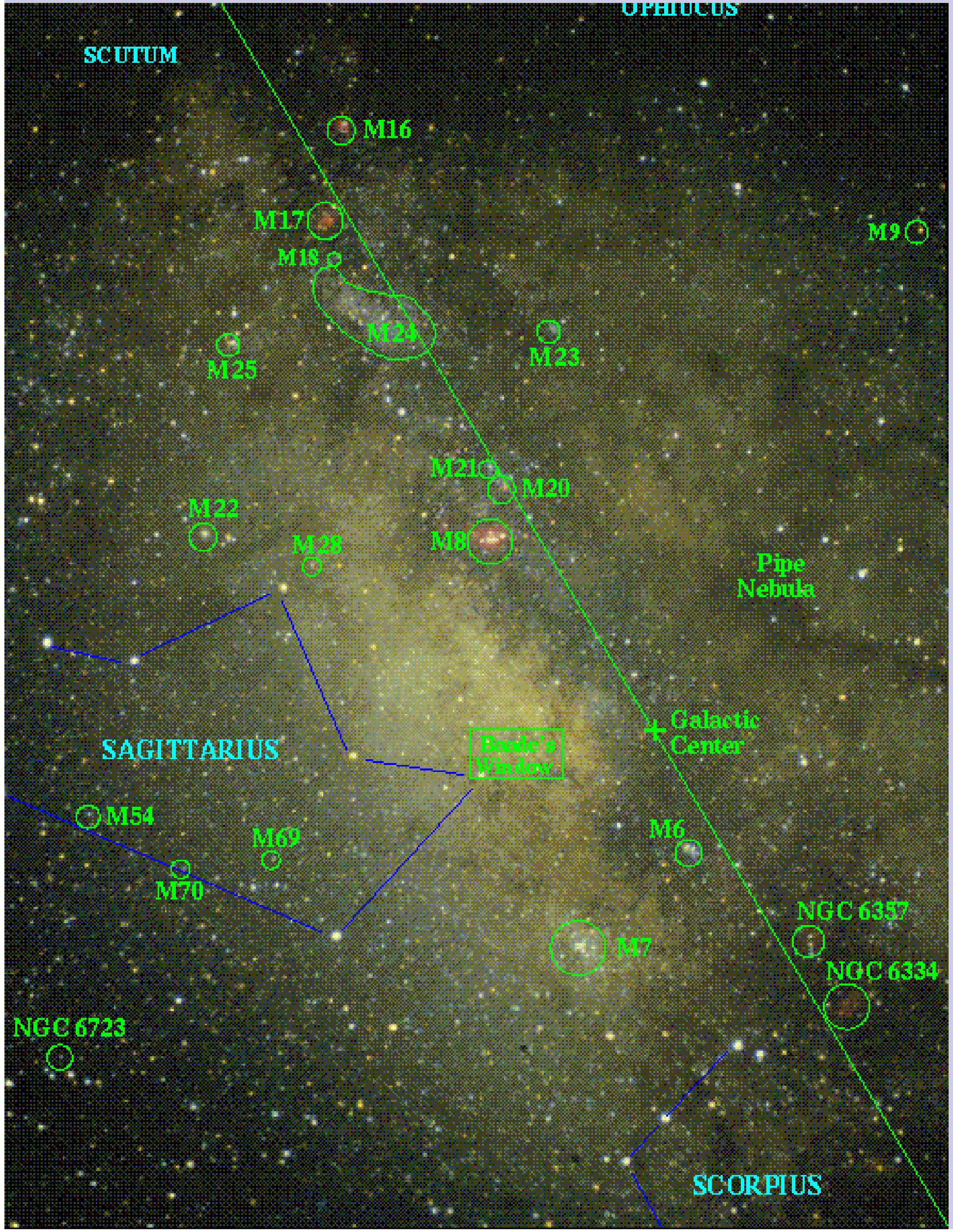
# Black Holes in the Universe

## Galactic Nuclei

[Animation from amazing-space.stsci.edu]

## Dead stars

# The biggest nearby black hole: Sgr A*




Center of our galaxy: radio source Sgr A*

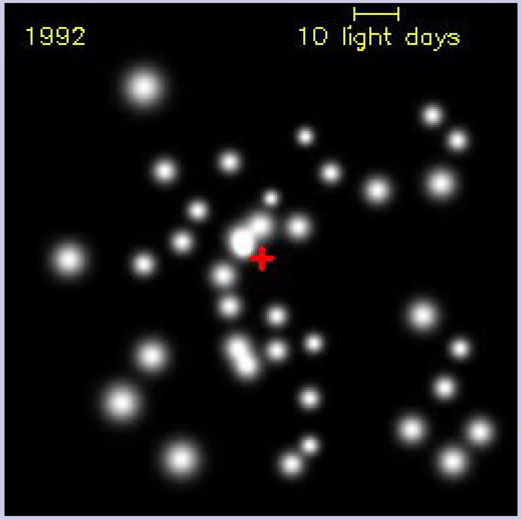
Distance: 8 kpc = 8000 pc, 1pc = 3.3 lt-yr

Highly obscured in optical

Dense central star cluster visible in infrared

Photo/illustration from A. Tanner, UCLA

# Stellar Dynamics of Sgr A*




From R. Genzel et al., Max-Planck-Institut für extraterrestrische Physik

$$\frac{GM}{a} = \left(\frac{2\pi a}{P}\right)^2$$

M = combined mass

a = semimajor axis of stellar orbit

P = orbital period

Star S0-2 has a=920 AU, P=14.5yr

→M = $3.7 \times 10^6$ solar masses (±20%)

A. Ghez et al. (2005)

# How do we know it's a black hole?

*No firm proof yet*: Black holes are indistinguishable from Newtonian bodies at large distances r:

$$\frac{GM}{rc^2} = \left(\frac{M}{M_{\mathrm{solar}}}\right)\left(\frac{1.5\ \mathrm{km}}{r}\right) \ll 1$$

*Lack of alternative*: Any plausible alternative would lead quickly to gravitational collapse

