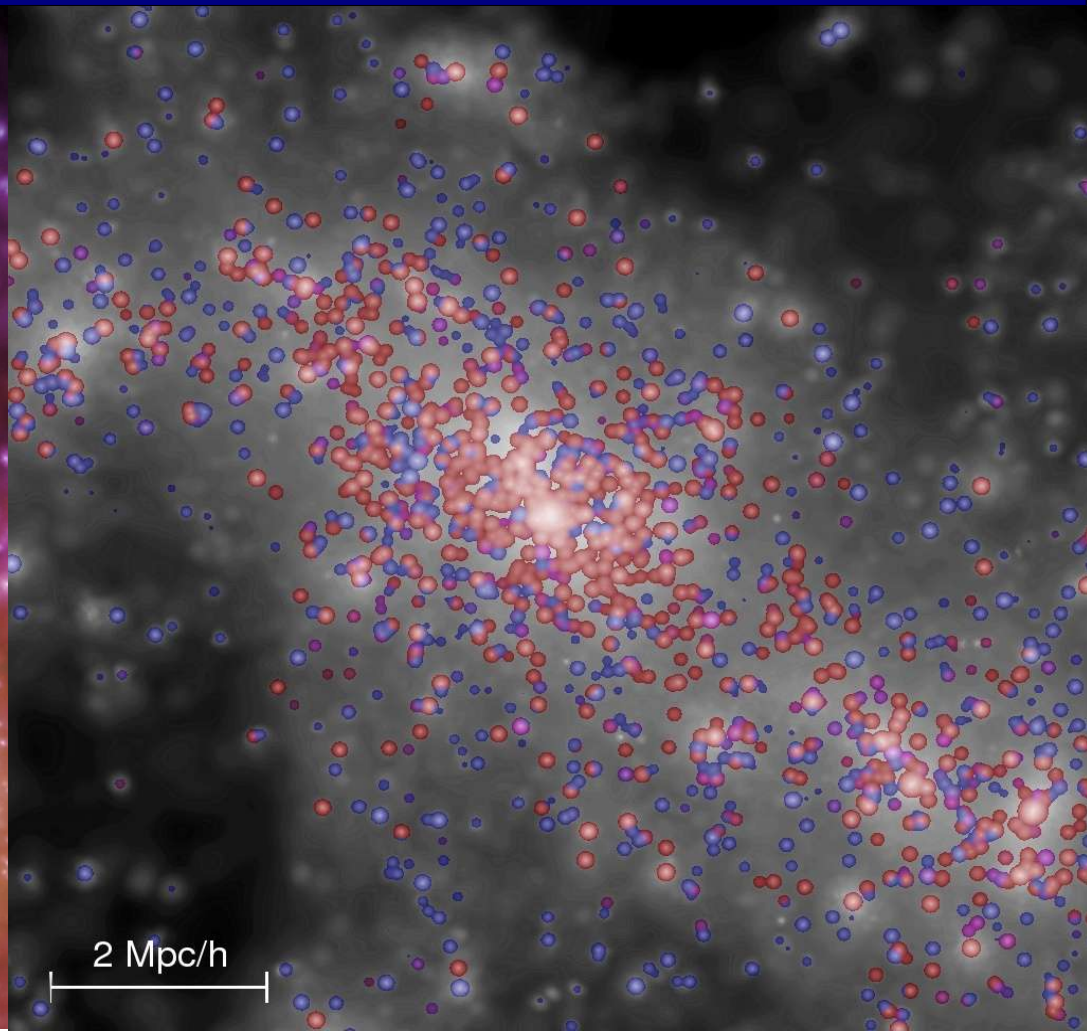
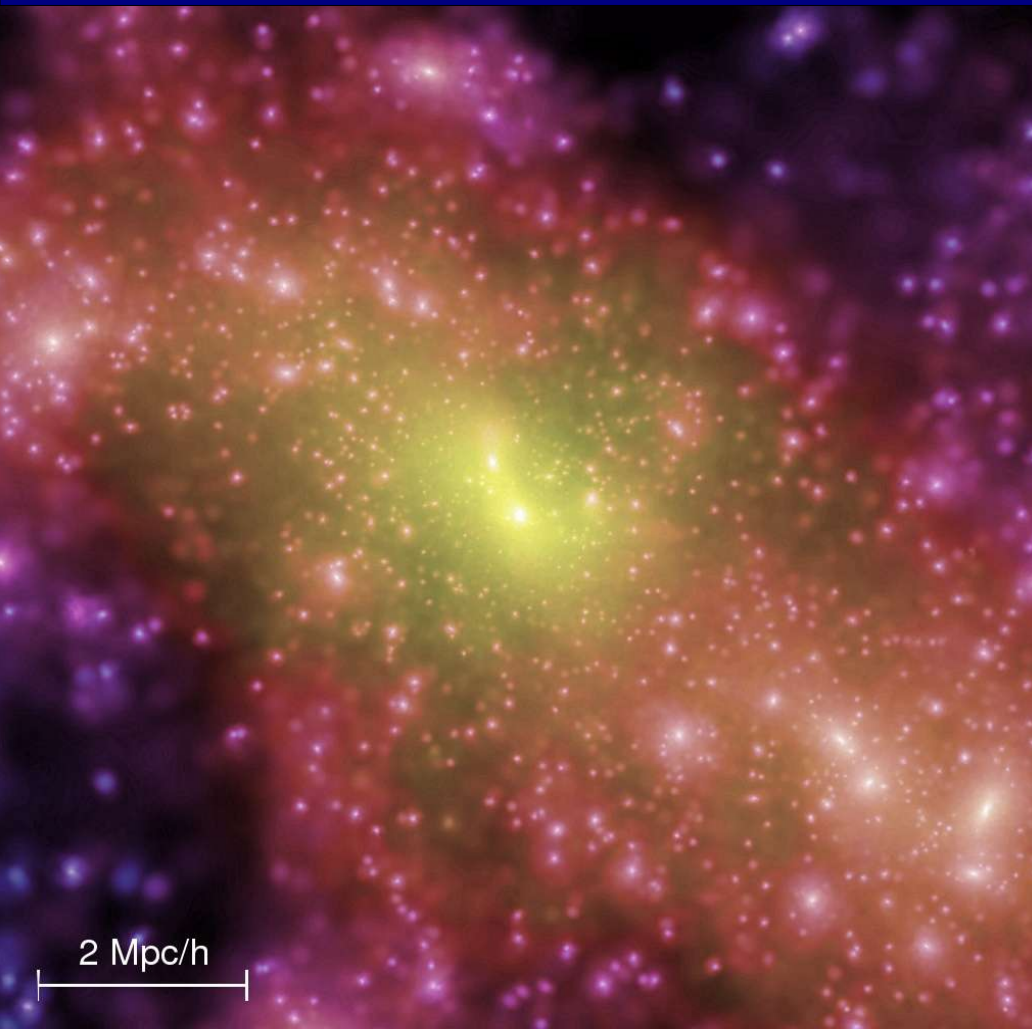


An Introduction to Phenomenological Models of Galaxy Formation

usually called “semi-analytic” models (SAMs)



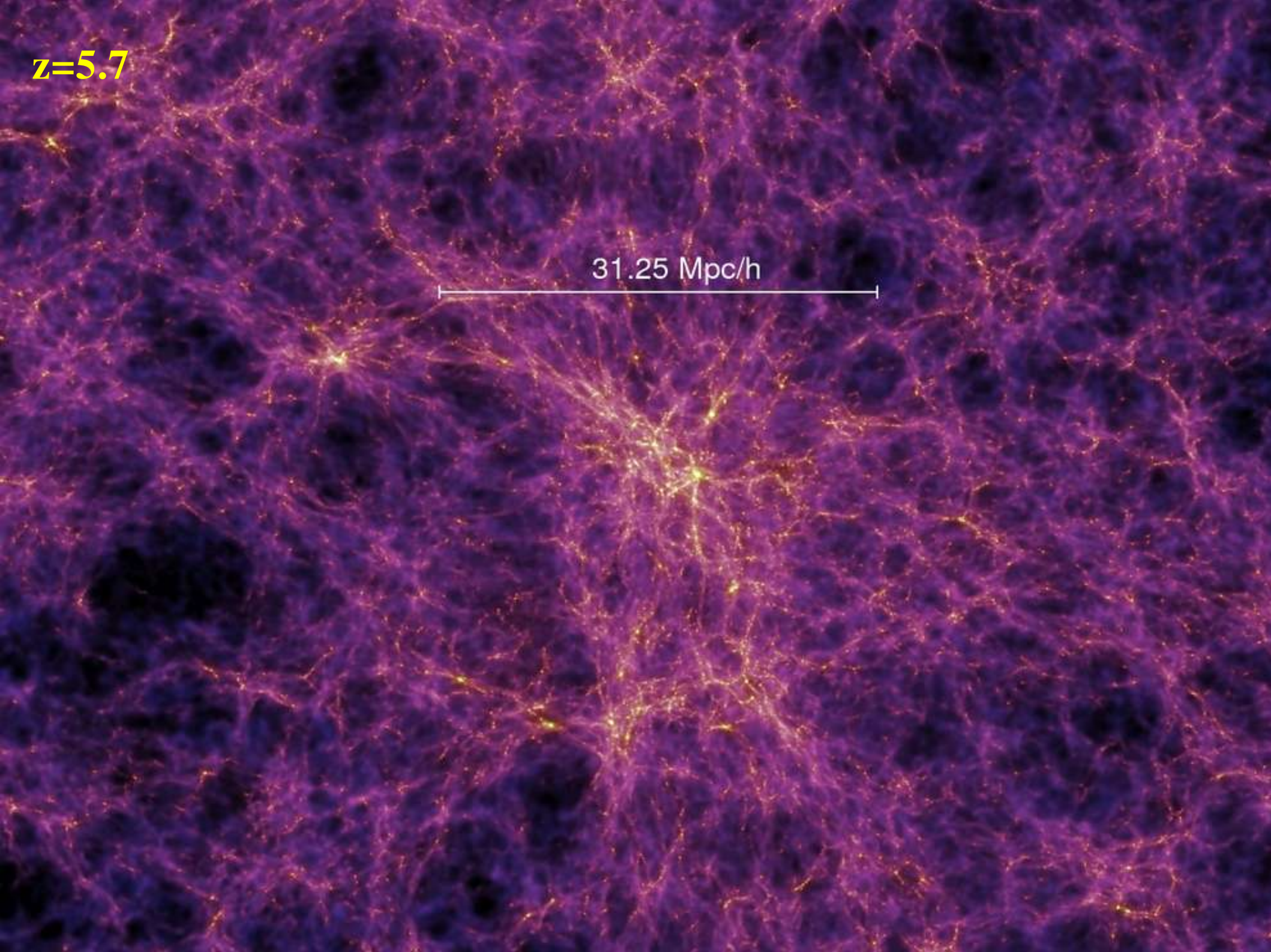
$z=18.3$

31.25 Mpc/h

A cosmological simulation snapshot at redshift $z=18.3$ showing a complex, filamentary structure. The image is dominated by a dense network of purple and blue filaments, with some brighter, yellowish-white regions. A horizontal scale bar is located in the upper-middle part of the image, labeled "31.25 Mpc/h". The overall appearance is that of a highly turbulent and interconnected web of matter.

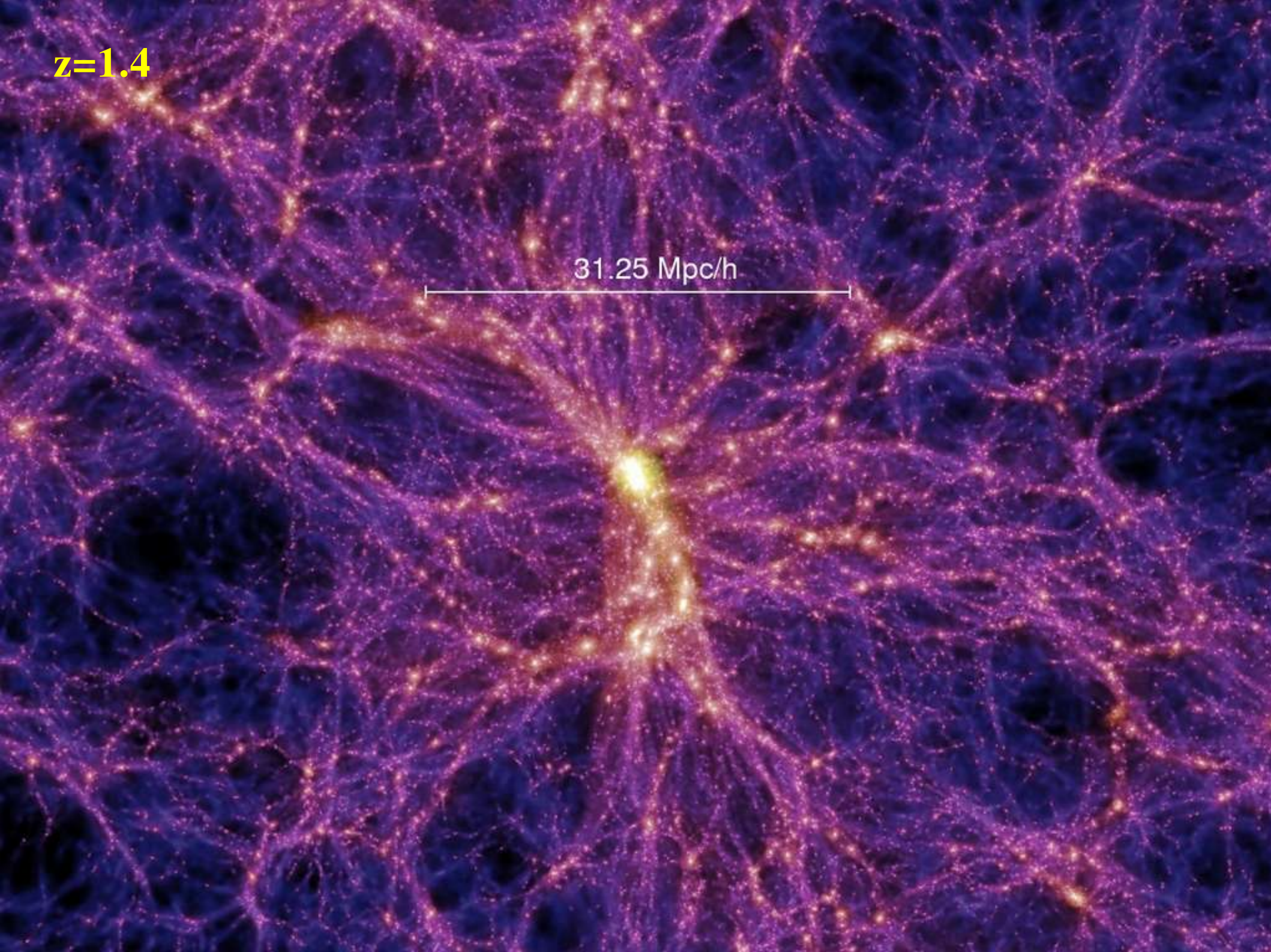
$z=5.7$

31.25 Mpc/h

A visualization of the cosmic web at redshift $z=5.7$. The image shows a complex network of filaments and nodes, with a central, more densely populated region. The filaments are colored in shades of purple and blue, while the nodes are highlighted in yellow and orange. A horizontal scale bar is located in the upper-middle part of the image, labeled "31.25 Mpc/h".

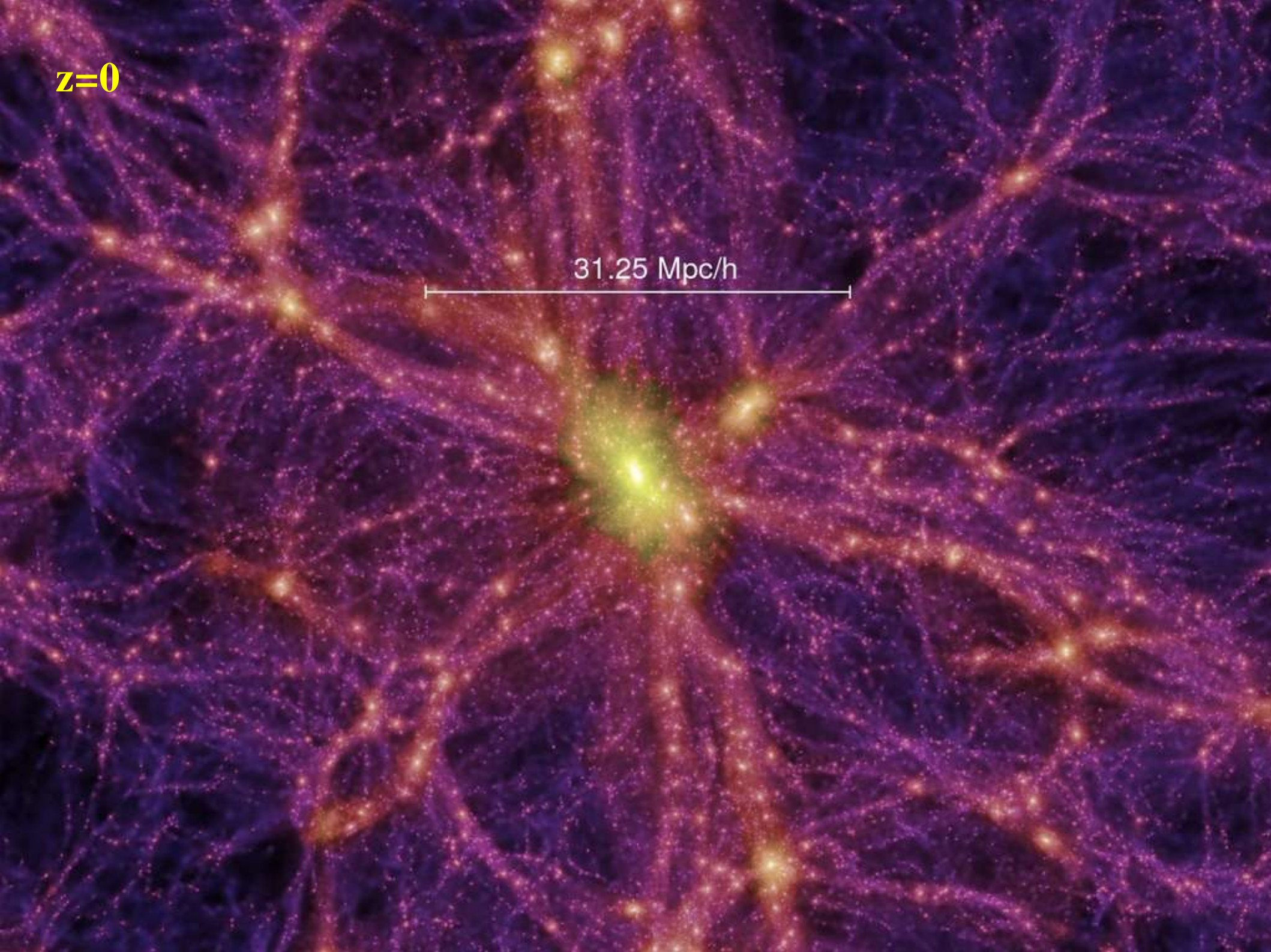
$z=1.4$

31.25 Mpc/h

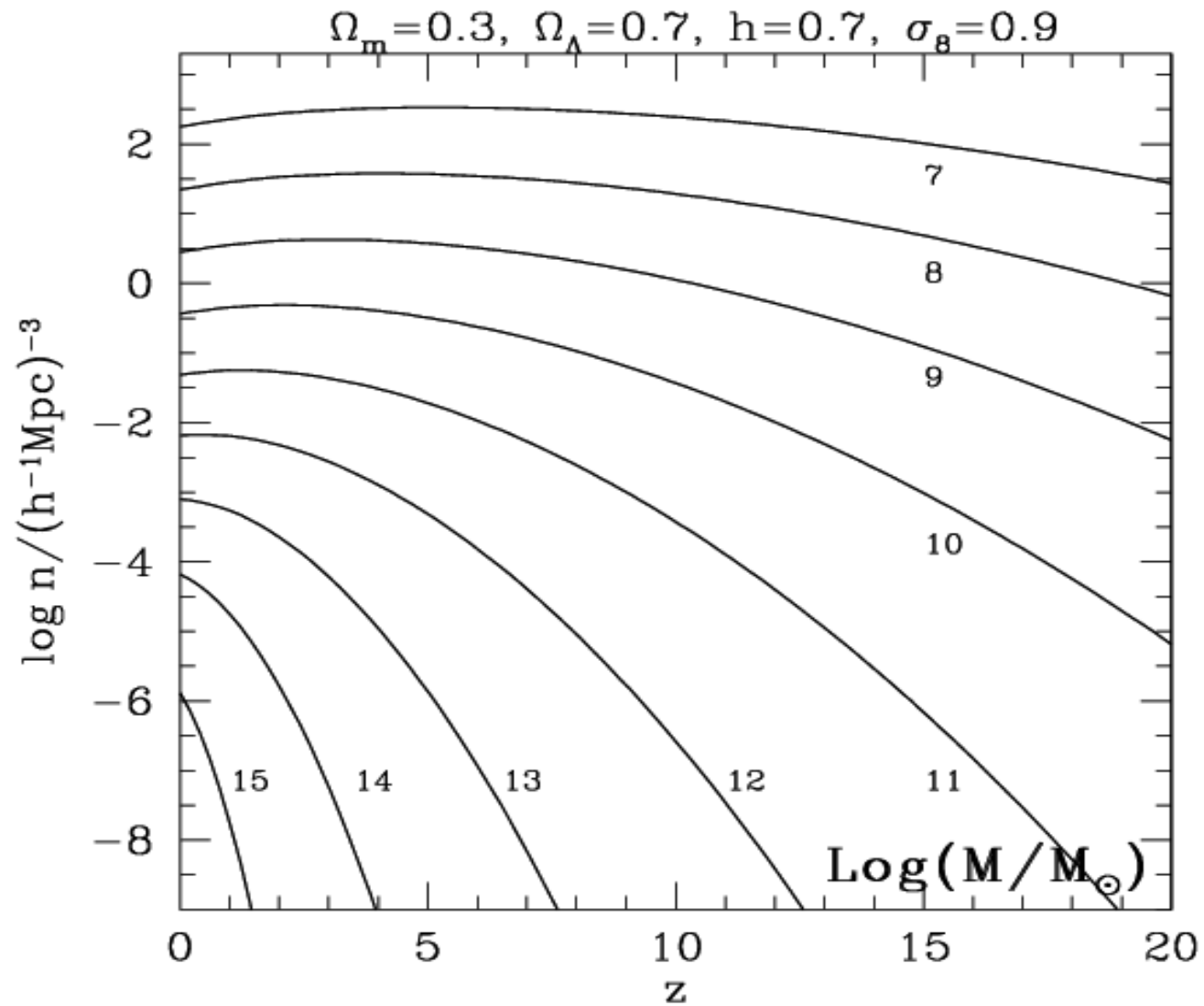
A visualization of the cosmic web at redshift $z=1.4$. The image shows a complex network of dark matter filaments and clusters, rendered in shades of purple and blue. A central, bright yellow-green cluster is the most prominent feature. A horizontal white line with tick marks at each end is positioned above the center, indicating a scale of 31.25 Mpc/h. The overall structure is highly filamentary and interconnected, with many smaller clusters and voids scattered throughout the field.