# Pinning down a black hole: Observe it close in

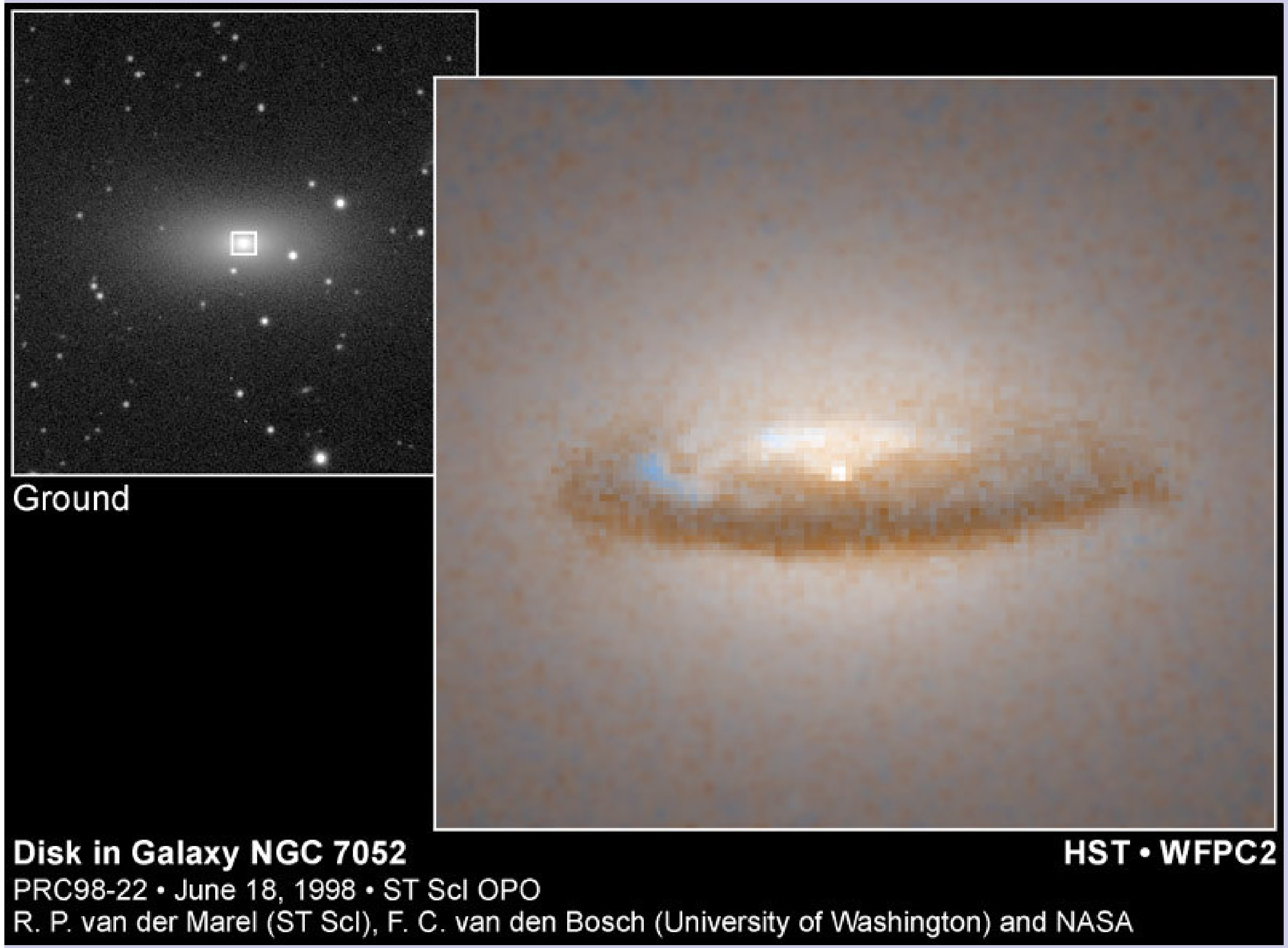



Figure from STScI

<u>Accretion disk</u>

Angular momentum forces gas falling toward the BH to orbit in a disk

Friction causes the gas to slowly spiral in toward the BH, and makes the gas very hot

<u>The problem</u>: Relativistic disks much too distant to resolve!

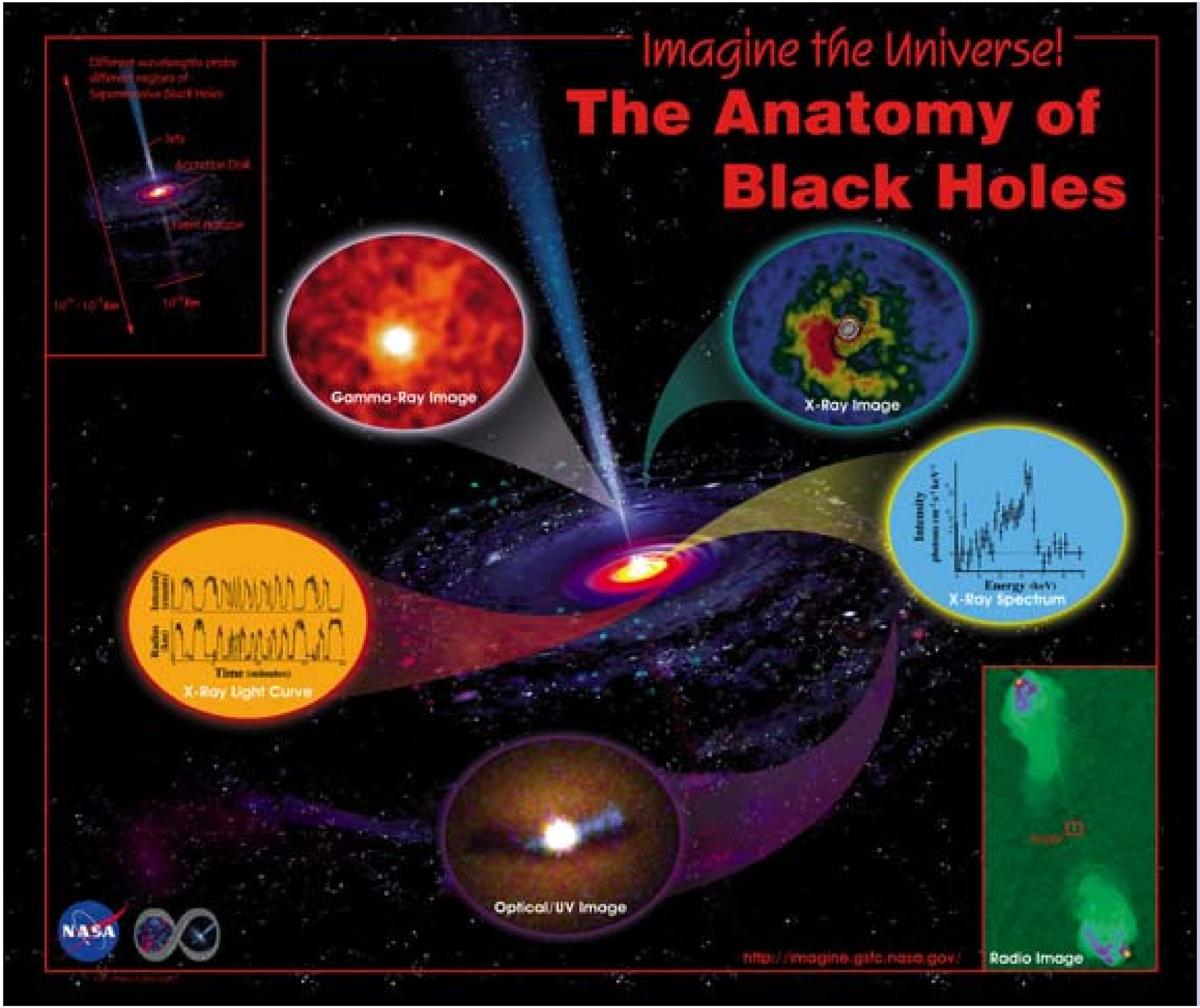



Figure from NASA/ GSFC Imagine

# MCG–6-30-15

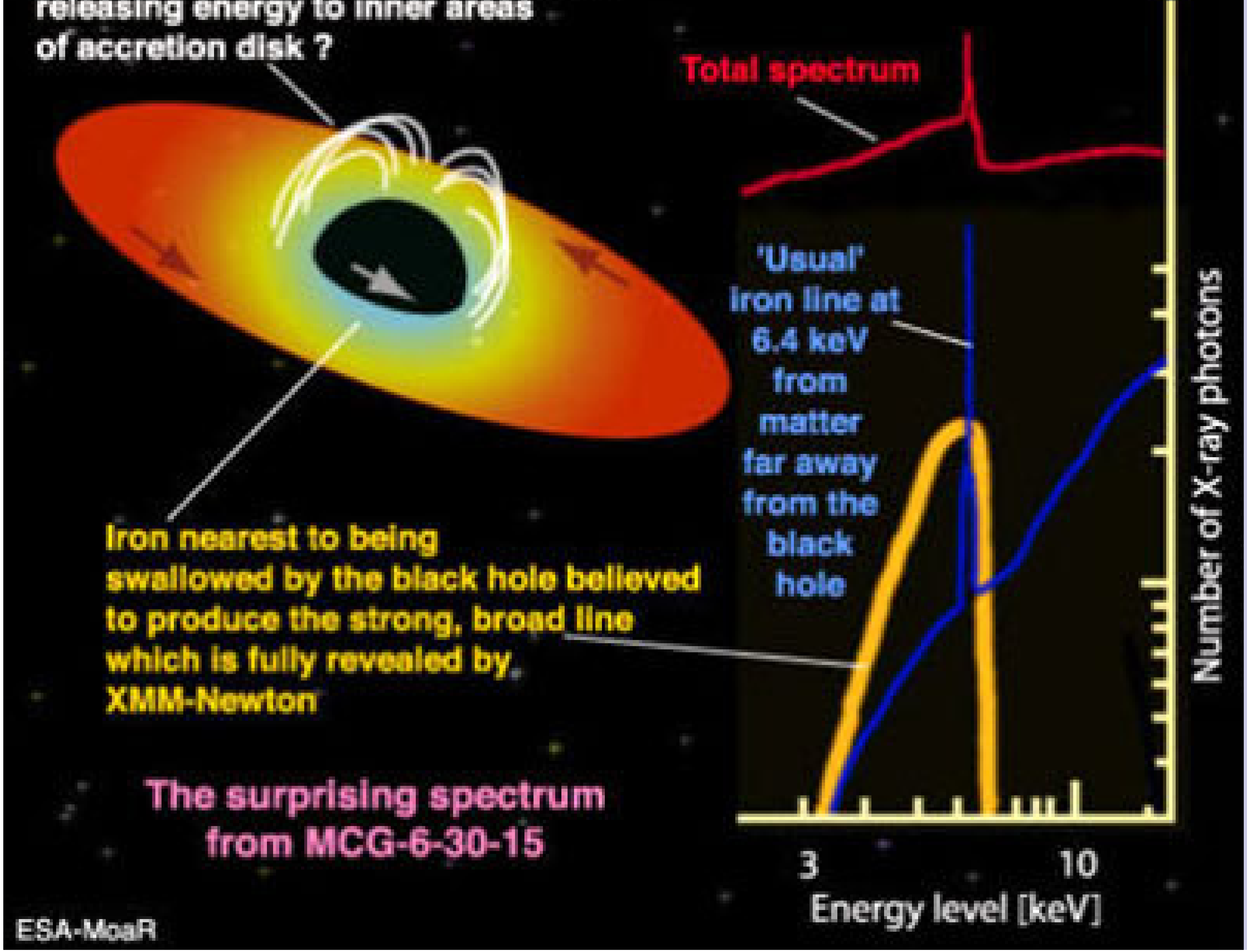
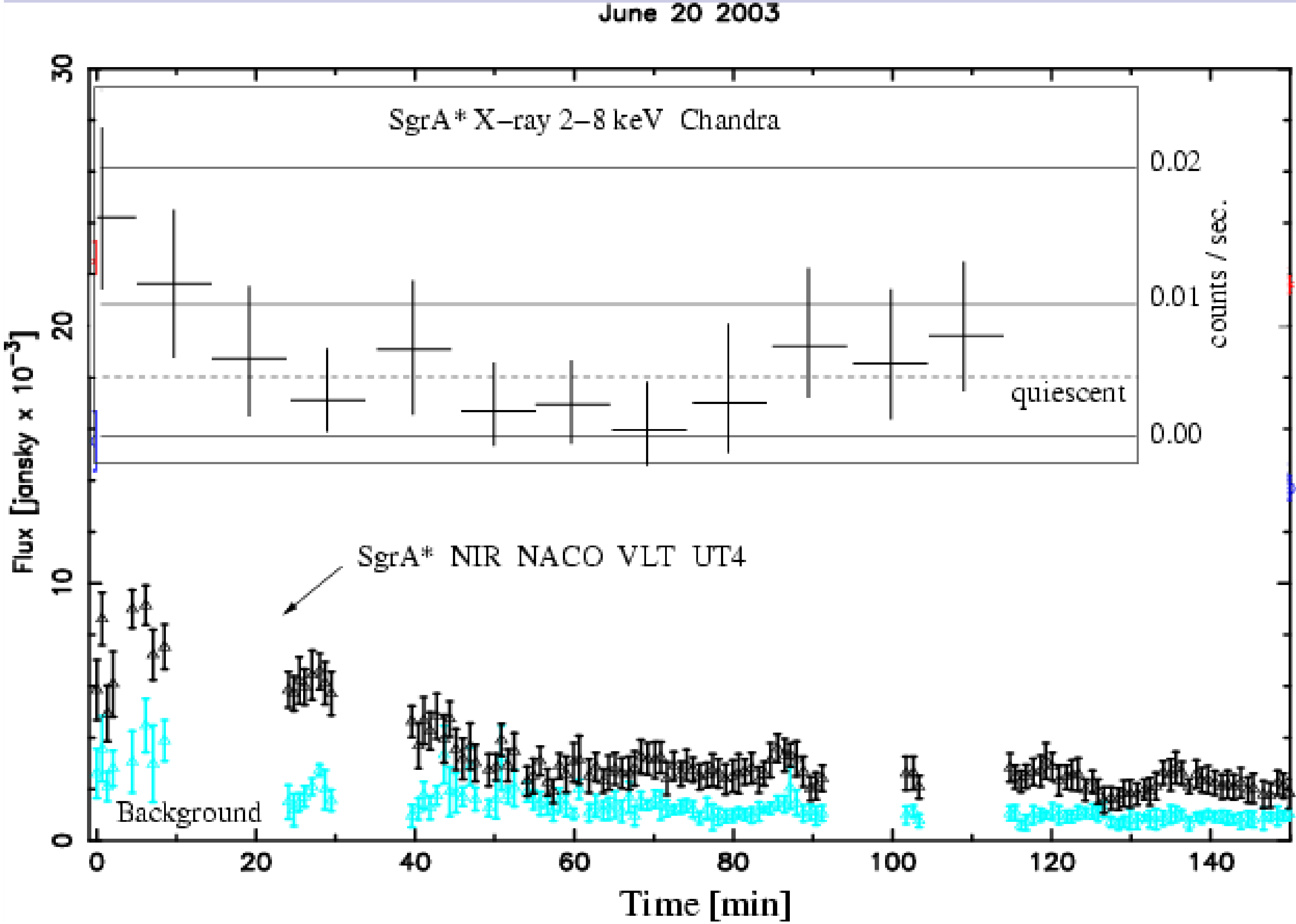



Figure from ESA/ XMM-Newton

# Time variability




A. Eckart et al., A&A 427, 1 (2005)

# Causality and cause of flickering

*Causality*:

Emitting region of size R cannot switch on and off in time less than R/c.