$z=0$

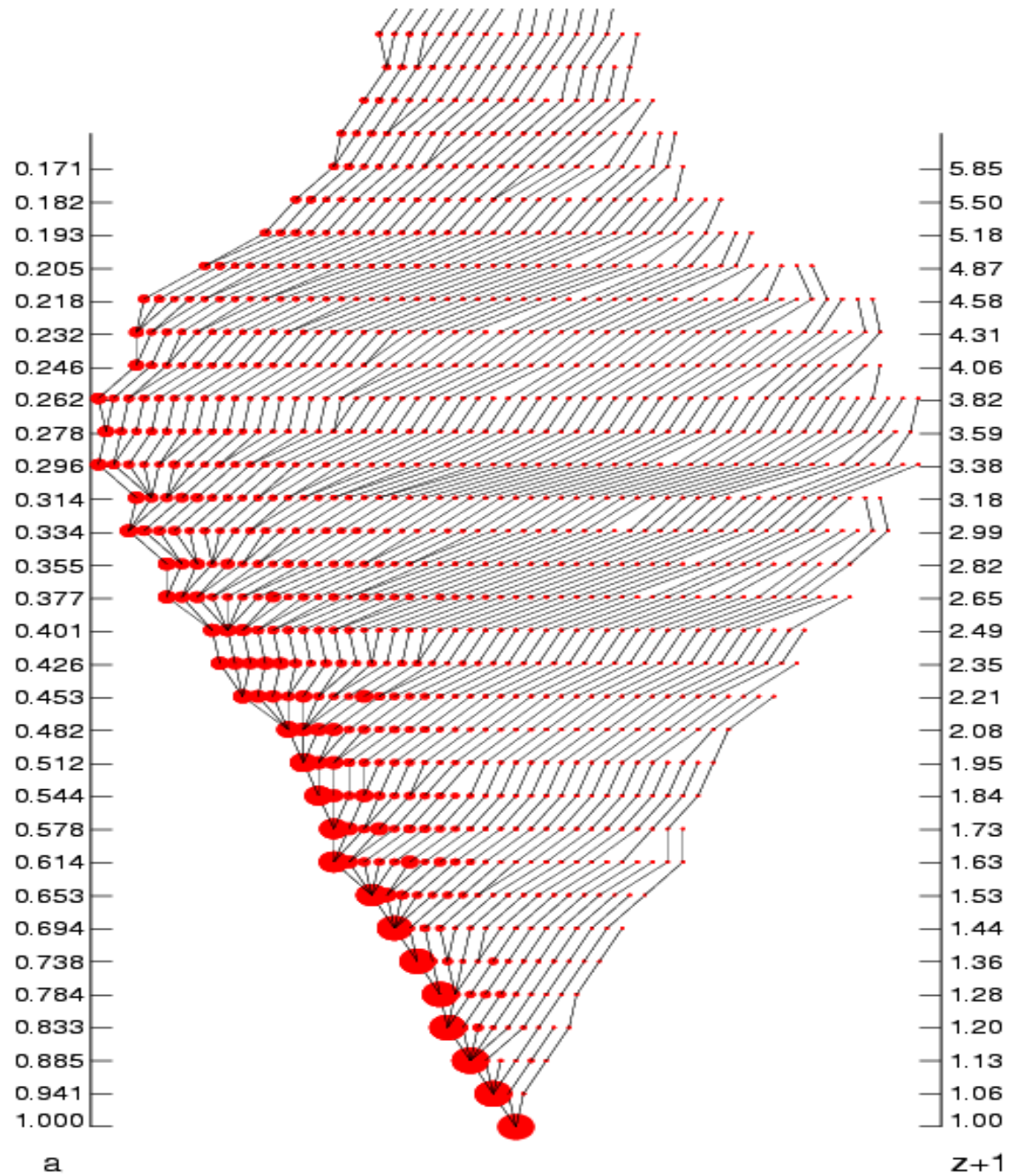
31.25 Mpc/h



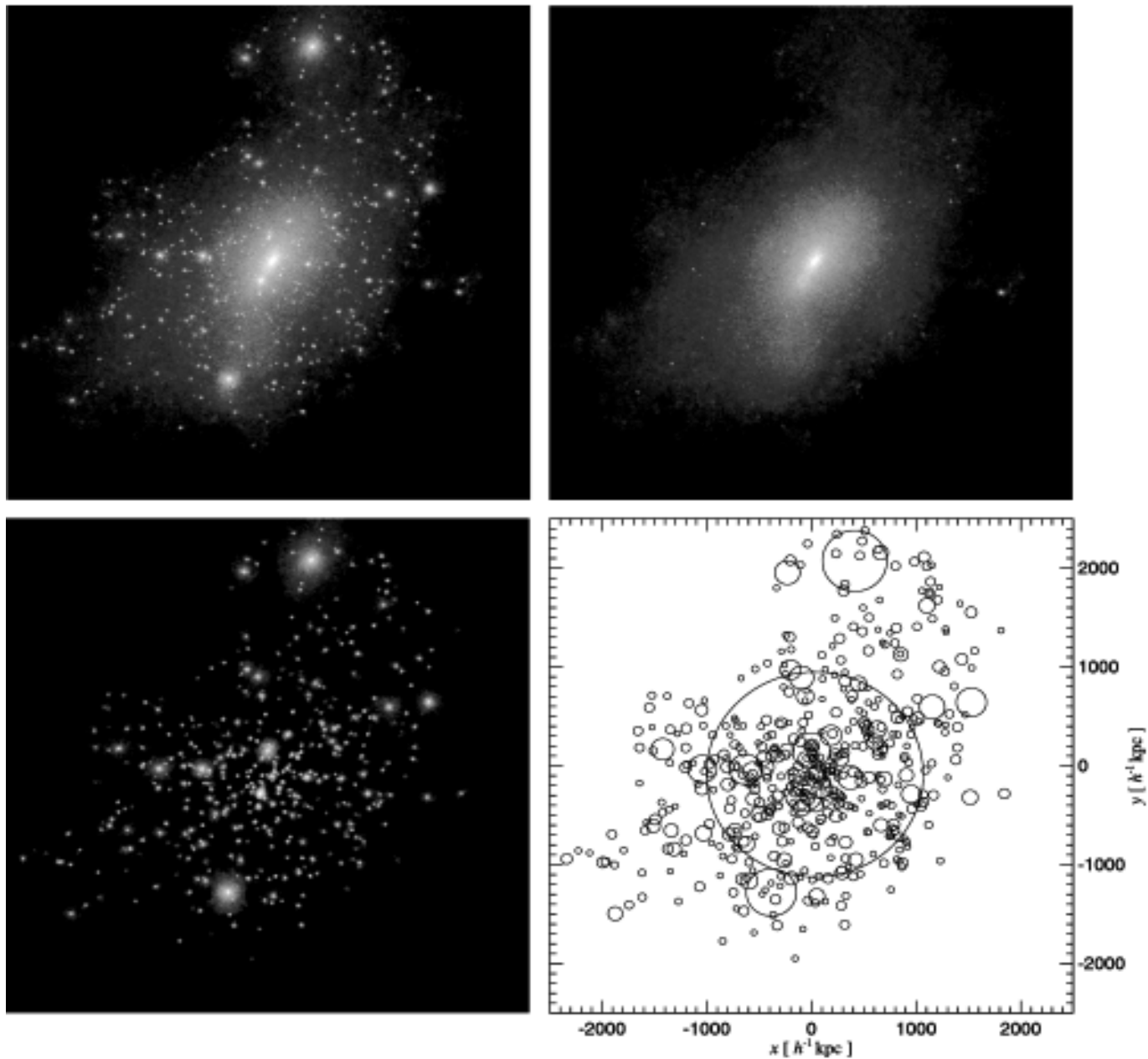
Evolution of the abundance of dark matter halos of different mass as a function of redshift.



THE MERGING TREE



Substructures within Dark Matter Halos



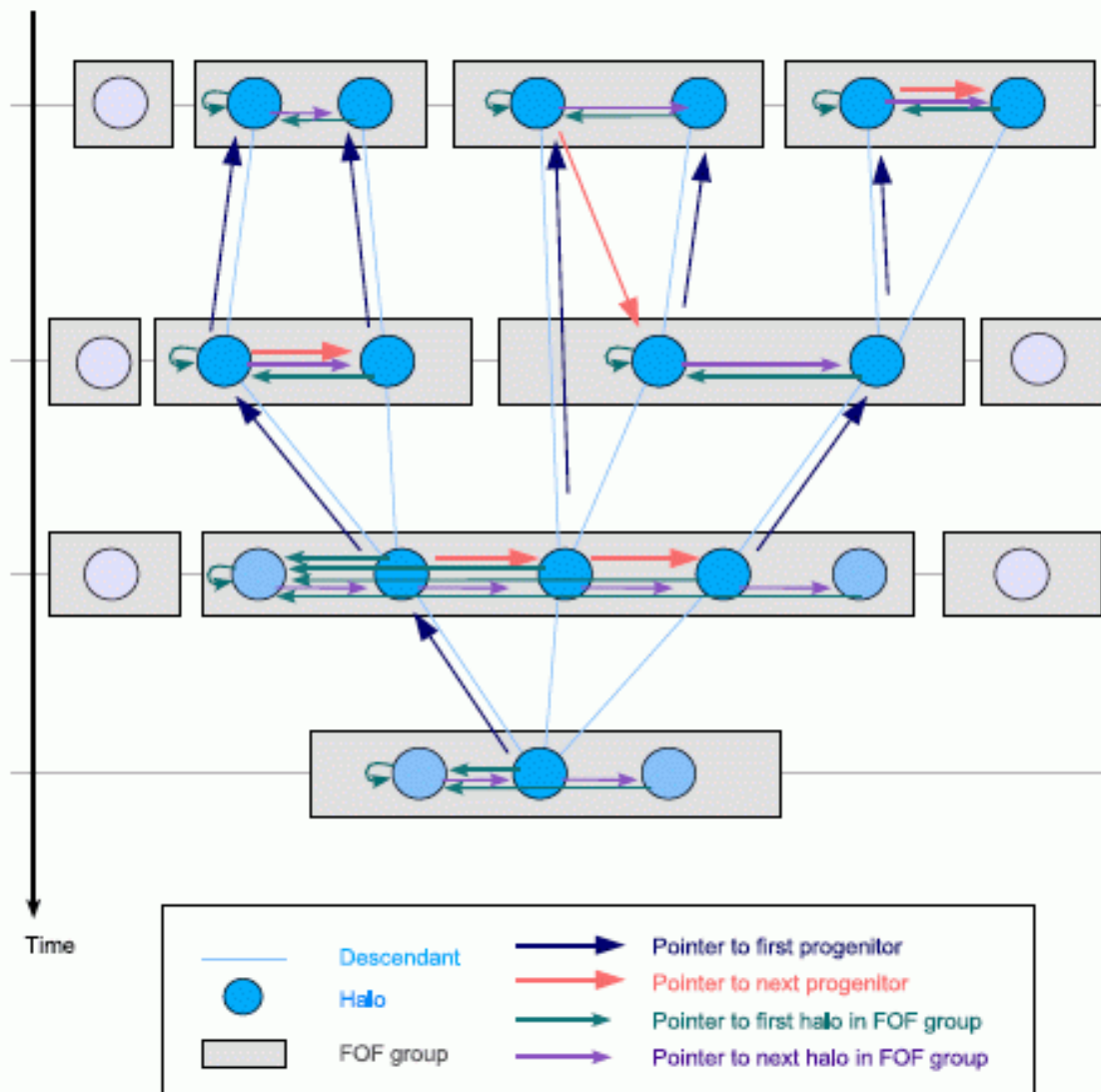


Figure 11: Schematic organisation of the merger tree in the Millennium Run. At each output time, FOF groups are identified which contain one or several (sub)halos. The merger tree connects these halos. The FOF groups play no direct role, except that the largest halo in a given FOF group is the one which may develop a cooling flow according to the physical model for galaxy formation implemented for the trees. To facilitate the latter, a number of pointers for each halo are defined. Each halo knows its descendant, and its most massive progenitor. Possible further progenitors can be retrieved by following the chain of ‘next progenitors’. In a similar fashion, all halos in a given FOF group are linked together.

Baryonic Processes Important in Understanding Galaxy Formation

- 1) Radiative cooling of gas within dark matter halos
- 2) The formation of stars
- 3) Supernova-driven feedback processes
- 4) Stellar evolution and evolutionary population synthesis
- 6) galaxy-galaxy merging; morphological transformations

Putting the baryonic processes together with the evolution of the dark matter: semi-analytical models of galaxy formation.

RADIATIVE COOLING

Above 10^6 K (primordial) and 10^7 K (enriched)

Gas is ionized. Gas cools by bremsstrahlung due to the acceleration of electrons as they encounter atomic nuclei.

$$dE/dT \sim n_e n_H T^{1/2}$$

Below 10^6 K

Electrons recombine with ions, emitting a photon.

Partially ionized atoms excited by collisions with electrons, then decay radiatively emitting an electron. This is the dominant process and causes the “double peak” in the cooling curve at 15000 K (for H) and 100,000 K (for He+).

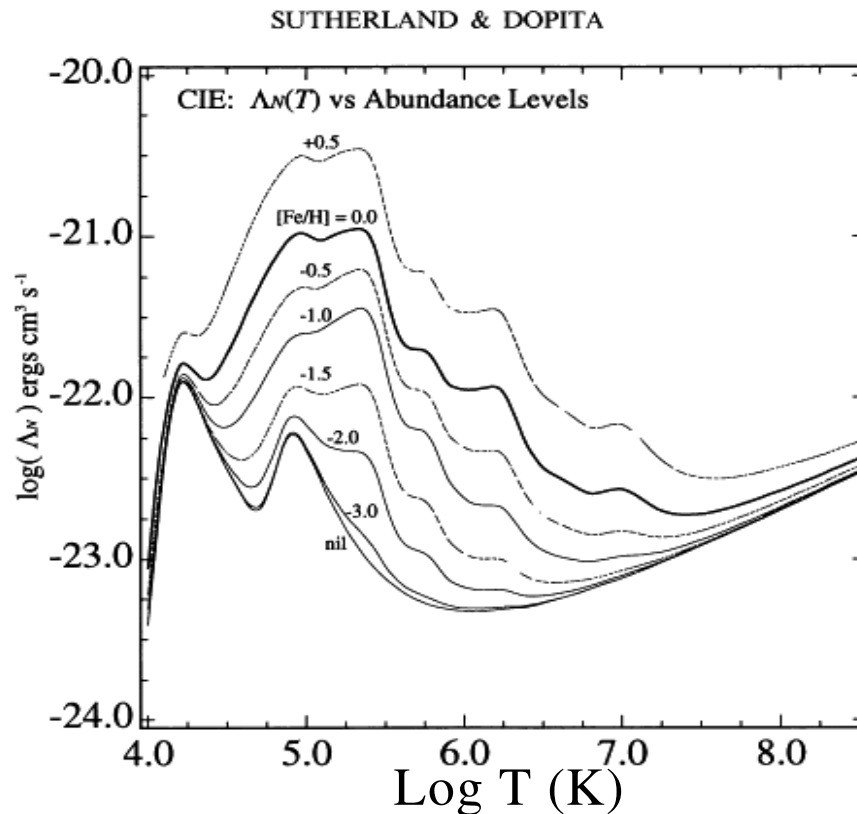
$$dE/dT \sim n_e n_H f(T)$$

Below 10^4 K

Gas is neutral and the cooling rate drops. If molecules are present, gas cools by collisional excitation of molecular vibrations.

In presence of strong UV background

Cooling by collisional excitation and radiative decay is suppressed because the abundance of partially ionized elements is reduced.



A SIMPLE MODEL FOR COOLING

Let us assume that gas is shock heated during the collapse of a dark matter halo and is then in hydrostatic equilibrium with a density profile $\rho_g(r)$ that follows that of the dark matter. The temperature of the gas may be written

$$T = 35.9 \left(\frac{V_{vir}}{100 \text{ km s}^{-1}} \right)^2 K$$

The local cooling time $t_{cool}(r)$ can be defined as the ratio of the specific thermal content of the gas, and the local cooling rate per unit volume

$$t_{cool}(r) = \frac{3}{2} \frac{kT \rho_g(r)}{\bar{\mu} m_p n_e^2(r) \Lambda(T, Z)},$$

where $\bar{\mu} m_p$ is the mean particle mass, $n_e(r)$ is the electron density and $\Lambda(T, Z)$ is the cooling function described in the previous section (as explained, it depends on gas temperature T and metallicity Z).