Sgr A* : 10 min → $R < 34(GM/c^2)$

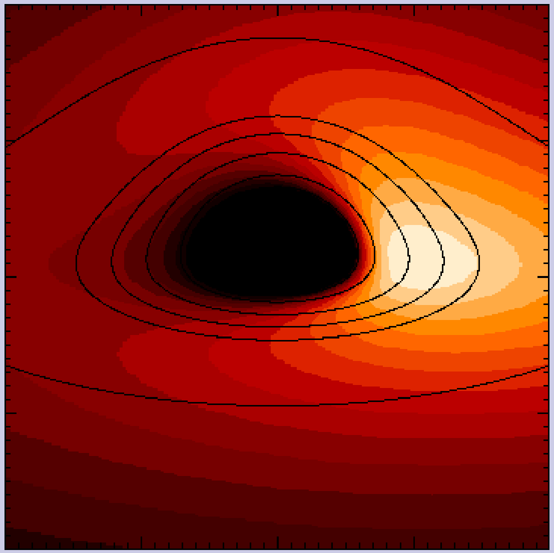
*Causes of flickering:*

Sporadic accretion

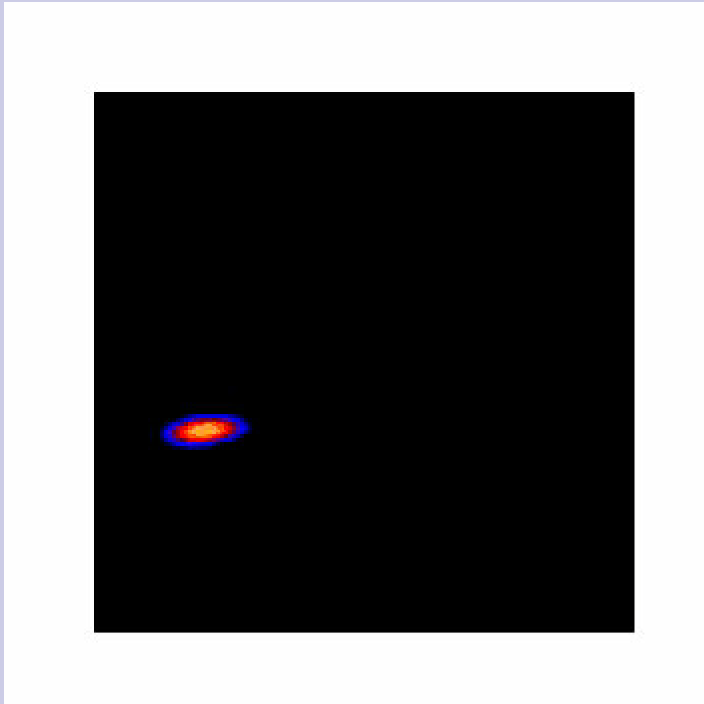
Accretion disk instabilities

Orbiting hot spots or spiral waves

# Optical illusions predicted



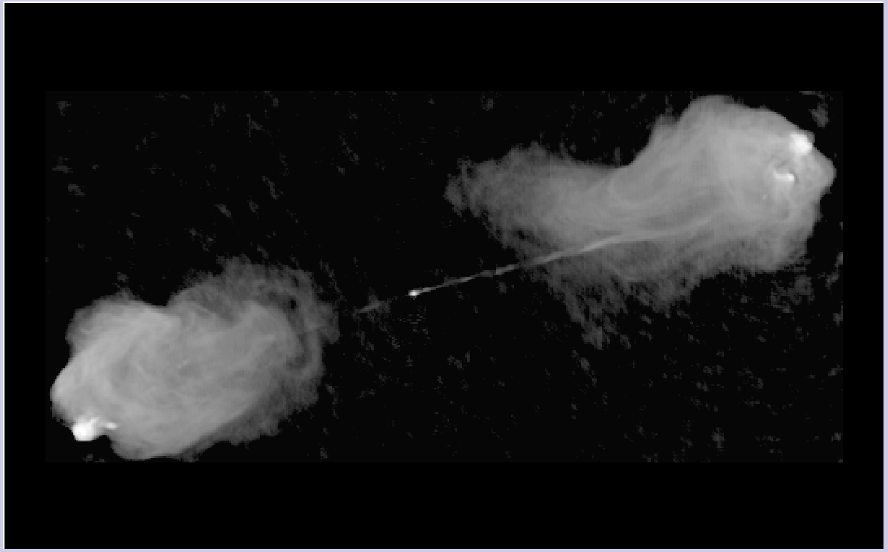
Visual appearance of an accretion disk close to the event horizon (solid lines show projected orbits) (J. Schnittman, MIT)



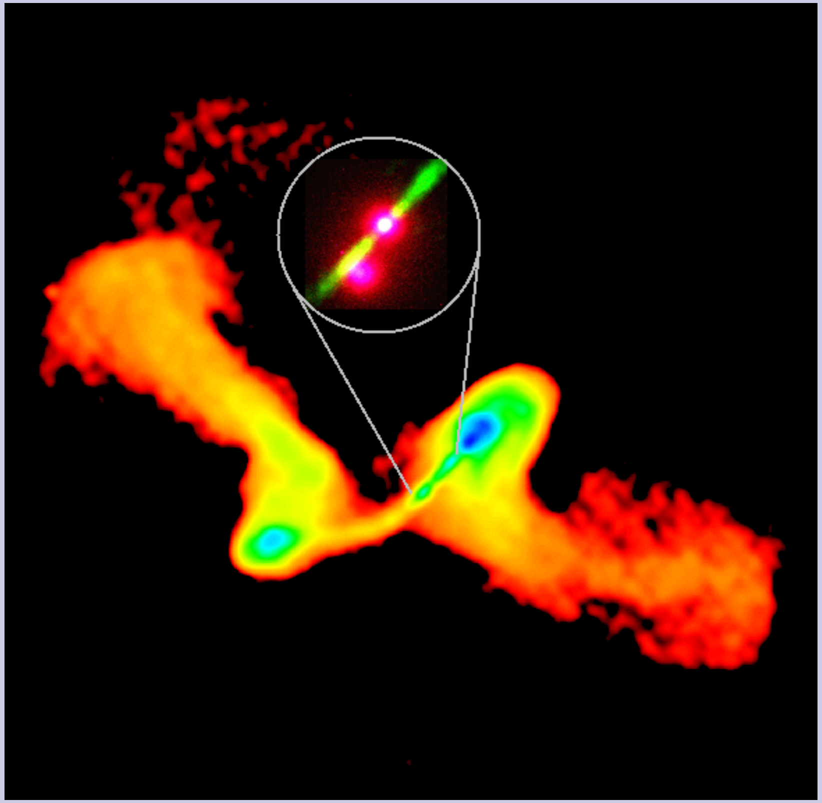
Visual appearance of an orbiting hot spot (J. Schnittman, MIT)

[show APOD2002Oct8]

# Relativistic Jets



From NRAO/AUI: R. Perley, C. Carilli, J. Dreher



From NRAO/AUI and STScI: D. Merritt and R. Ekers

We don't know how the jets form!
Magnetic fields? BH spin?

# Gamma-Ray Bursts

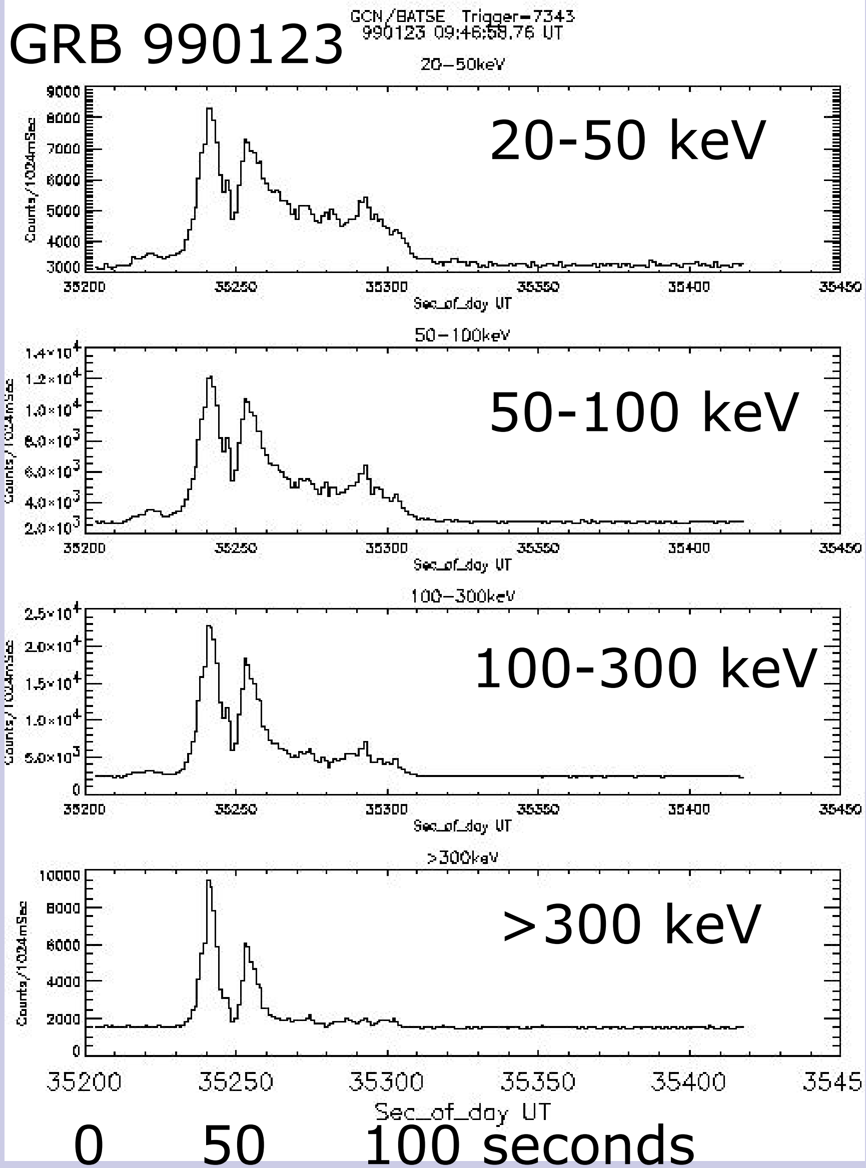



Figure from NASA CGRO/BATSE

Long-duration (> 2s) GRBs caused by relativistic jets of a newly born BH, pointing toward earth

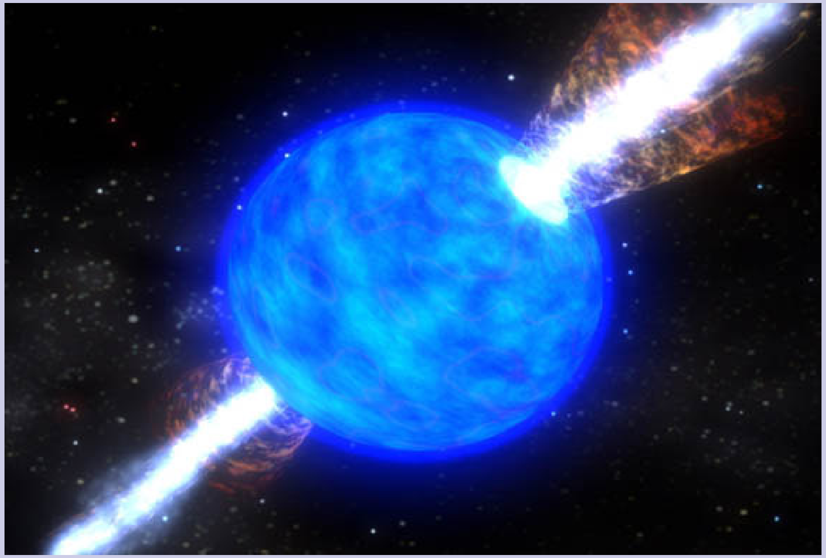


Figure from NASA/ SkyWorks Digital

# Outline

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# Are Black Holes Giant Spacetime Trampolines?