We define the cooling radius r_{cool} as the radius for which t_{cool} is equal to the age of the Universe at the epoch of interest. If the cooling radius lies within the virial radius of the halo (defined as the radius within which the overdensity is 178), then:

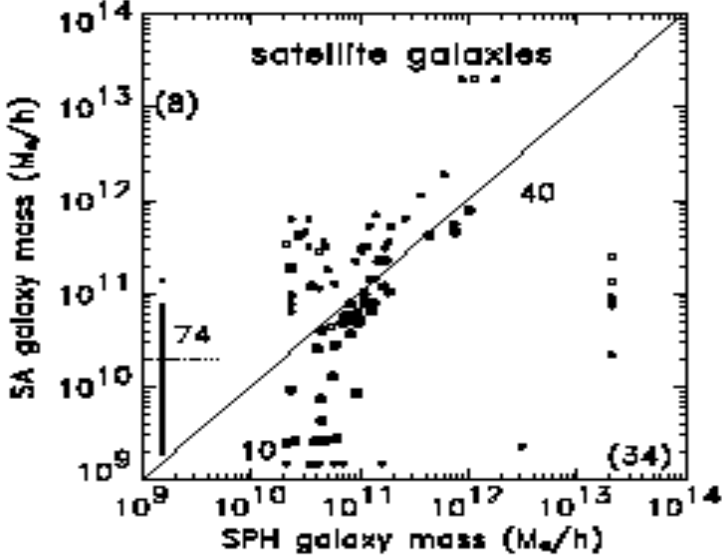
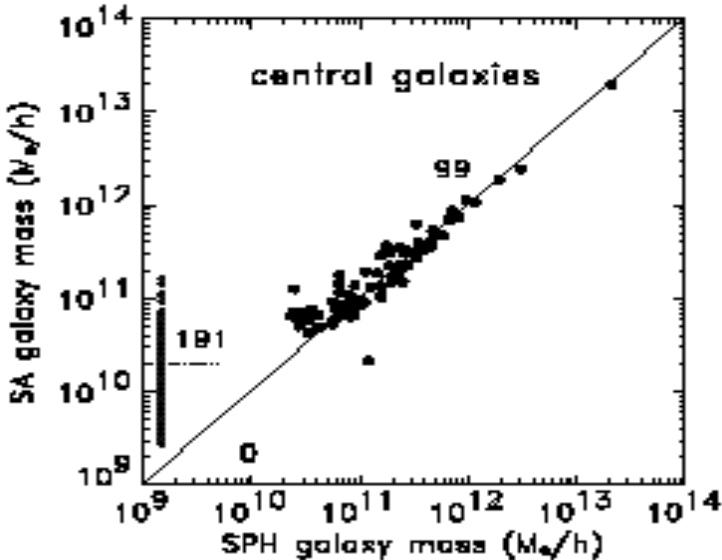
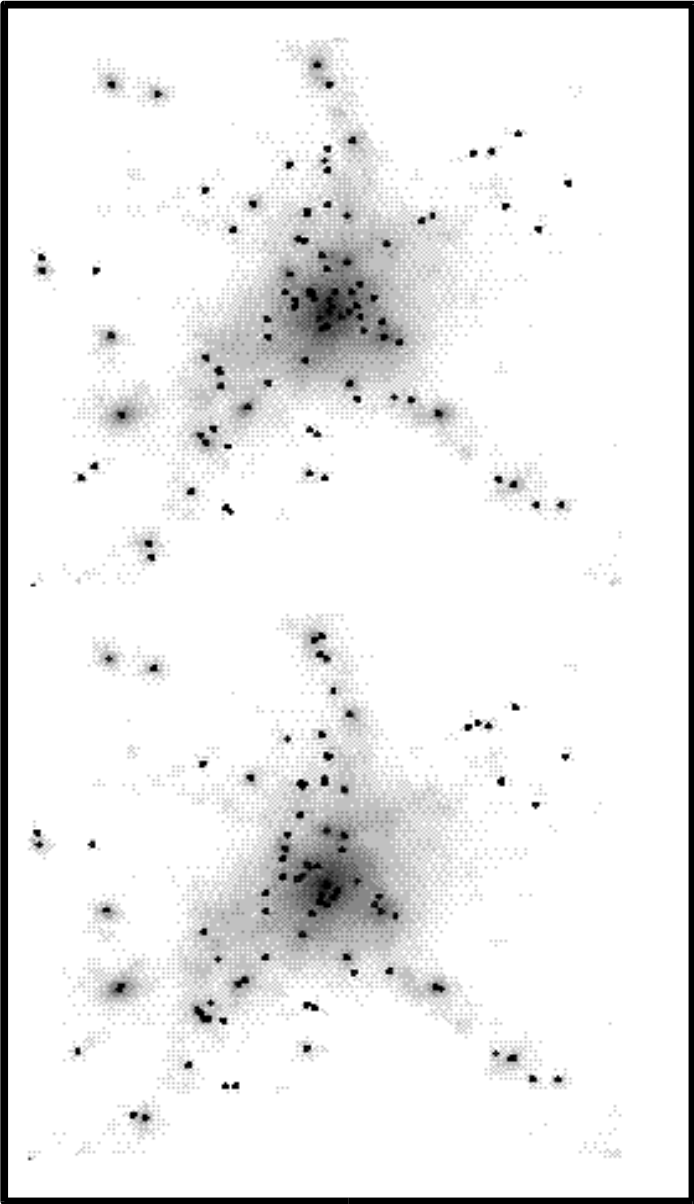
$$\frac{dM_{cool}}{dt} = 4\pi\rho_g(r_{cool})r_{cool}^2 \frac{dr_{cool}}{dt}.$$

At early times and in low-mass haloes, $r_{cool} > r_{vir}$. The hot gas is then never in hydrostatic equilibrium and the cooling rate is limited by the accretion rate, which can be approximated as

$$\frac{dM_{accr}}{dt} = \frac{M_{hot}V_{vir}}{R_{vir}}.$$

How well does the simple model work?

Yoshida et al 2002



STAR FORMATION IS A COMPLEX PROCESS

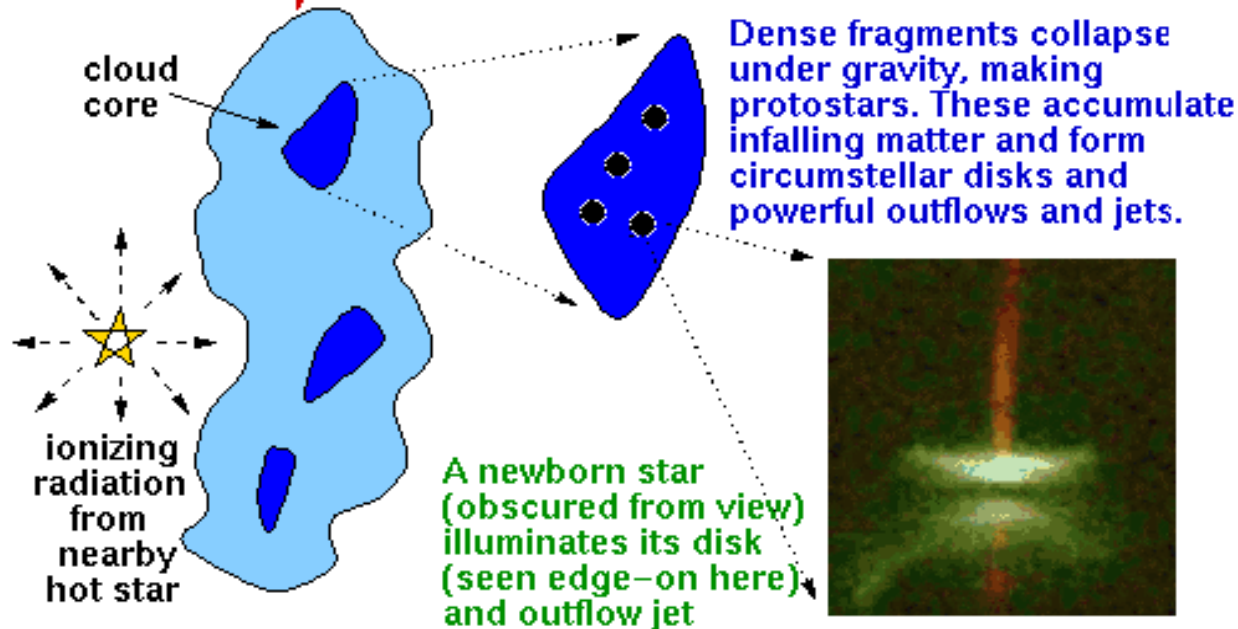


Molecular clouds are cold, dark, giant condensations of dust and molecular gas which serve as "stellar nurseries".

All stars are born in molecular clouds, including our Sun. Molecular clouds are the "stuff" we're made of!

Because of their dusty content, visible light cannot penetrate into a molecular cloud. Thus, infrared and submillimeter observations are needed to "see" the star-forming process.

molecular cloud



Schmidt/ Kennicutt “law” for star formation in disk galaxies.

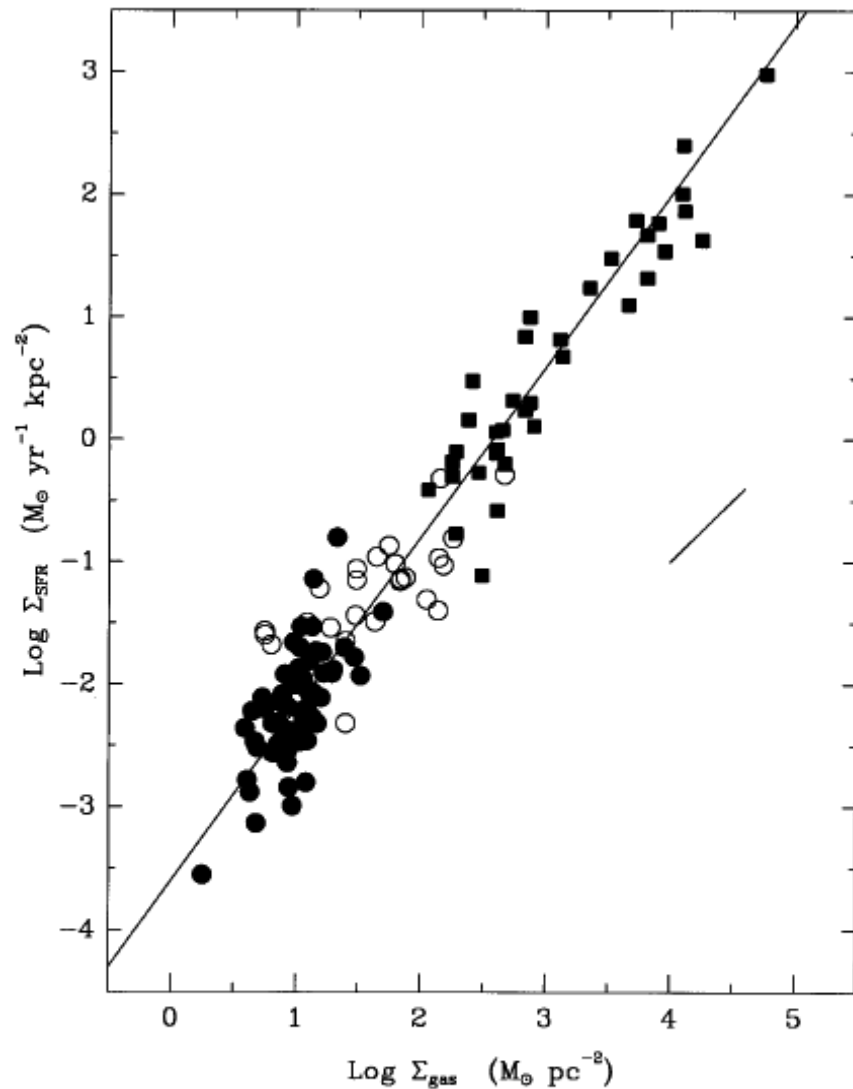


FIG. 6.—Composite star formation law for the normal disk (*filled circles*) and starburst (*squares*) samples. Open circles show the SFRs and gas densities for the centers of the normal disk galaxies. The line is a least-squares fit with index $N = 1.40$. The short, diagonal line shows the effect of changing the scaling radius by a factor of 2.

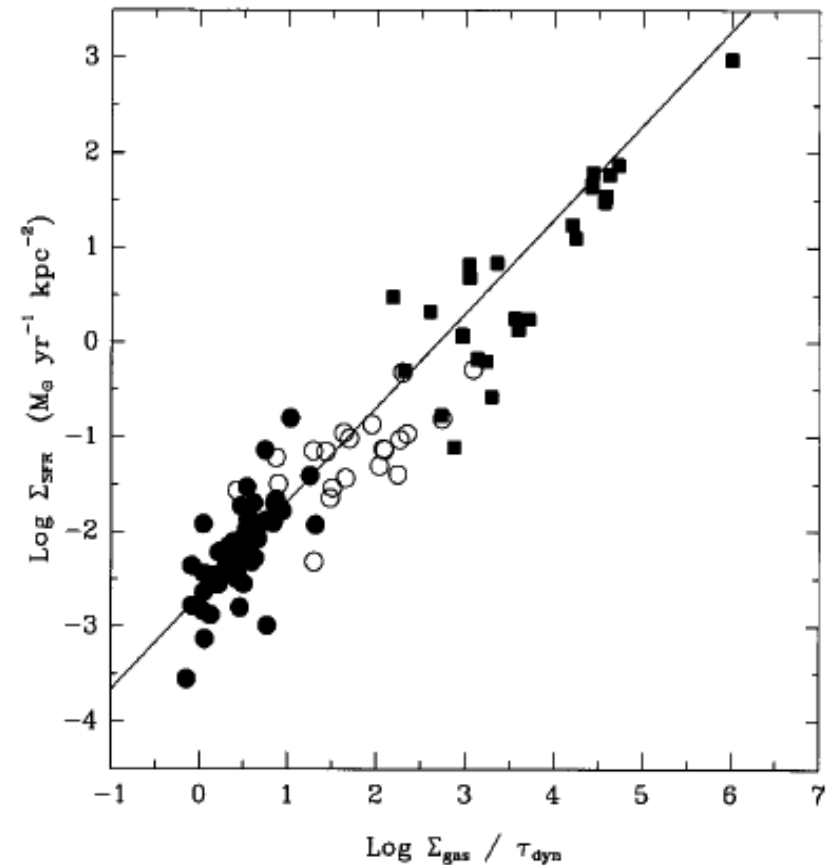


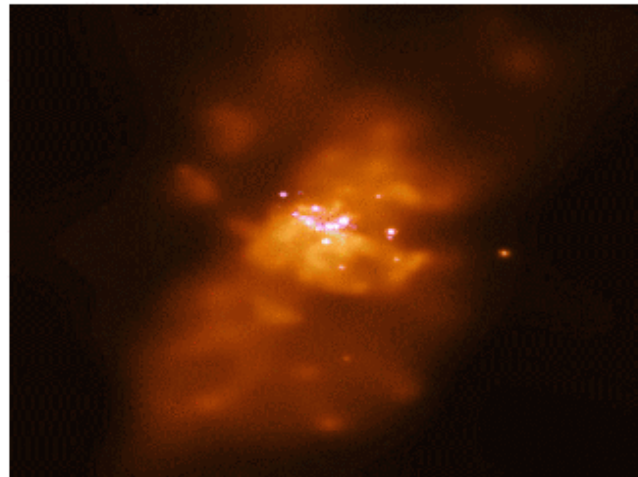
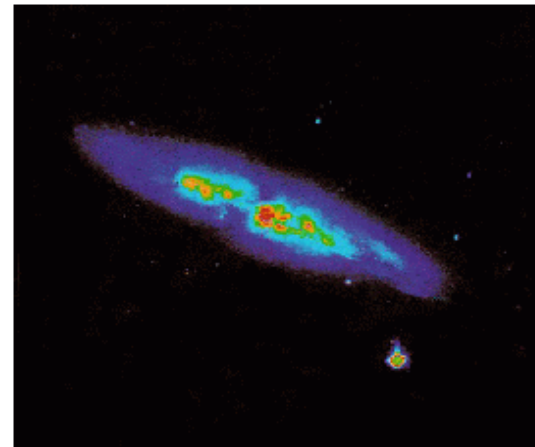
FIG. 7.—Relation between the SFR for the normal disk and starburst samples and the ratio of the gas density to the disk orbital timescale, as described in the text. The symbols are the same as in Fig. 6. The line is a median fit to the normal disk sample, with the slope fixed at unity as predicted by equation (7).

'Feedback' at work in the starburst galaxy M82

OPTICAL



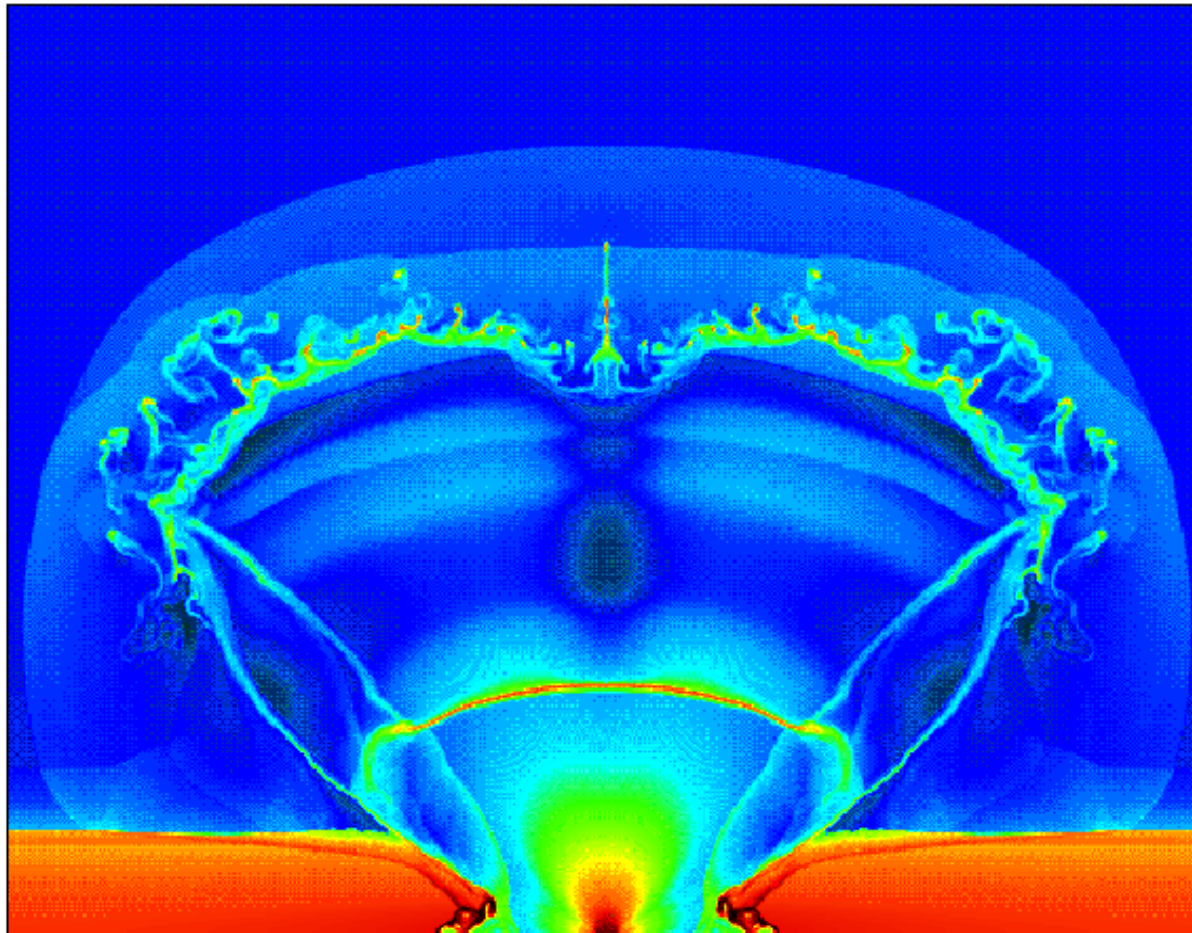
INFRARED



XRAY

Numerical Simulation of a Starburst-driven Wind

The picture shows gas density: red is high density, blue low density
Energy deposited into the ISM heats up the gas to millions of degrees, where it emits x-rays. The hot gas expands and breaks out of the disk and escapes into the halo at 1000 km/s.



STAR FORMATION

The star formation rate of a galaxy is modelled as

$$\frac{dM_*}{dt} = \alpha \frac{M_{cold}}{t_{dyn}} \quad (1)$$

M_{cold} is the mass of cold gas and t_{dyn} is the dynamical timescale, which can be written

$$t_{dyn} = \frac{0.1 R_{vir}}{V_{vir}} \quad (2)$$

FEEDBACK

Given a stellar initial mass function (IMF), the energy released by supernovae per unit mass of stars that are formed, is $\eta_{SN} E_{SN}$. Typically, $\eta_{SN} = 5 \times 10^{-3} M_{\odot}^{-1}$ and $E_{SN} = 10^{51}$ erg.

The major uncertainty is how this energy affects the evolution of the interstellar medium and how the star formation rate is regulated by it. In current models, it is assumed that some of the energy reheats cold gas back to the virial temperature of the dark halo. Invoking energy conservation, we can write

$$\Delta M_{reheat} = \frac{4}{3} \frac{\eta_{SN} E_{SN}}{V_{vir}^2} \Delta M_* \quad (3)$$

WHAT HAPPENS DURING A MERGER OF TWO GALAXIES?