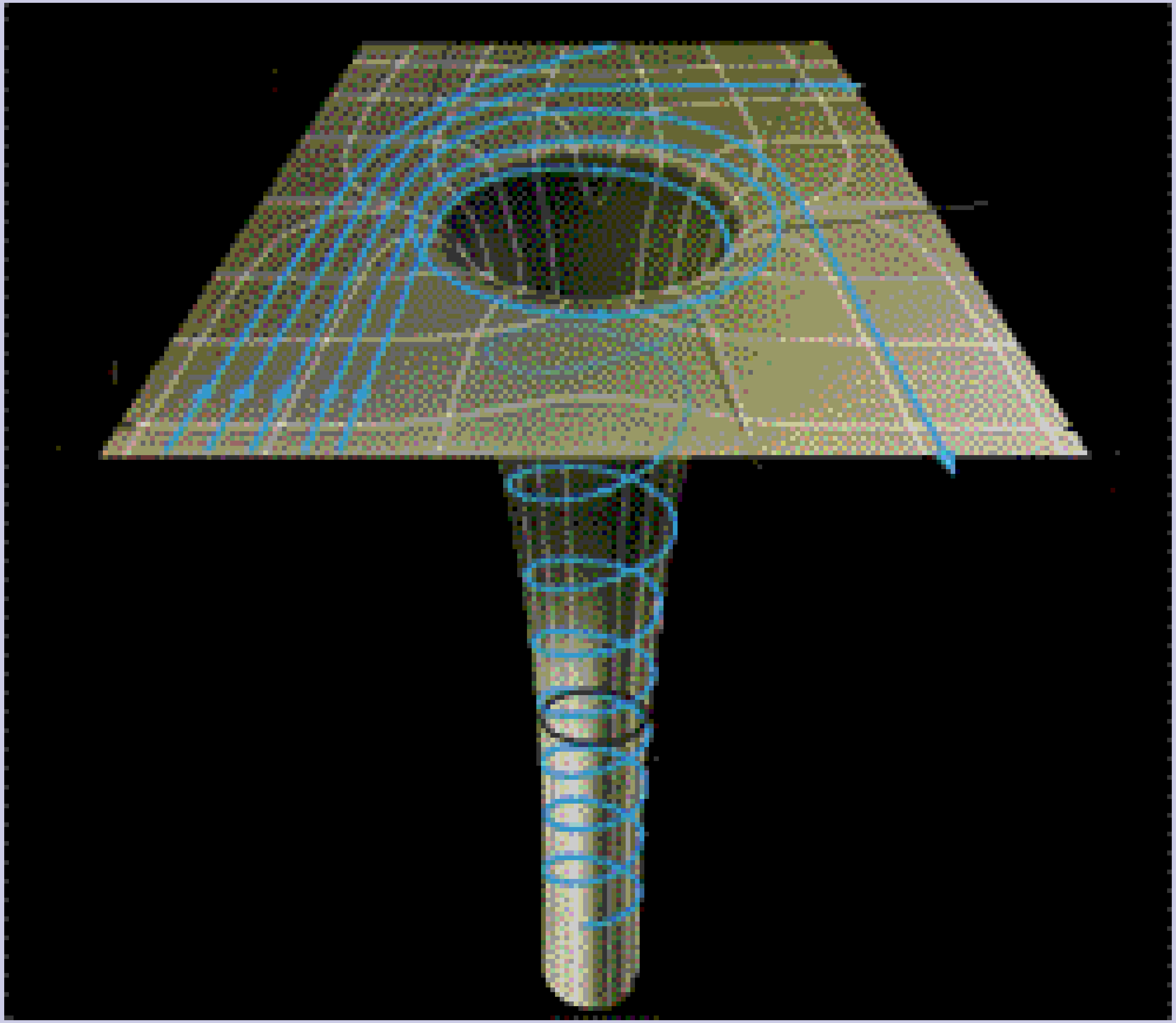
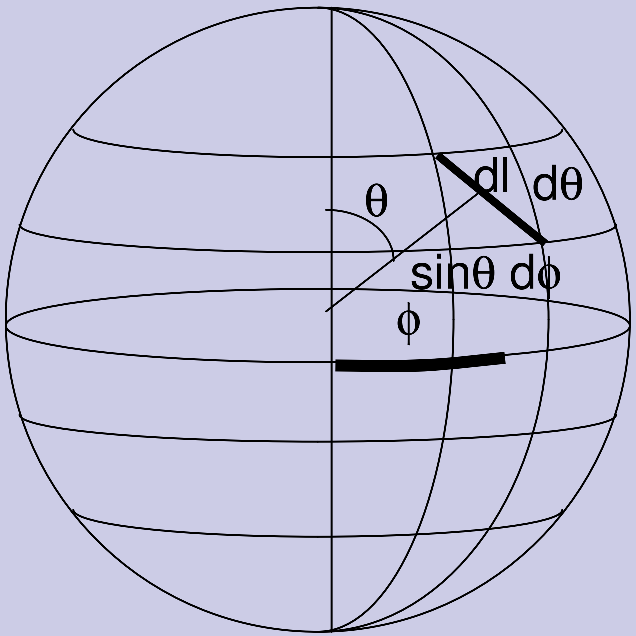


Figure from NASA/GSFC Imagine

What does Stephen say?

# Visualizing spatial curvature




Pythagorean Theorem on a sphere:

$$dl^2 = (\sin\theta\, d\phi)^2 + (d\theta)^2$$
$$\equiv d\theta^2 + \sin^2\theta\, d\phi^2$$

Change variables:

$$s = \sin\theta \;\rightarrow\; dl^2 = \frac{ds^2}{1 - s^2} + s^2\, d\phi^2$$

Three dimensions are unnecessary to describe a sphere, but they can help us to visualize it.

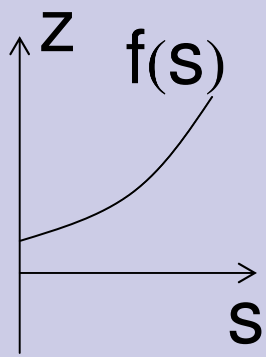
# Embedding a 2-sphere in Euclidean 3-space

Pythagorean Theorem in cylindrical coordinates

$$x = s\cos\phi\ , \quad y = s\sin\phi$$

$$\Rightarrow \quad dx^2 + dy^2 + dz^2 = ds^2 + s^2\, d\phi^2 + dz^2$$

Now define a surface of revolution $z = f(s)$




Distance between $(s,\phi)$ and $(s+ds,\phi + d\,\phi)$ is

$$dl^2 = [1 + (df/ds)^2]ds^2 + s^2\, d\phi^2$$

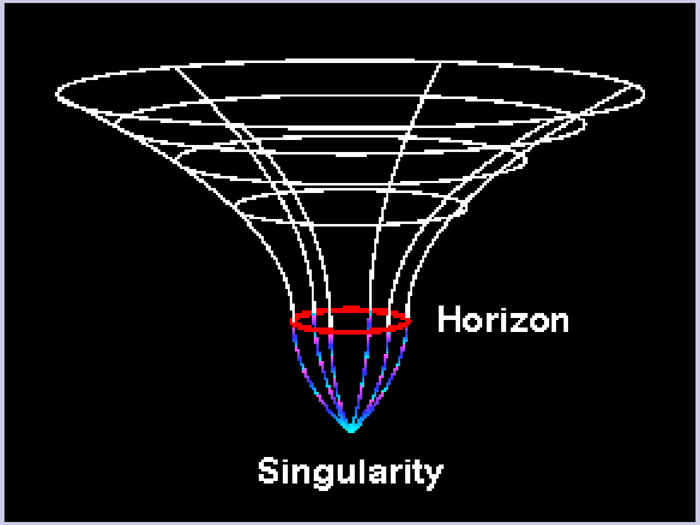
If $f(s) = \pm\sqrt{1 - s^2}$ , then $dl^2 = \dfrac{ds^2}{1 - s^2} + s^2\, d\phi^2$ .

$$z^2 = f^2 = 1 - s^2 = 1 - (x^2 + y^2)$$

is the equation of a unit sphere!