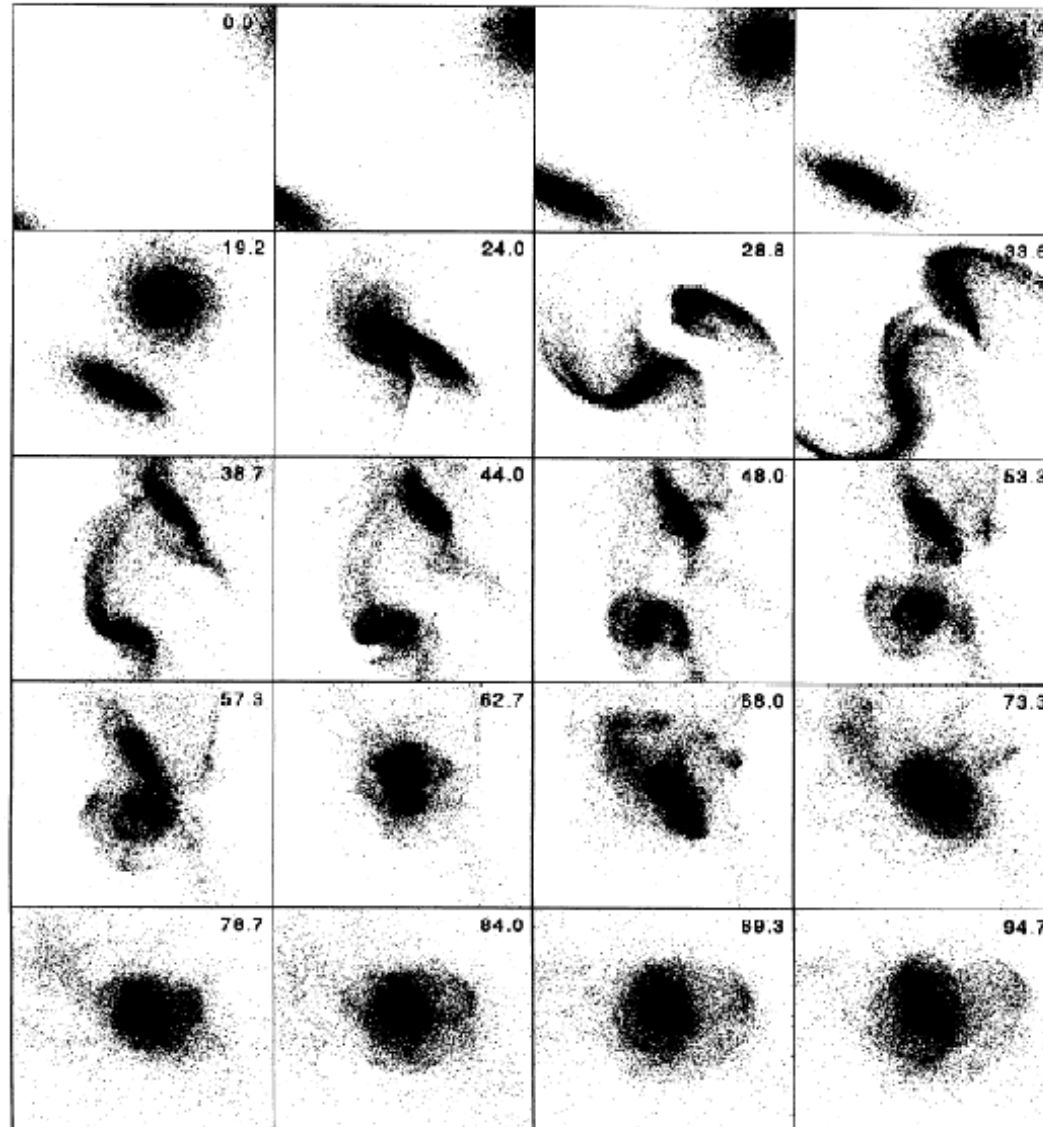
STARS:

The tidal forces from the passing companion distort the galaxy. It forms “tidal tails” and a bridge connecting the two objects. The inner regions of the disks can form linear barlike structures. The stellar component eventually relaxes to a $r^{1/4}$ profile.

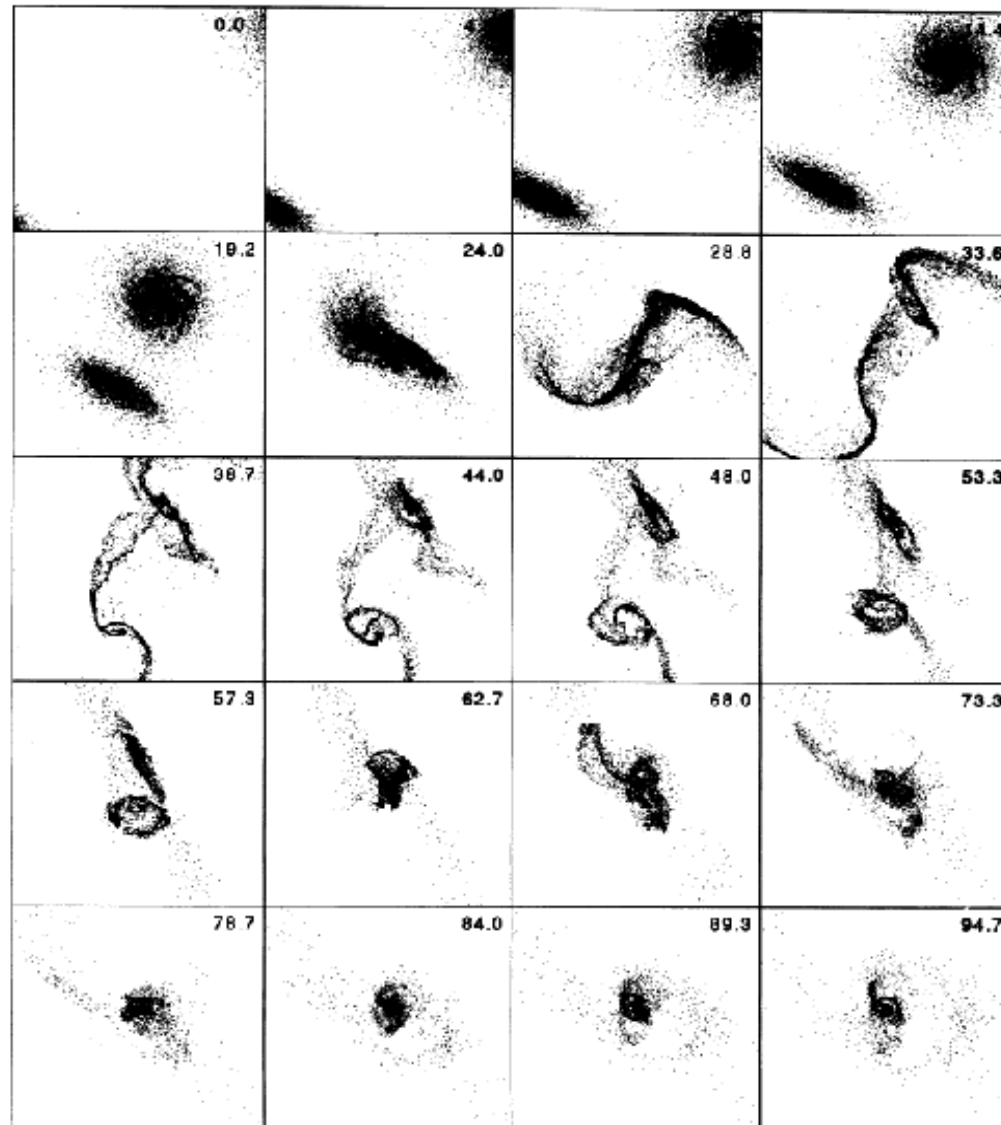
GAS:

Gas is subject to shocking, dissipation and loss of angular momentum. Gas shocks first at the interface between the two galaxies. At first the gas reacts like the stars and forms a bar, but then it flows inwards. By the end, 75% of the gas has ended up in a compact core in the remnant galaxy.

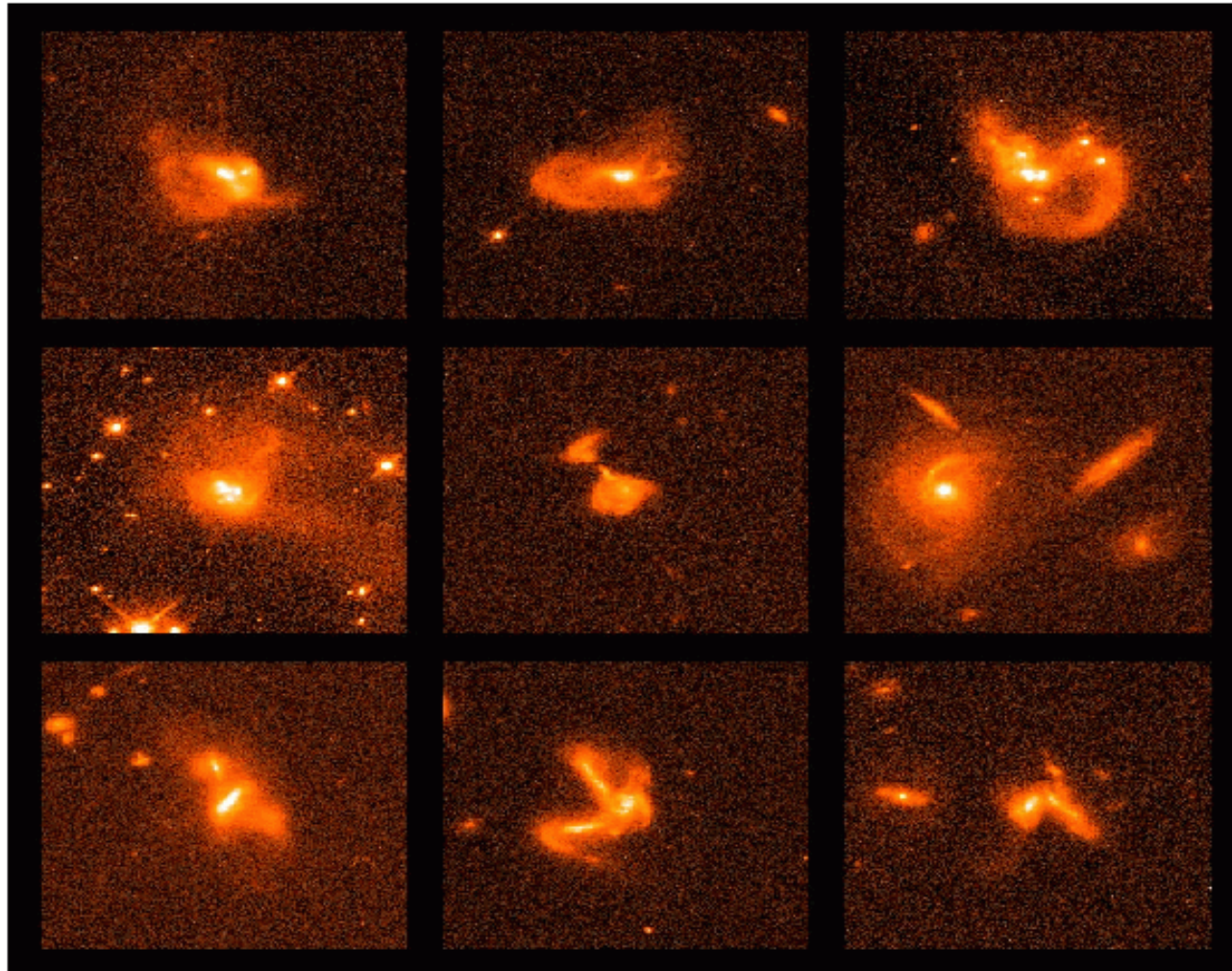
Evolution of the Stars during the merger of two equal mass disk galaxies
Mihos & Hernquist 1996



Evolution of the Gas during the merger of two equal mass disk galaxies
Mihos & Hernquist 1996



**Ultra-luminous Infrared Galaxies (ULIRGs):
Elliptical galaxies in formation today**



★ Evolutionary population synthesis

(Tinsley 1967, 1980; Bruzual 1983; Renzini & Buzzoni 1986; Guiderdoni & Rocca-Volmerange 1987; Buzzoni 1989; Charlot & Bruzual 1991; Bressan et al. 1994; Fritze & Gehrard 1994; Worthey 1994; Mayya 1995; Weiss, Peletier & Matteucci 1995; ...)

- Start from a library of stellar evolutionary tracks (from stellar evolution theory) and library of stellar spectra
- Main adjustable parameters

Initial Mass Function (IMF):

$\phi(m)dm$ = number of stars formed with masses between m and $m + dm$ (lower and upper cutoffs $m_L \approx 0.1 M_\odot$ and $m_U \approx 100 M_\odot$, typically)

→ Star Formation Rate (SFR):

$$\psi(t) = dM_*/dt$$

→ Chemical Enrichment Rate (CER):

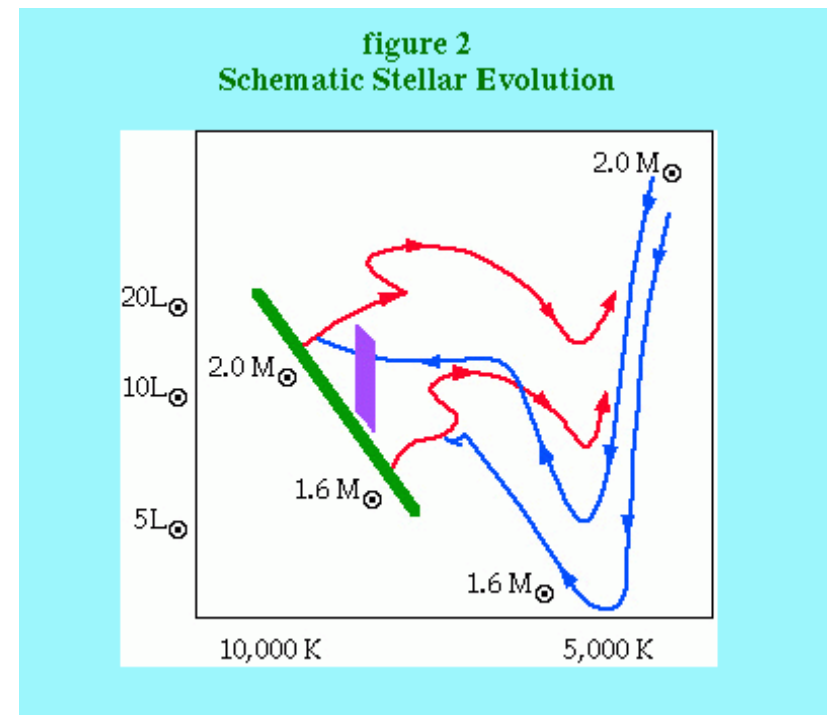
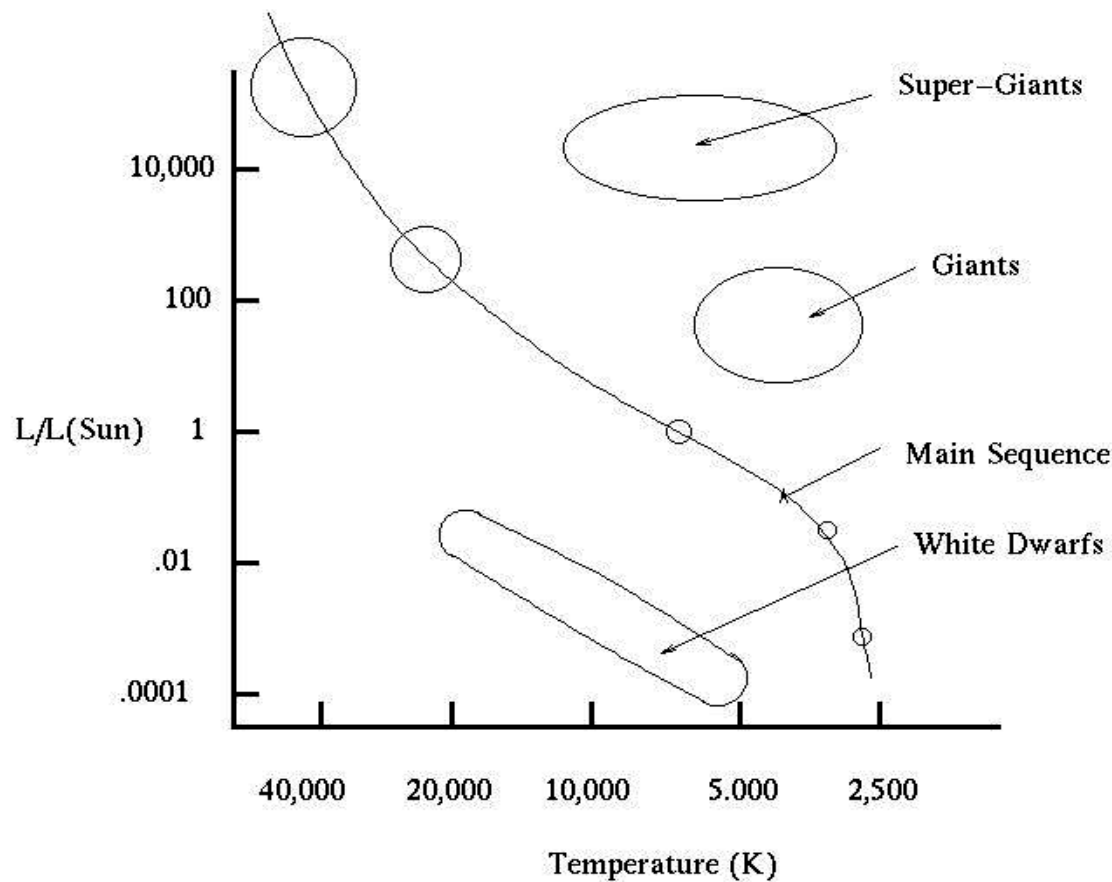
$$\chi(t) = dZ/dt \quad (Z = \text{mass fraction of elements heavier than He})$$

● Principle

1. compute distribution of stellar masses, ages, and metallicities as a function of time from ϕ , ψ , and χ
2. add up spectra of individual stars to obtain the integrated spectrum of the stellar population as a function of time

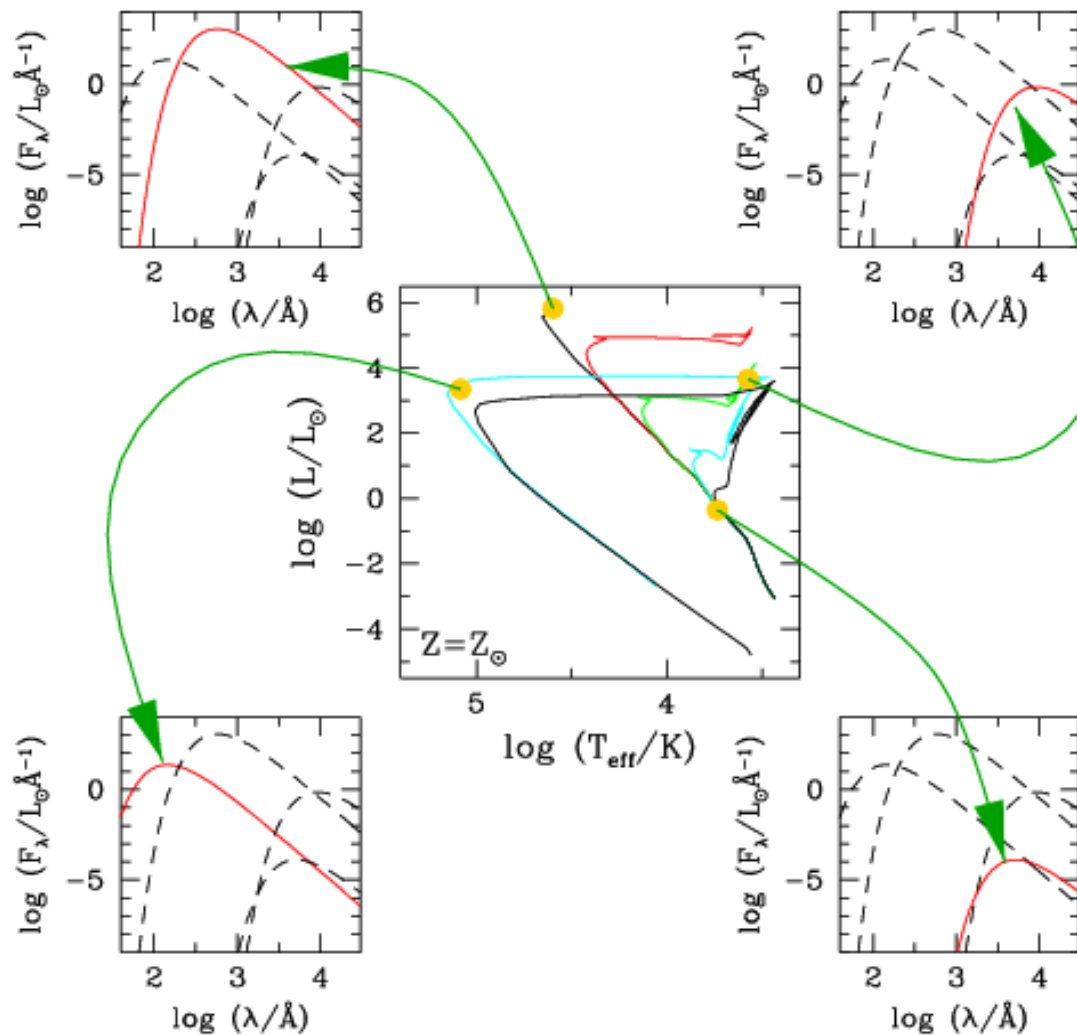
→ Possible predictions of past and future spectral evolution

Stars spend most of their lives on the main sequence, then go through a “giant” phase before ending their lives as white dwarfs, neutron stars or black holes.

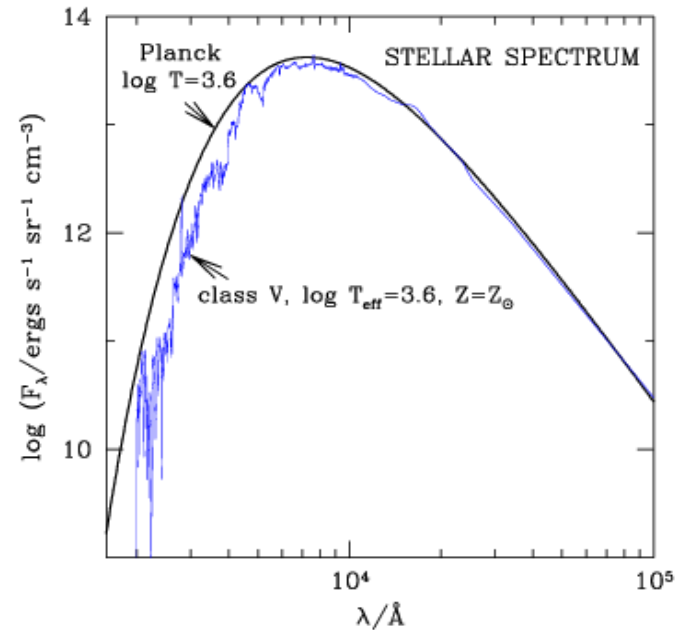


★ Assign individual spectra to stars along the isochrones...

$T_{\text{eff}} \nearrow \longrightarrow F_{\text{UV}}/F_{\text{NIR}} \nearrow \longrightarrow$ star bluer



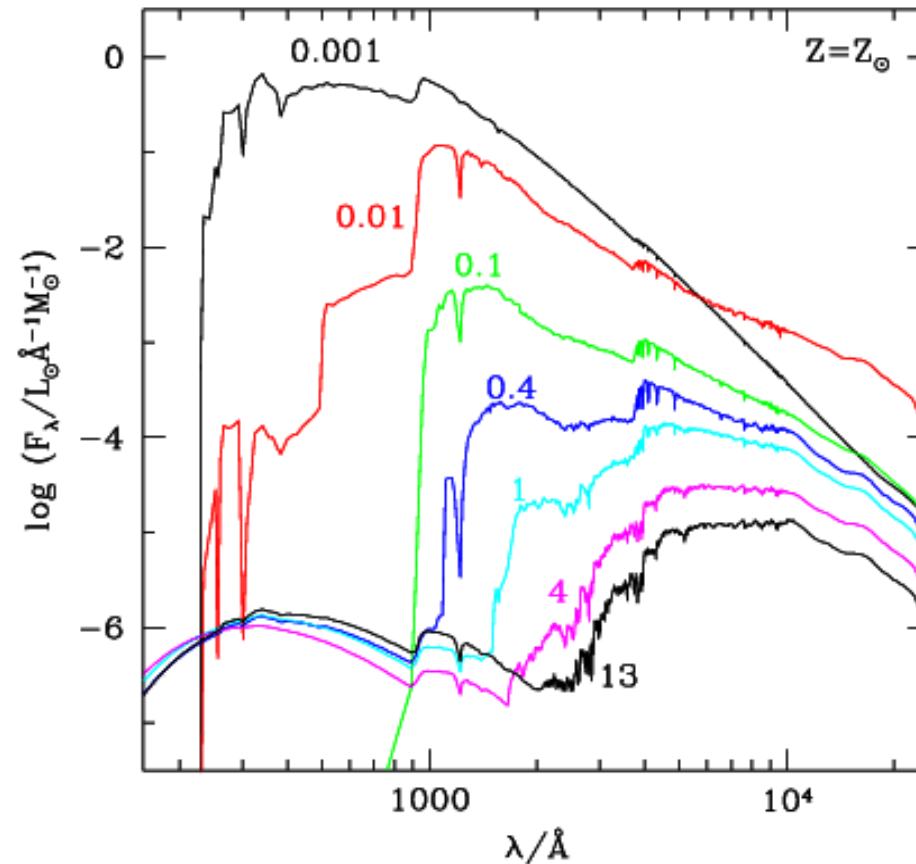
... in reality, libraries of stellar spectra are a critical input of population synthesis models



Stellar spectra taken from:

- **Observations**
So far mostly for solar metallicity & Galactic abundance ratios; need to patch together spectra in various λ ranges; missing domains...
- **Theoretical model atmospheres**
Full wavelength & metallicity domains, but line blanketing (by atomic & molecular transitions) difficult to model...
Calibration against observations in good progress

After all this... → evolution of the integrated light of an instantaneous-burst stellar population



- The integrated UV luminosity fades considerably during the first ~ Gyr
- The shape of the spectral energy distribution does not evolve much after ~ 4 Gyr

★ ...compute photometric evolution from spectra

- Frequently used magnitude systems (colors, color indices):
(magnitudes are *absolute* when the object is placed at 10 pc)

Standard (UBVRIJKL) magnitudes:

$$m^{\text{BP}} = -2.5 \log \left[\left(\int d\lambda R_{\lambda}^{\text{BP}} f_{\lambda} \right) / \left(\int d\lambda R_{\lambda}^{\text{BP}} f_{\lambda, \text{ref}} \right) \right]$$

$$= -2.5 \log \left(\int d\lambda R_{\lambda}^{\text{BP}} f_{\lambda} \right) + C^{\text{BP}},$$

R_{λ}^{BP} is the filter response function

$\int d\lambda f_{\lambda}$ and $\int d\lambda f_{\lambda, \text{ref}}$ in $\text{ergs s}^{-1} \text{cm}^{-2}$

Reference $f_{\lambda, \text{ref}}$ is that of the star Vega (A0V) (zero colors)

Gunn (*uvgriz*) magnitudes:

Same as above, but the reference $f_{\lambda, \text{ref}}$ is that of the star
BD+17°4708 (F6) ($g = 9.50$ and zero colors)

Filters are non-overlapping and exclude strongest sky lines

AB magnitudes:

$$m_{\text{AB}}^{\text{BP}} = -2.5 \log f_{\nu}^{\text{BP}} - 48.60$$

f_{ν}^{BP} is the filter-averaged flux in $\text{ergs s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$,

$$f_{\nu}^{\text{BP}} = \left(\int d\nu R_{\nu}^{\text{BP}} f_{\nu} / h\nu \right) / \left(\int d\nu R_{\nu}^{\text{BP}} / h\nu \right)$$

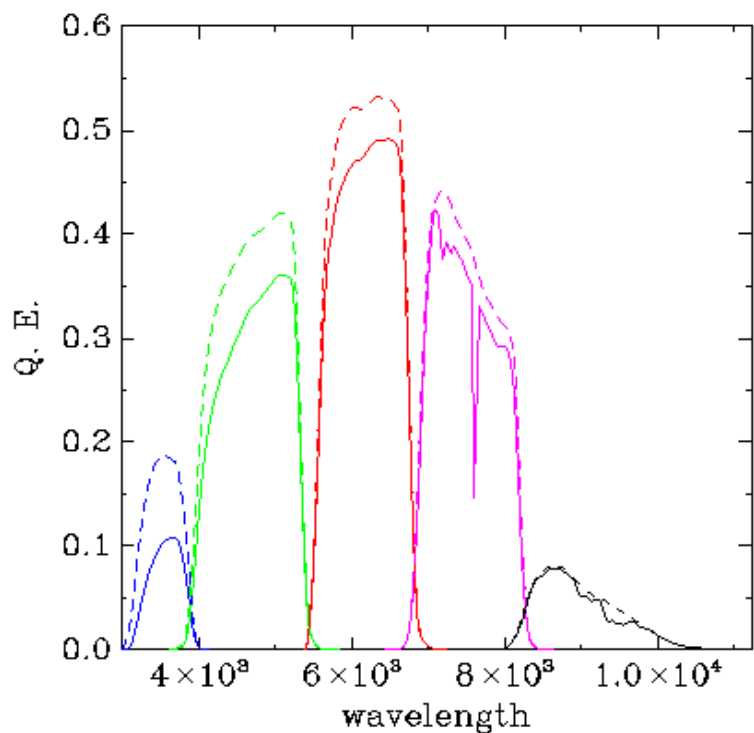
“Flat” spectrum has zero colors

- Reminder on colors...

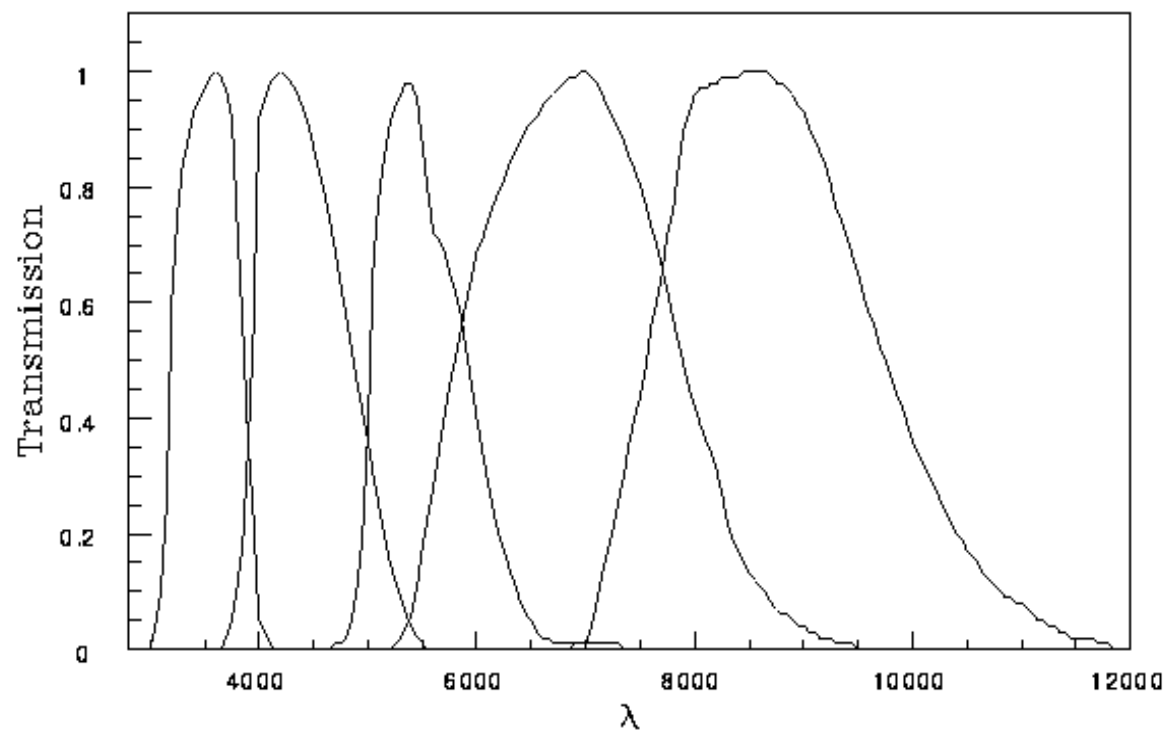
$$F_B / F_V \nearrow \longrightarrow B - V = m_B - m_V \searrow \longrightarrow \text{bluer}$$

EXAMPLES OF COMMON FILTER SYSTEMS

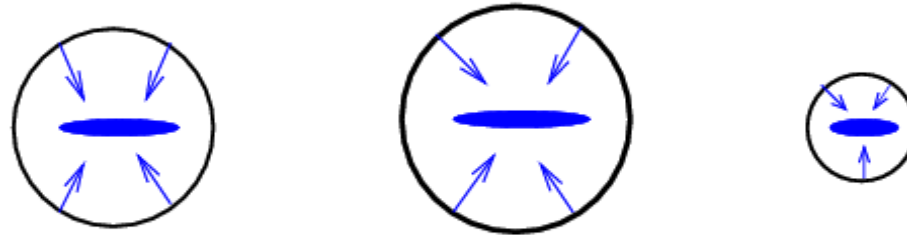
SDSS ugriz



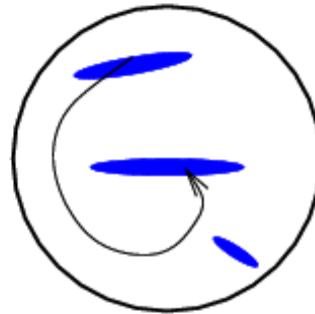
Johnson UBVRI



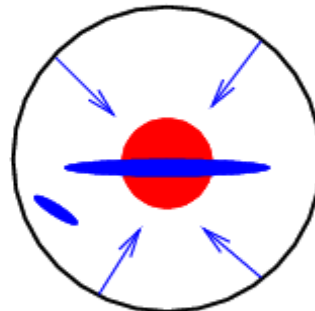
Formation of Different Hubble Types in Semi-Analytic Models



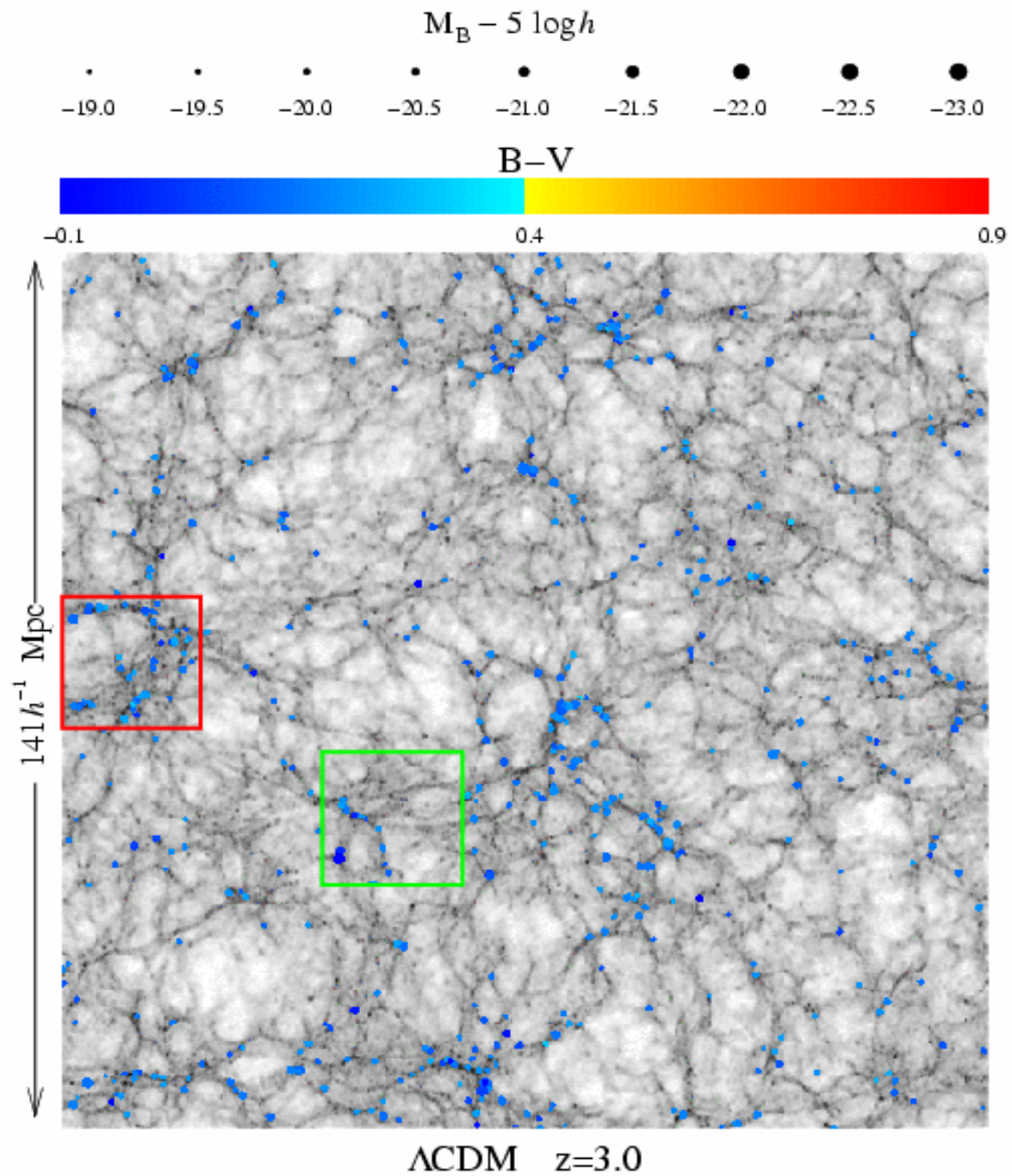
Gas cools and forms a rotationally-supported disk



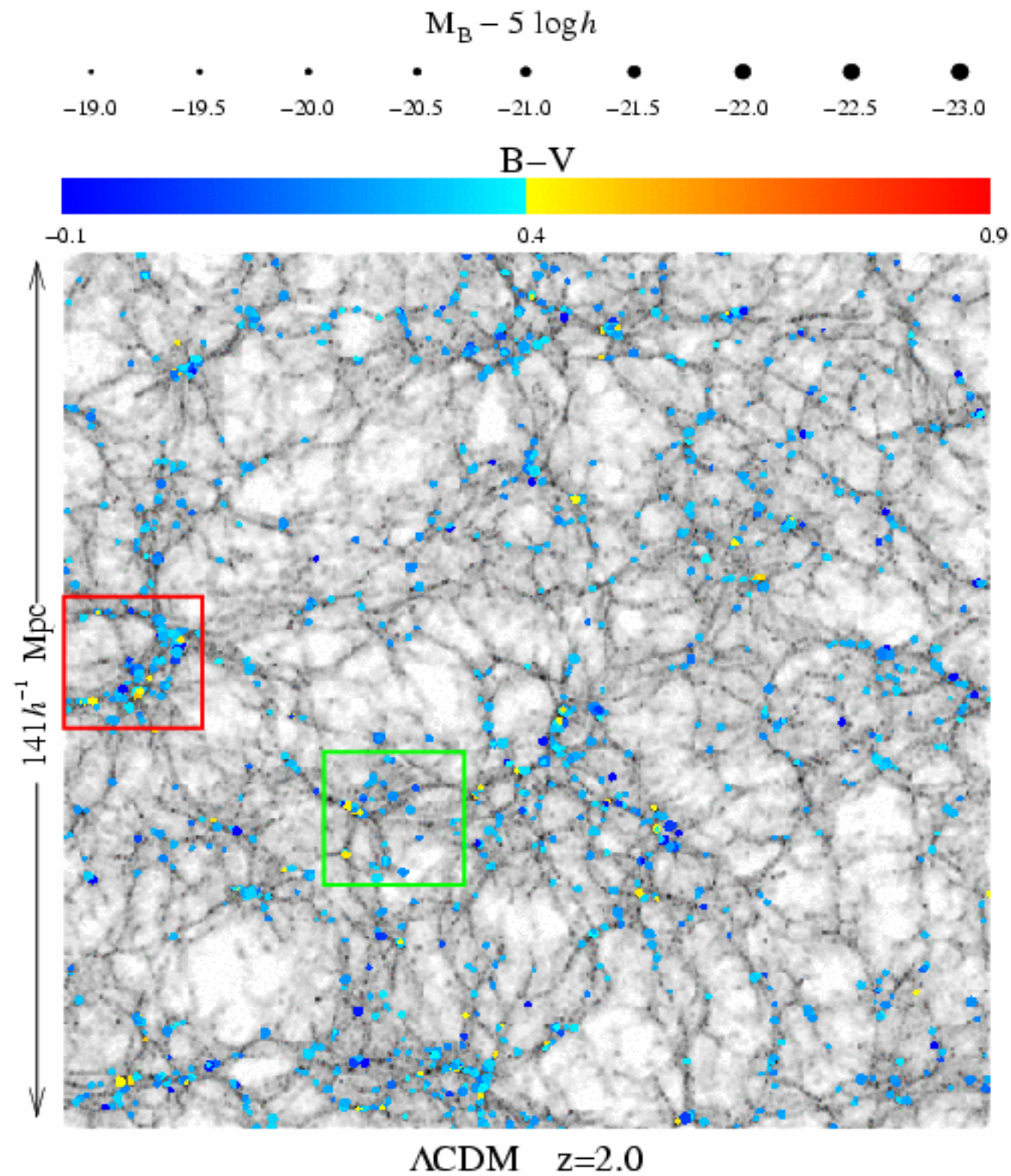
Galaxies merge on a dynamical friction time-scale