# Black Hole Geometry

Embedding diagram for $z = f(s) = \sqrt{\frac{2GM}{c^2}\left(s - \frac{2GM}{c^2}\right)}$




From Andrew Hamilton, U. Colorado

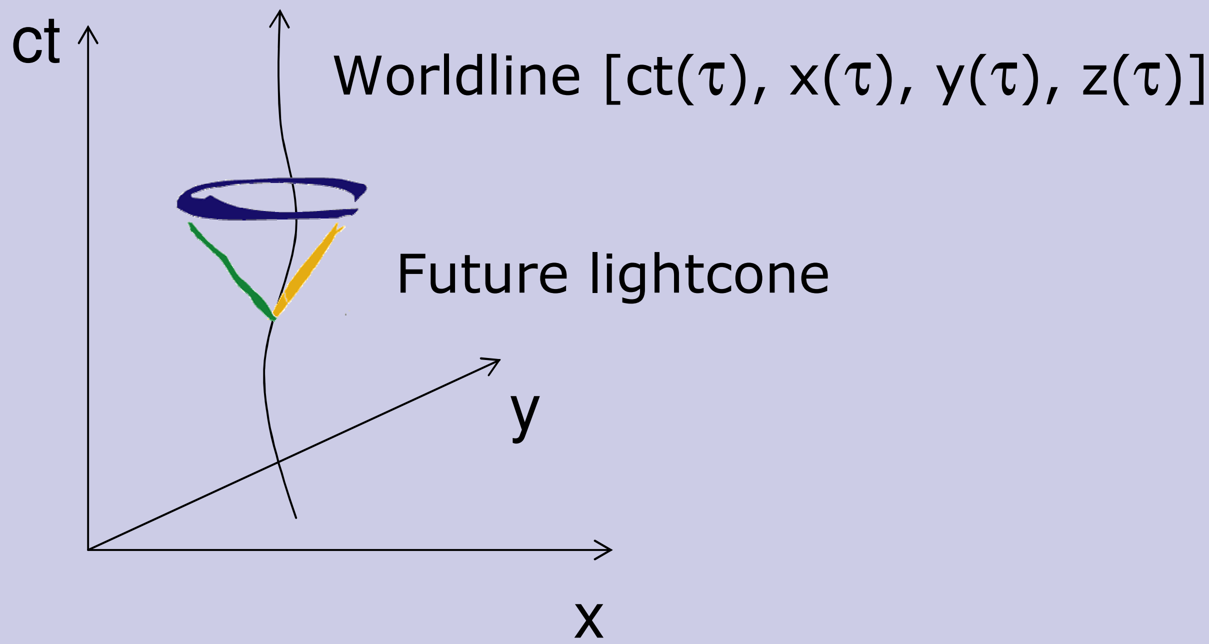
With s → r, this gives the spatial part of the Schwarzschild metric in the equatorial plane!

$$dl^2 = \frac{dr^2}{1 - 2GM/rc^2} + r^2 d\phi^2$$

The given embedding works only for $r > 2GM/c^2$

Blackhole geometry is Spacetime geometry!

# Spacetime Geometry (Flat)




Minkowski Diagram

Spacetime Pythagorean Theorem

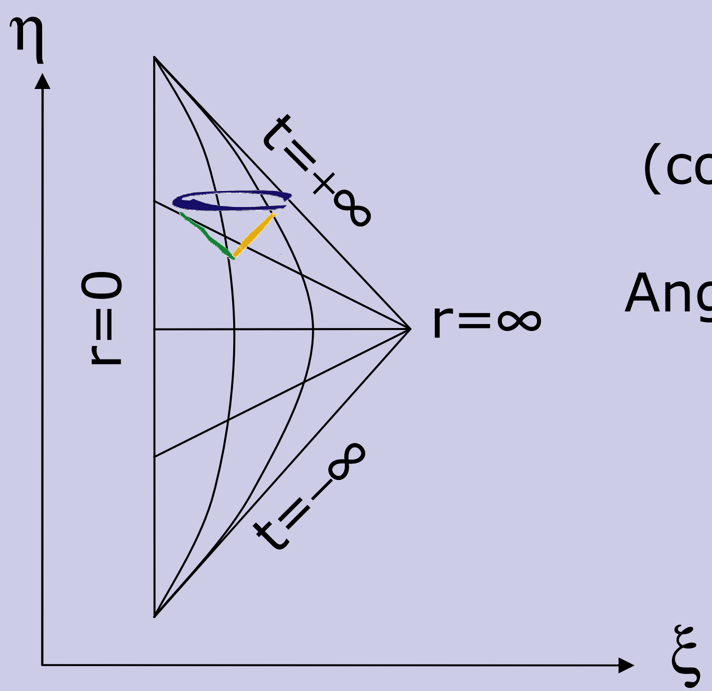
$$
\begin{aligned}
-c^2 d\tau^2 &= -c^2 dt^2 + dx^2 + dy^2 + dz^2 \\
&= -c^2 dt^2 + dr^2 + r^2\, d\phi^2 \quad \text{for } \theta = \pi/2.
\end{aligned}
$$

# Penrose Diagrams

Map infinity to boundary of a compact domain:

$$ct + r = \tan(\xi + \eta) \;, \quad ct - r = \tan(\xi - \eta)$$

Minkowski diagram becomes a Penrose Diagram.