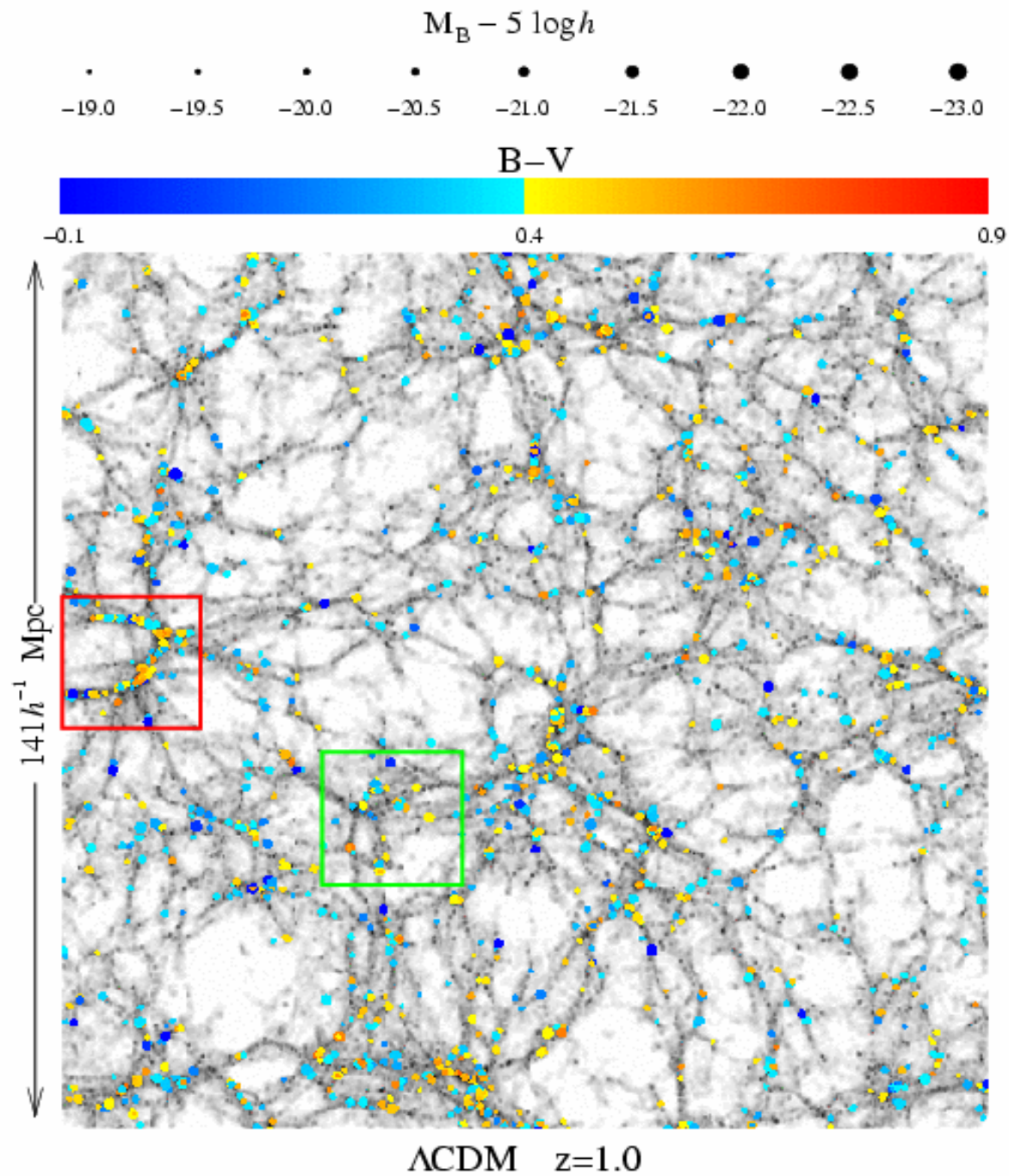
Major merger leads to formation of bulge; new disk forms when gas cools again



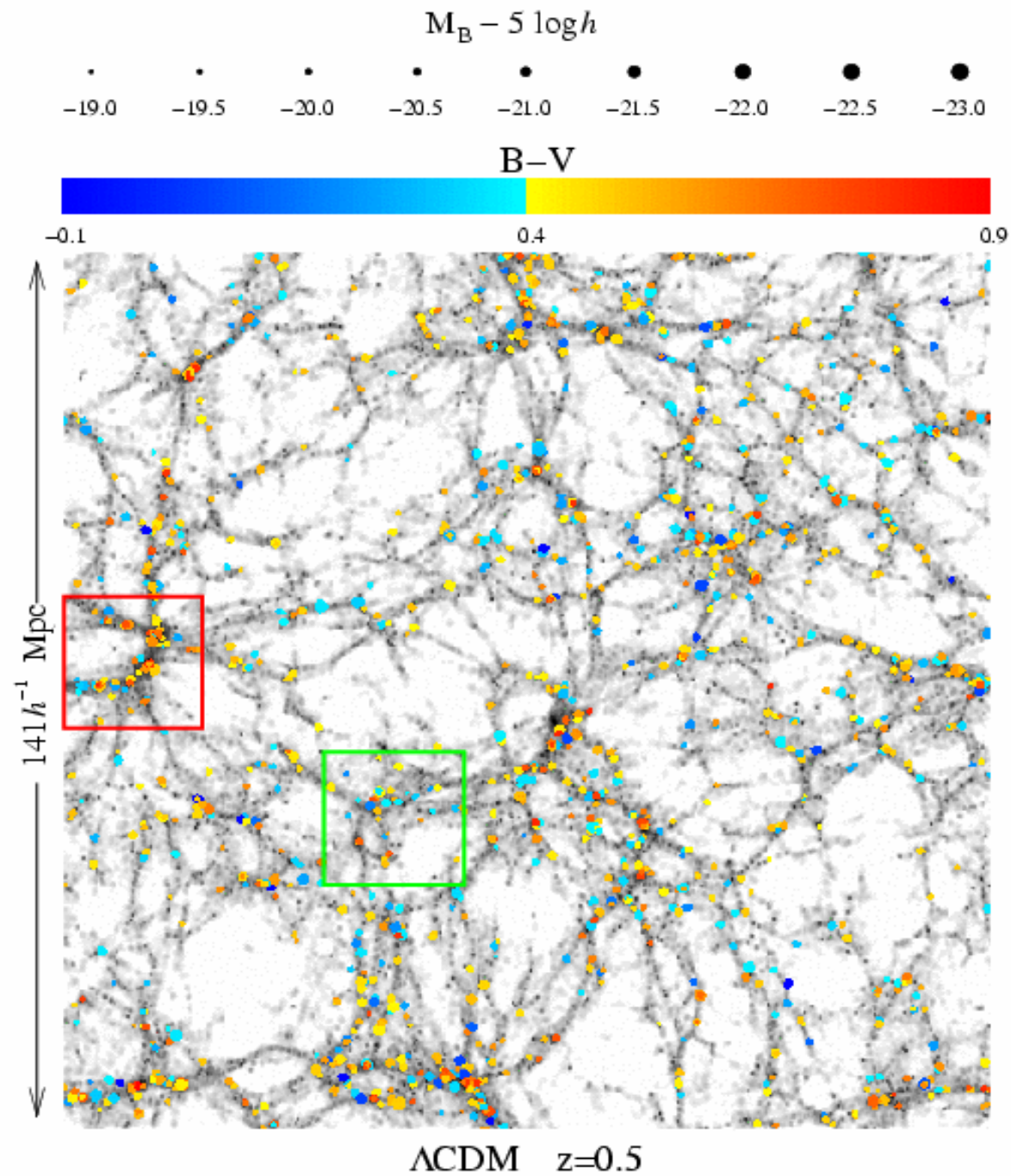
Benson, Frenk, Baugh, Cole & Lacey (2001)



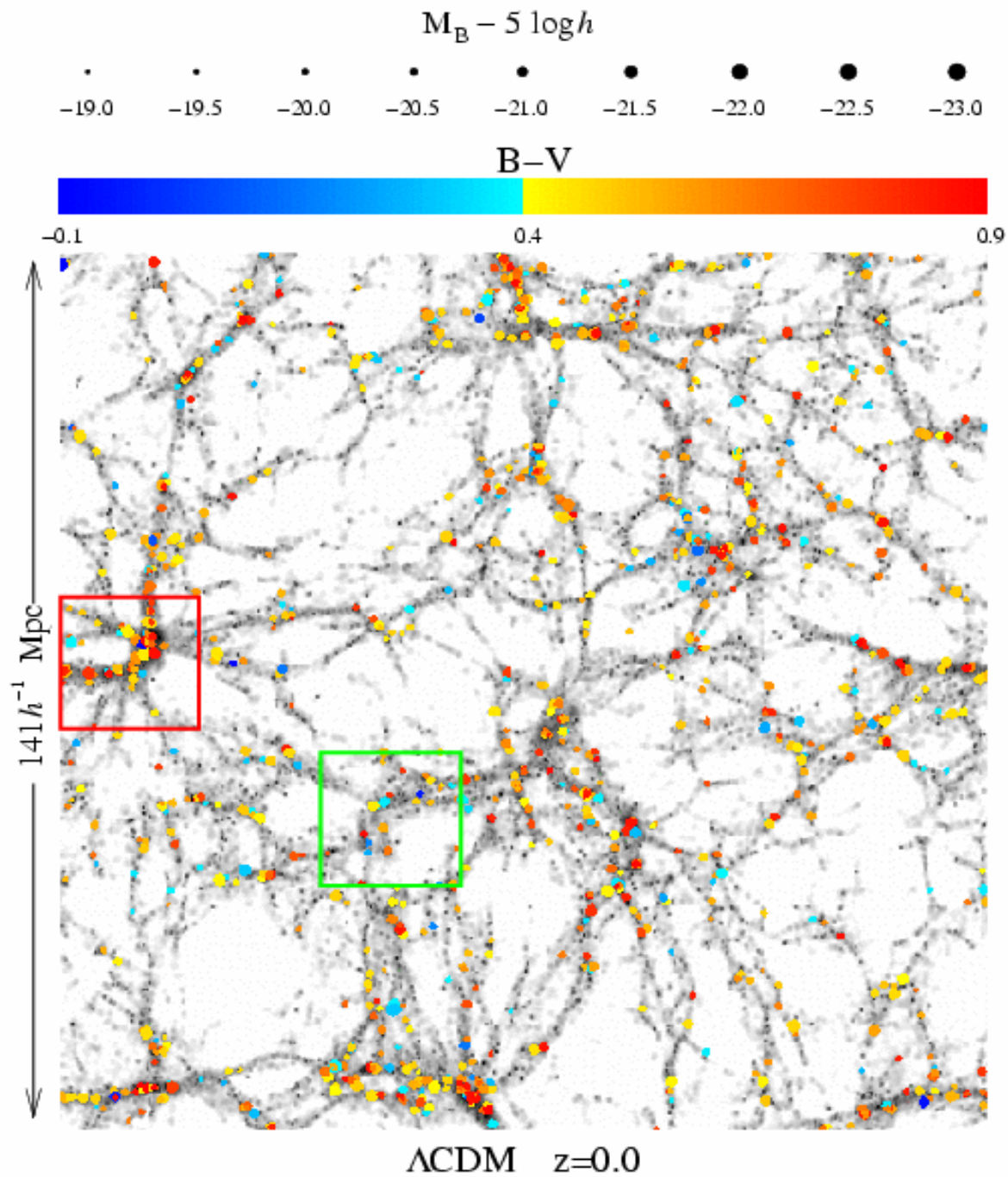
Benson, Frenk, Baugh, Cole & Lacey (2001)



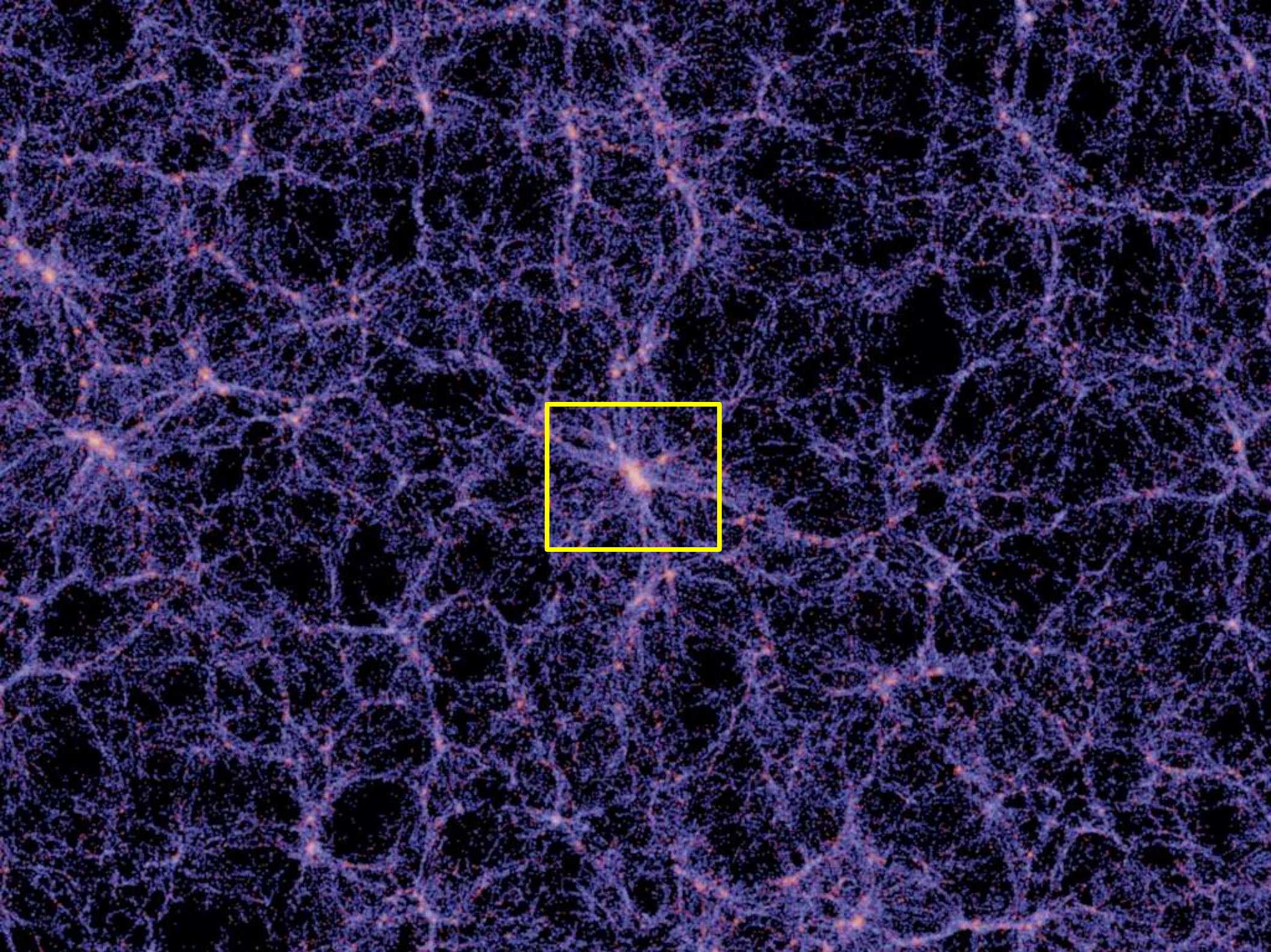
Benson, Frenk, Baugh, Cole & Lacey (2001)

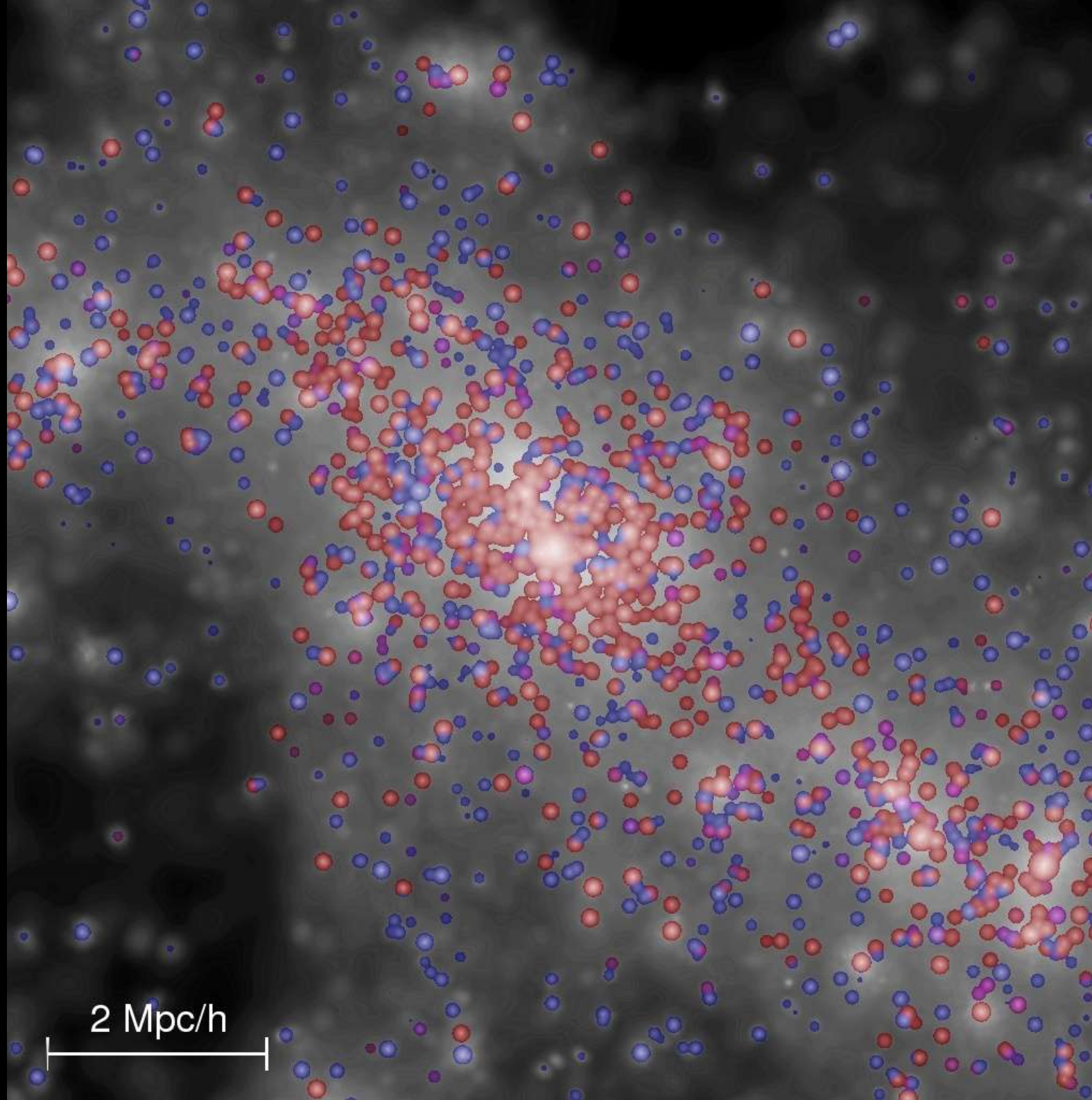


Benson, Frenk, Baugh, Cole & Lacey (2001)



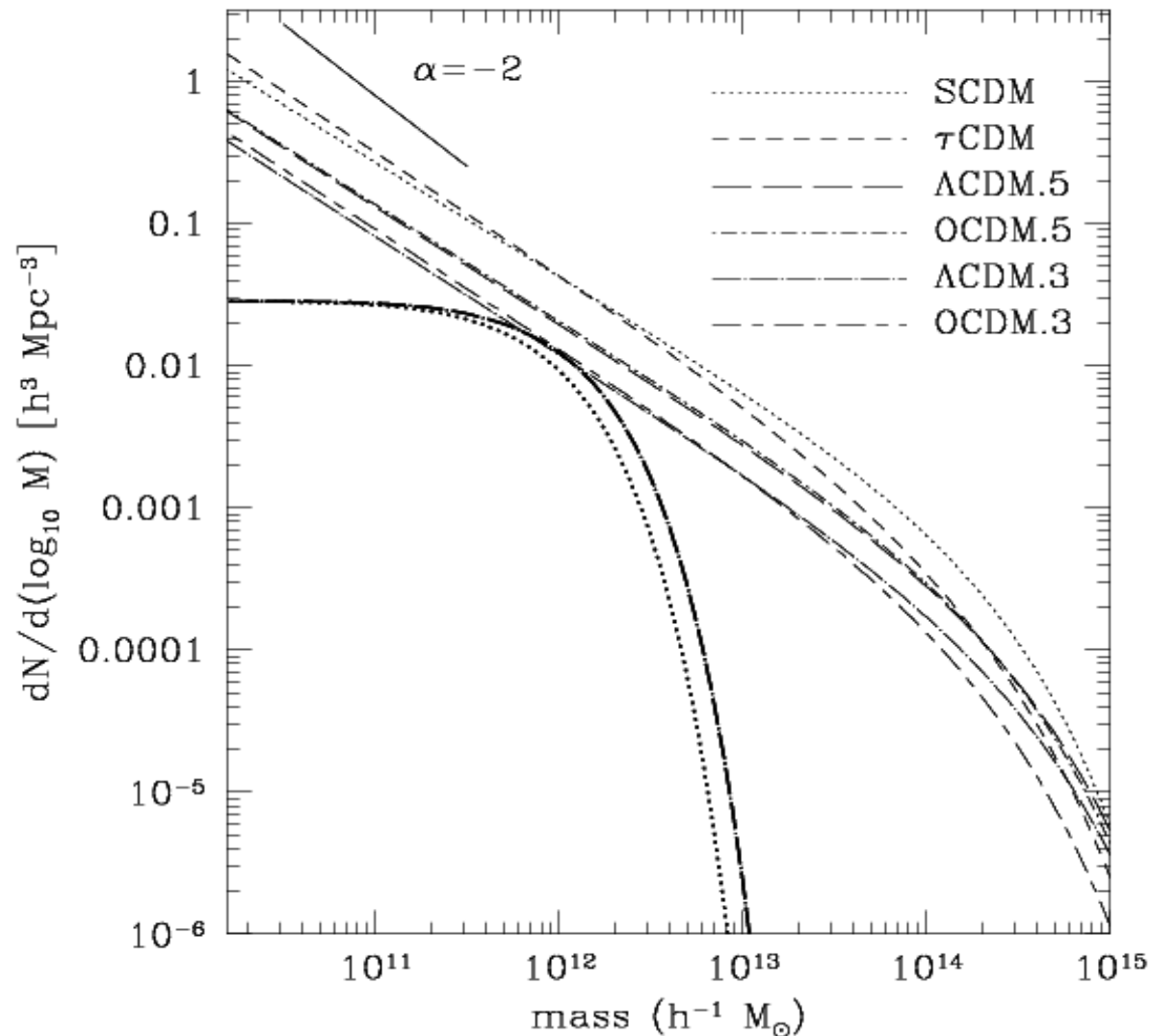
Benson, Frenk, Baugh, Cole & Lacey (2001)



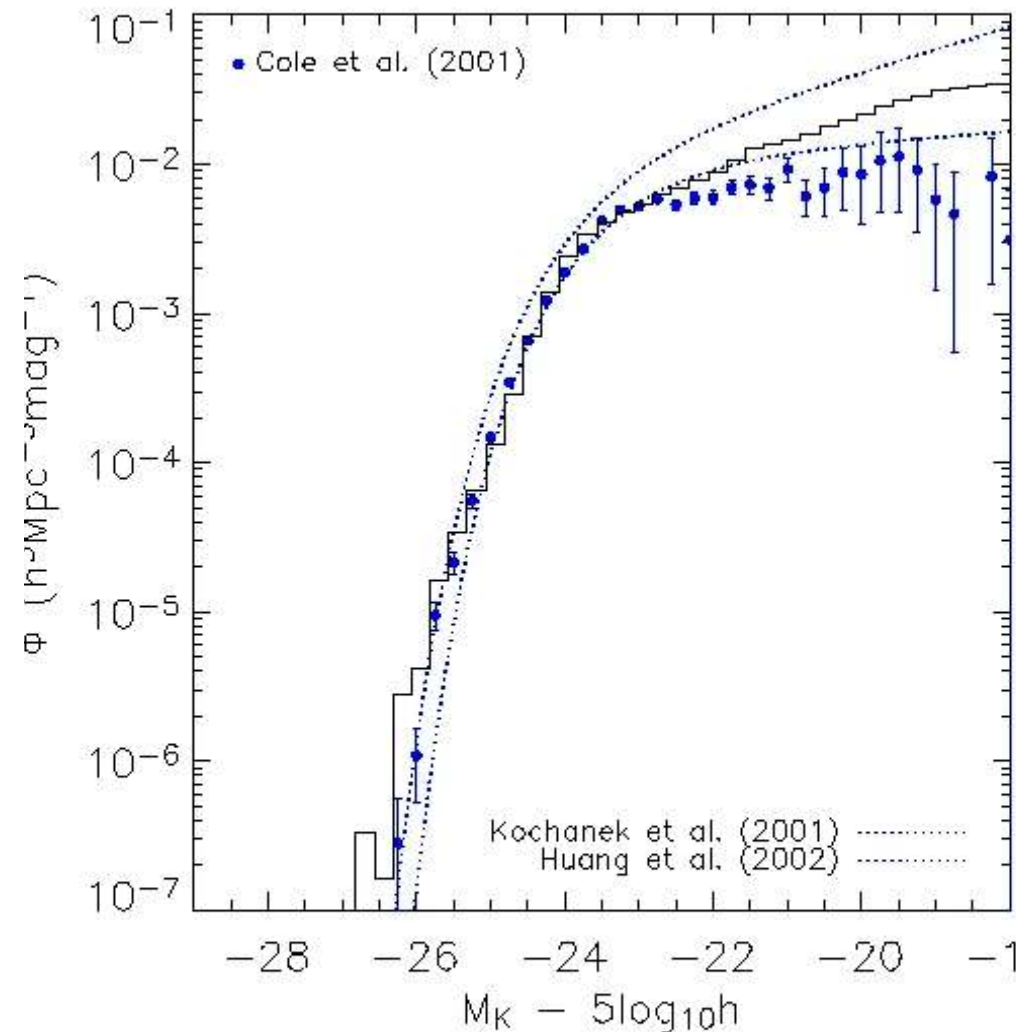
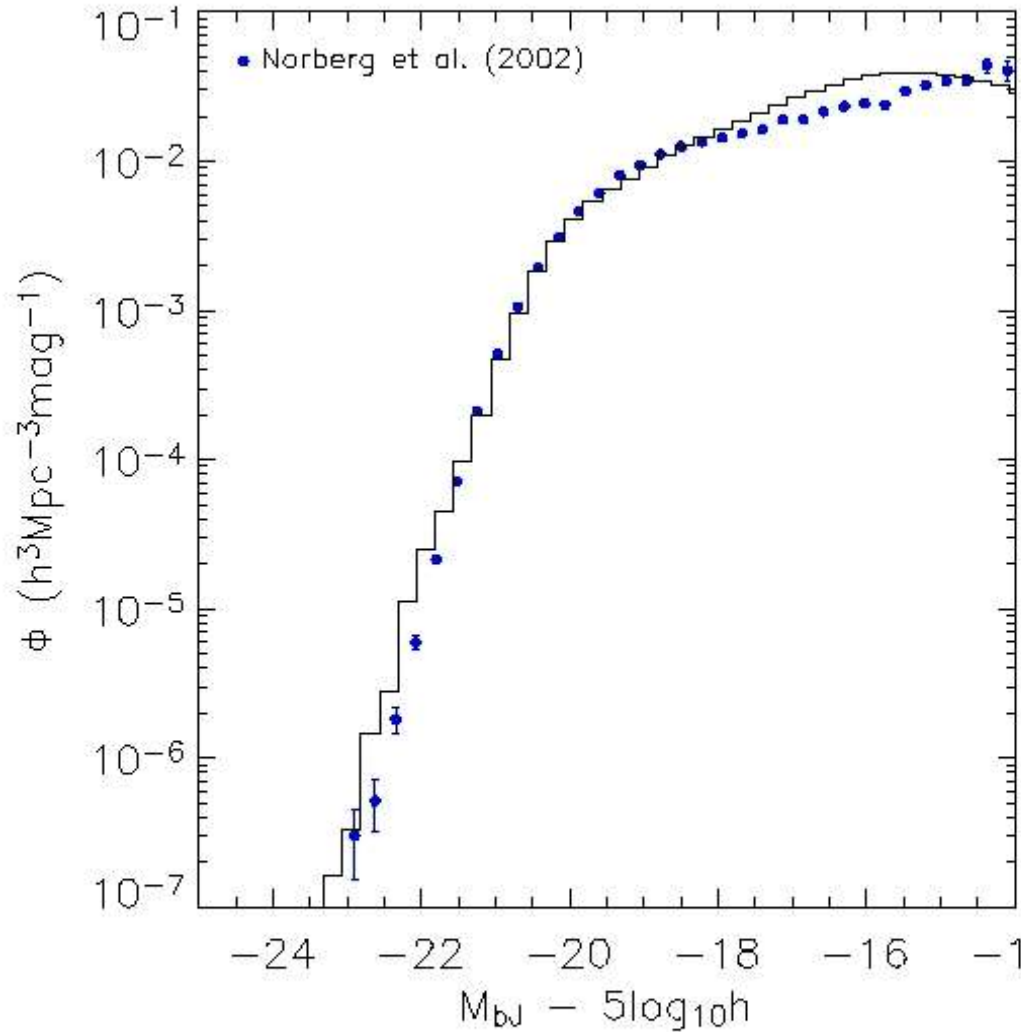


2 Mpc/h

Why does the observed luminosity function have the “Schechter” form with flat faint-end slope and exponential cut-off at high luminosities/masses?

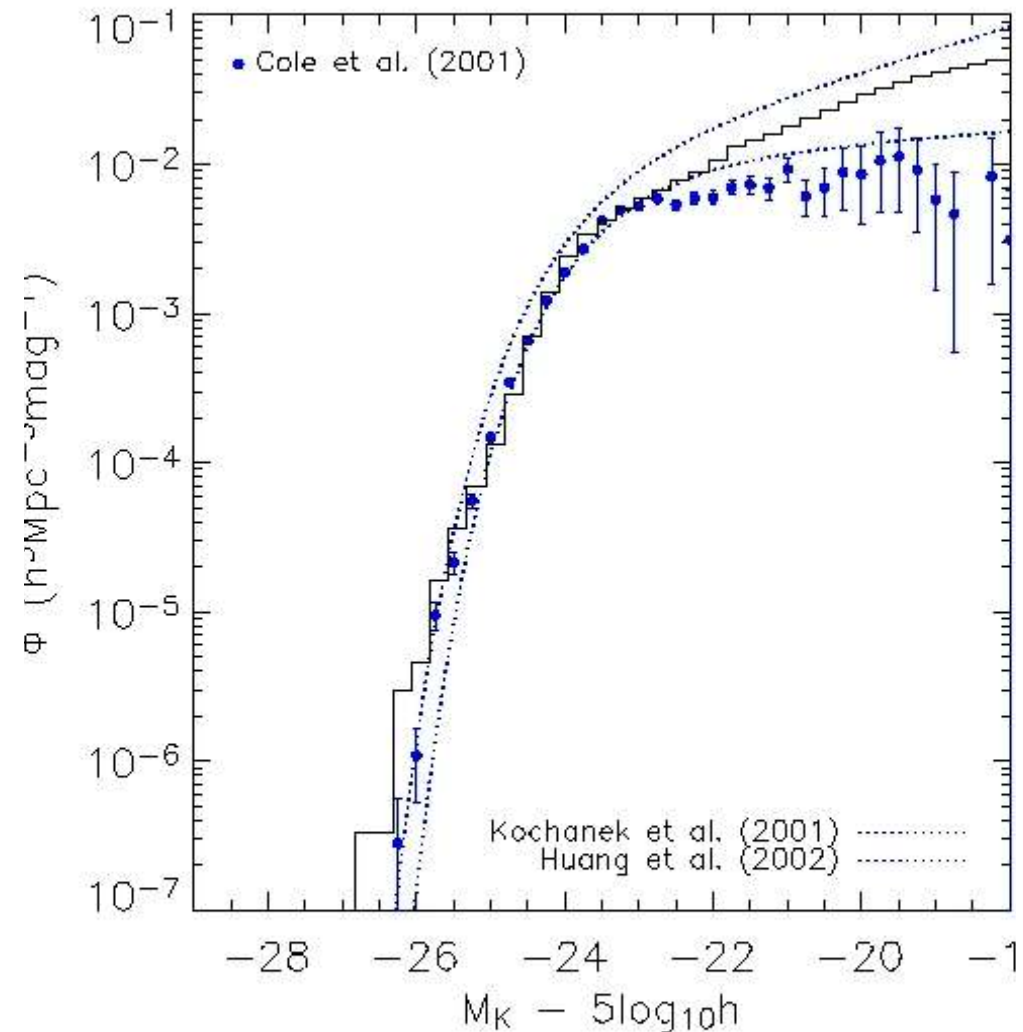
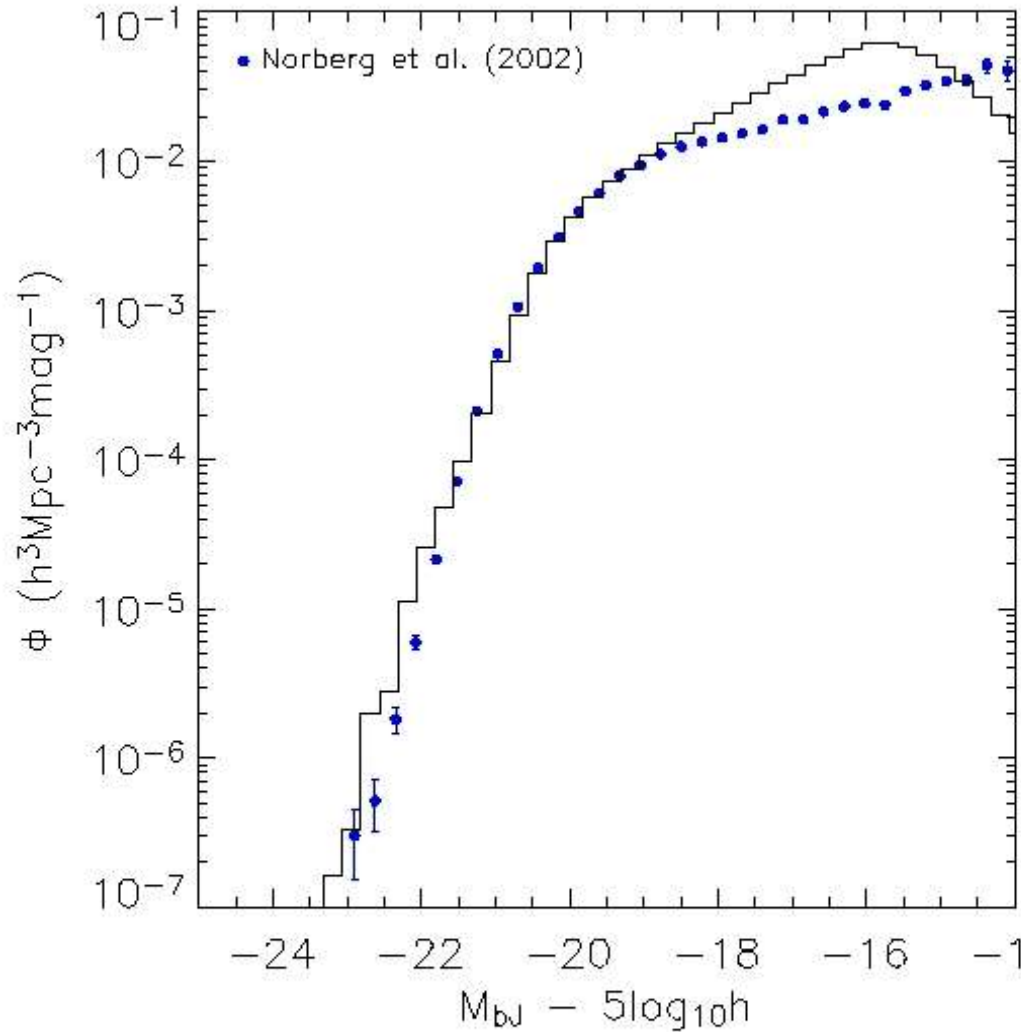


Effect of feedback on the Luminosity Function



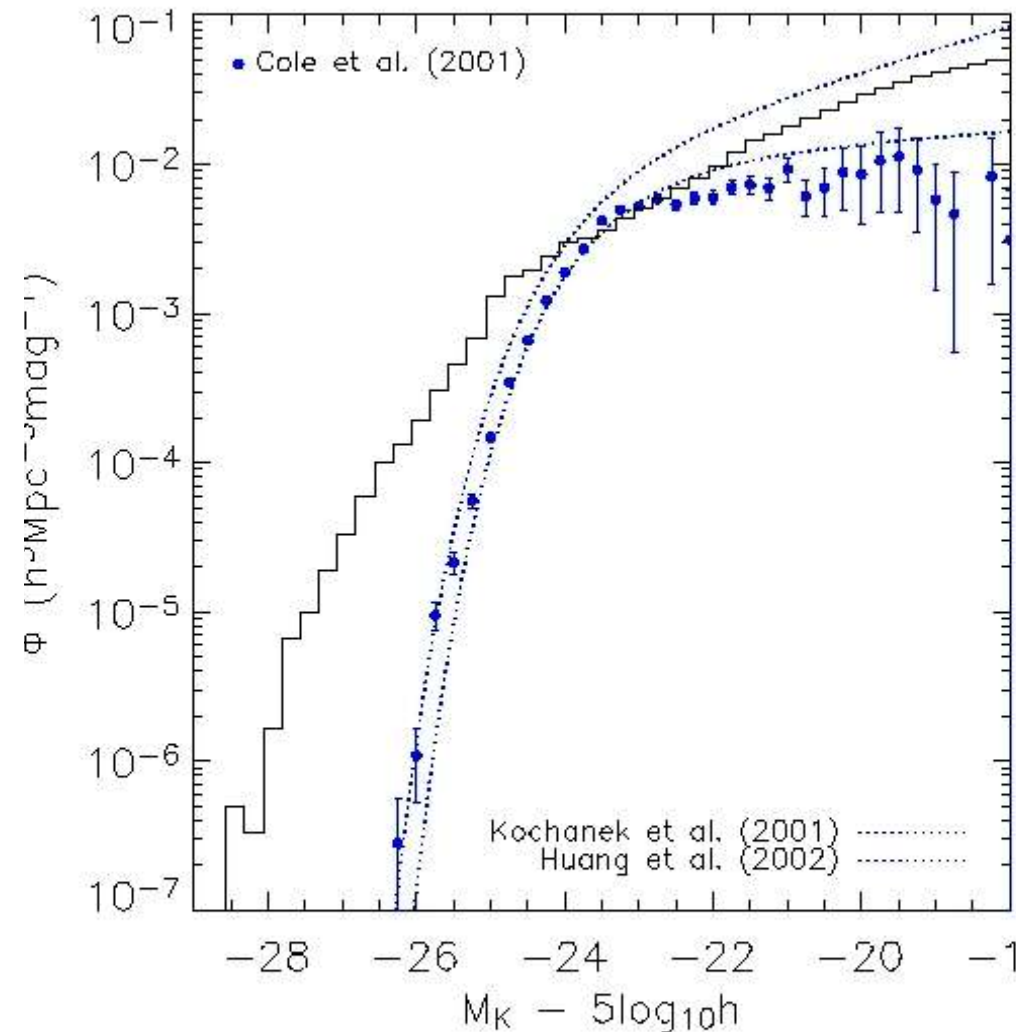
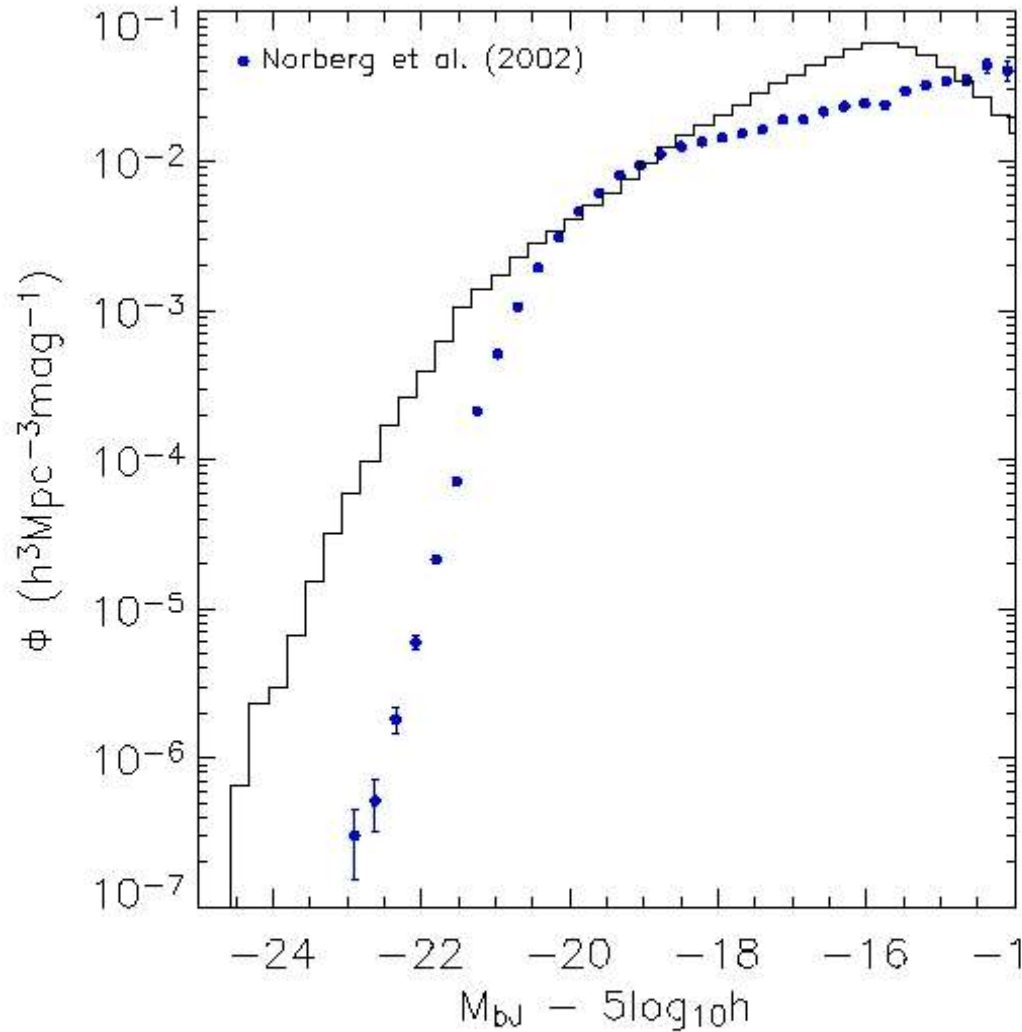
Full model with reionisation, AGN and SN feedback Croton et al 2005