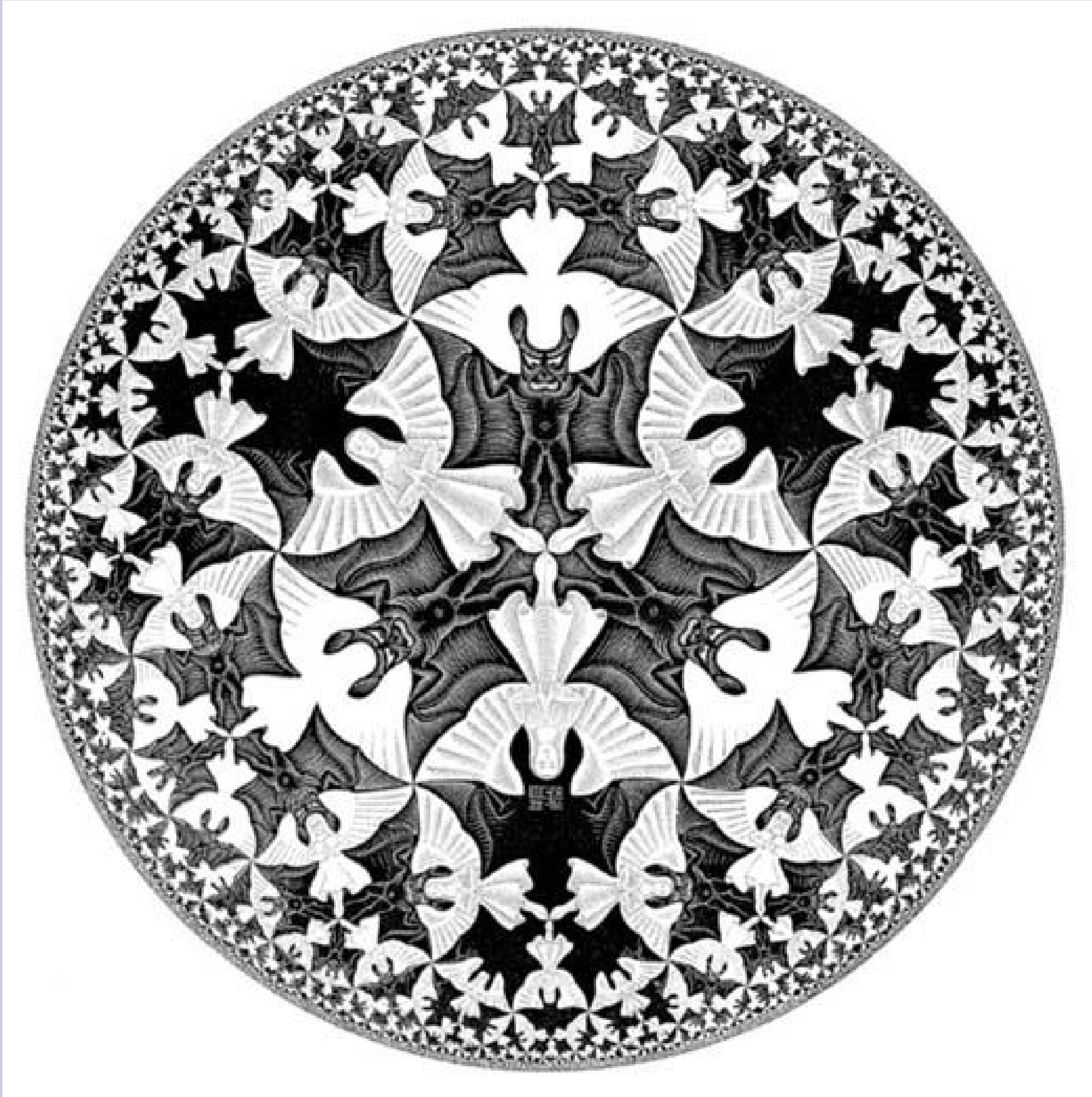



Light cones preserved (conformal transformation).

Angular coordinates (θ,φ) not shown in the diagram.

Severe distortion near boundary.

# M.C. Escher, Circle Limit IV



# Spherical, Uncharged Black Hole

## Oppenheimer and Snyder, 1939

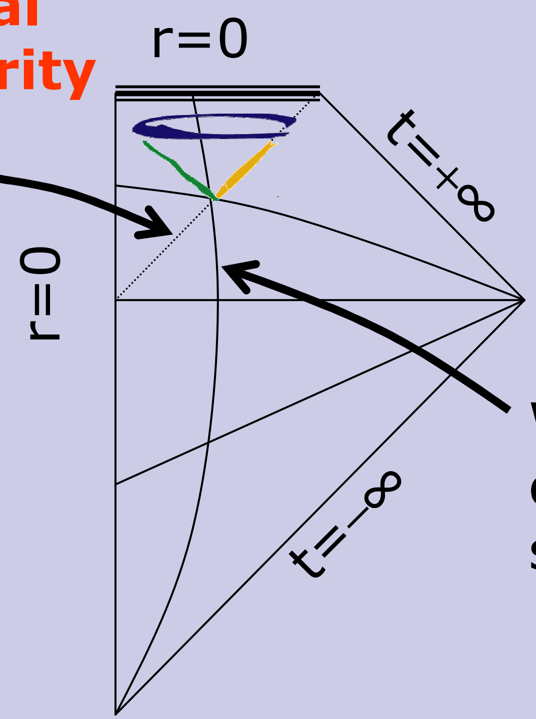
**Penrose diagram**:
Global geometry
Topology
Causal structure

**Metric**:
Local geometry
Curvature

Schwarzschild metric

$$-c^2 d\tau^2 = -A(r)c^2 dt^2 + \frac{dr^2}{A(r)} + r^2 d\phi^2 \,,$$

$$A(r) = 1 - \frac{2GM}{rc^2}$$




Schwarzschild t breaks down at $r=2GM/c^2$:
Event horizon!

# The Schwarzschild singularity is a *time*, not a *place*

Across the horizon,all of space is crushed, like a big bang singularity in reverse.

# Gravity as an effect of curvature

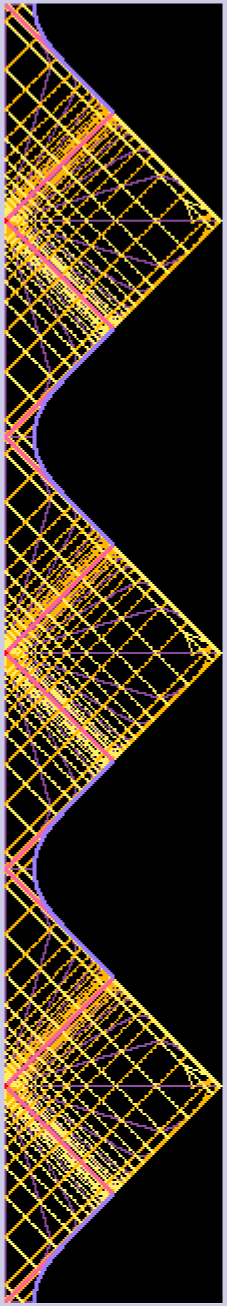
In flat spacetime, freely-falling bodies follow paths of *maximal* proper time.

Einstein equivalence principle: the same is true in curved spacetime.

JKerrOrbits was developed by S. Tuleja, T. Jezo, and J. Hanc based on earlier Schwarzschild integrator of A. Riess and E. Taylor

# What happens at the event horizon?

- Classically, nothing.
  - Event horizon is determined by global, not local, structure
- Quantum mechanically, Hawking radiation
  - Particle vacuum state fluctuates with creation/annihilation of virtual pairs of particles
  - Negative energy particles fall in, positive energy ones escape, black hole loses mass as blackbody radiation
  - Completely negligible for astrophysical black holes



# Interstellar travel by BH?

Penrose diagram of a spherical, charged BH (half of Reissner-Nordstrom).

Gravitational repulsion of electric fields causes freely-falling observer to bounce back to increasing r, crossing into another universe!

Problem: instability of the second horizon destroys the connections (Poisson and Israel "mass inflation").

Cool diagram from Andrew Hamilton, U. Colorado

# Nature of the Singularity

- Classical gravity: spacetime geometry near the singularity not fully understood
  - Oscillatory curvature singularity (Belinsky, Khalatnikov, Lifshitz): “spaghettification” or “eggbeater”?
  - Spacelike, timelike, or null?
  - Horizon structure?  Singularities without horizons?
- Quantum gravity: singularity resolved, but at Planck scale or much larger (Mathur’s stringy fuzzballs)?
- Unitary quantum evolution: whence information?  String states?

# Questions for the future

- Are the BH of nature exactly the BH predicted by Einstein's theory?
- How do supermassive BH form? What role do they play in galaxy formation?
- How and why do relativistic jets form?

# More Questions

- What is the nature of the singularity?
- What is the wavefunction of a black hole spacetime?
- Is four spacetime dimensions enough to properly describe a black hole?
- ???