Effect of feedback on the Luminosity Function



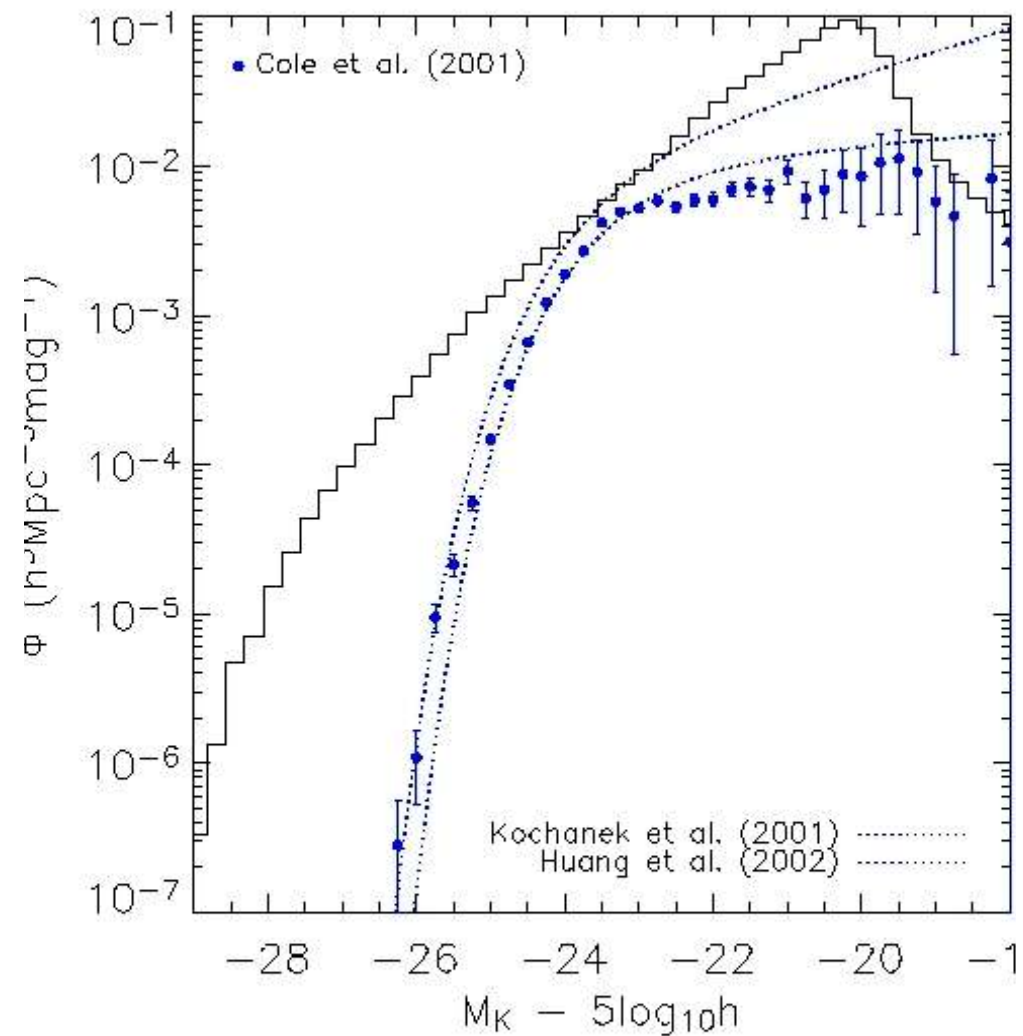
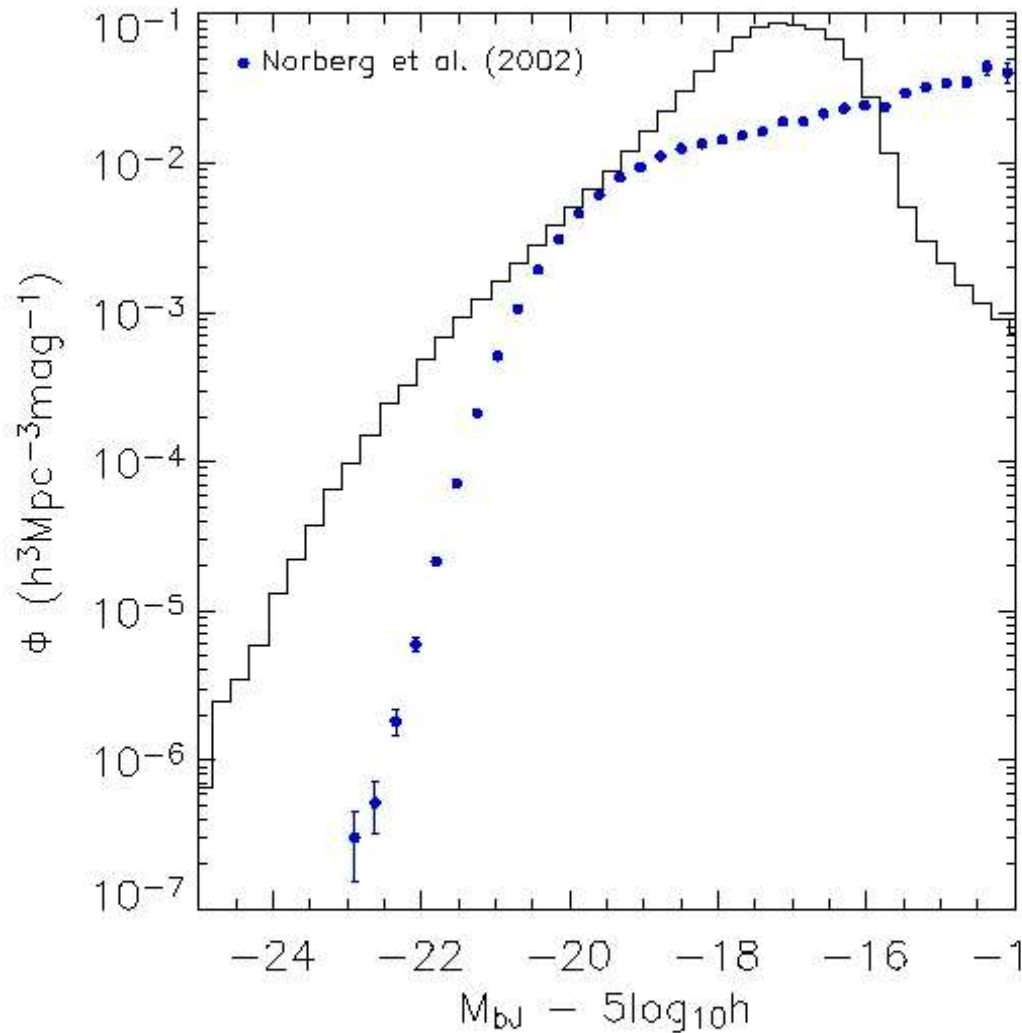
Full model with ~~reionisation~~, AGN and SN feedback Croton et al 2005

Effect of feedback on the Luminosity Function



Full model with ~~reionisation~~, ~~AGN~~ and SN feedback Croton et al 2005

Effect of feedback on the Luminosity Function



Full model with ~~reionisation~~, ~~AGN~~ and ~~SN~~ feedback Croton et al 2005

Which aspects of feedback are critical?

- Reionisation filtering has little effect on any but the faintest galaxies. May be relevant for faint Local Group dwarfs?
- SN feedback can progressively reduce the star formation efficiency in galaxies fainter than L^* and so flatten the faint end slope of the LF. Hard to get a strong enough effect.
- An additional mechanism is needed (radio AGN?) to suppress star formation in massive “cooling flow” systems. It should not involve star formation since most massive galaxies are red.

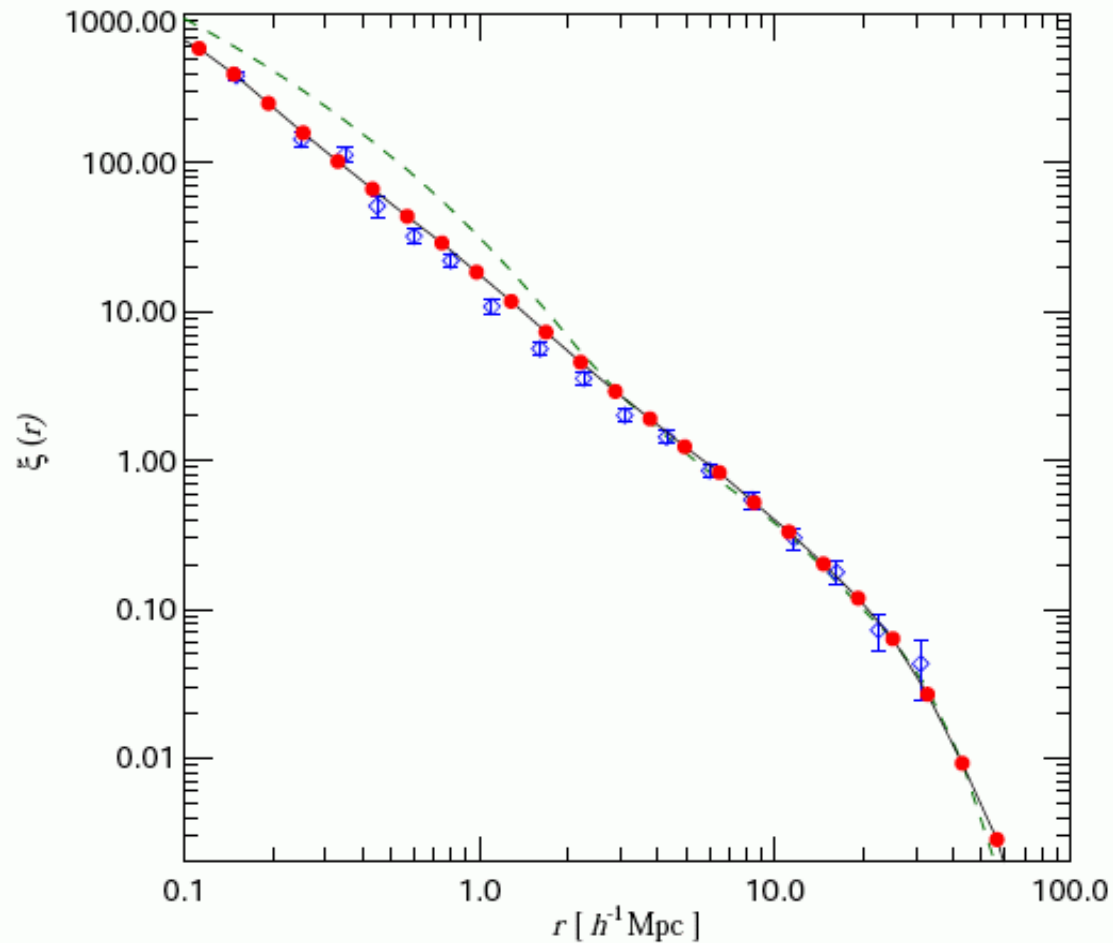
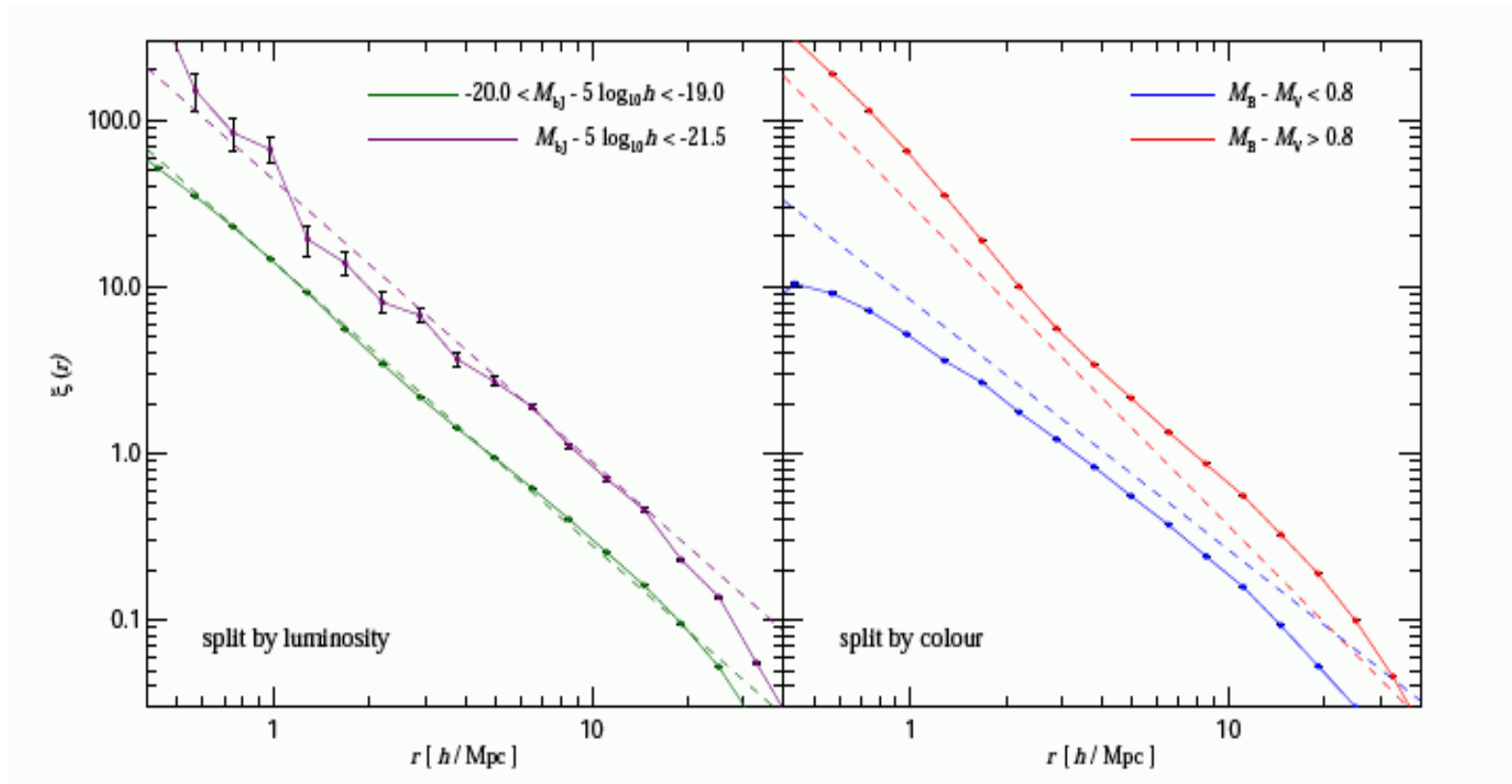


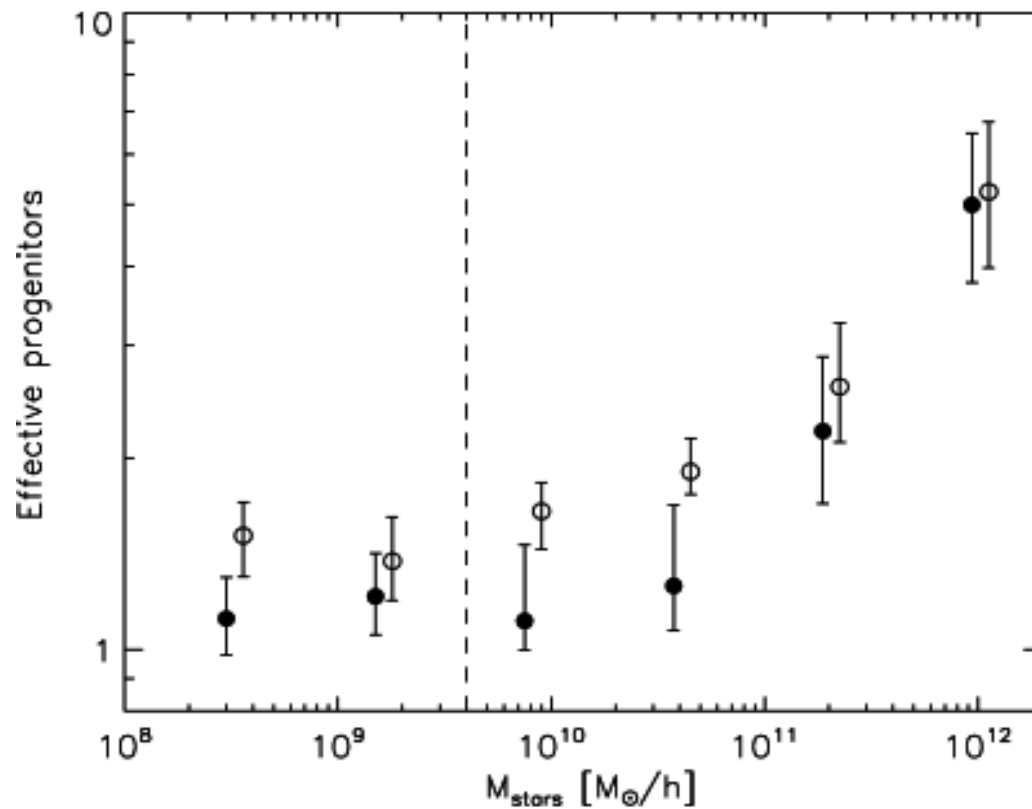
Figure 4: Galaxy 2-point correlation function at the present epoch. Red symbols (with vanishingly small Poisson error-bars) show measurements for model galaxies brighter than $M_K = -23$. Data for the large spectroscopic redshift survey 2dFGRS²⁸ are shown as blue diamonds. The SDSS³⁴ and APM³¹ surveys give similar results. Both, for the observational data and for the simulated galaxies, the correlation function is very close to a power-law for $r \leq 20h^{-1}\text{Mpc}$. By contrast the correlation function for the dark matter (dashed line) deviates strongly from a power-law.

Clustering as a function of luminosity and colour

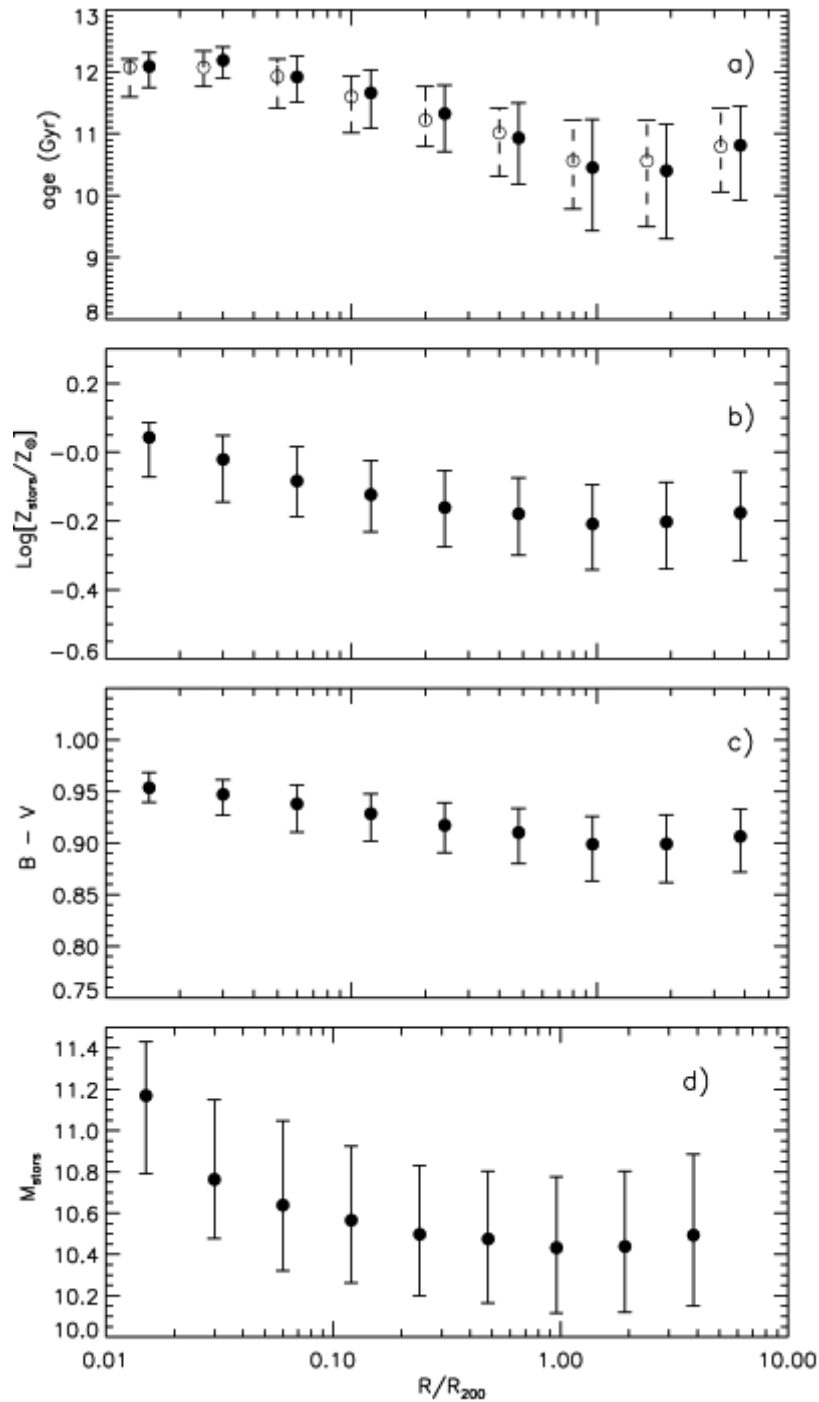


How are Elliptical Galaxies Formed?

Figure 9. Effective number N_{eff} of progenitors as a function of galaxy stellar mass. Filled circles represent the median of the distribution in our default model, while the error bars indicate the upper and lower quartiles. Empty circles and the corresponding error bars are for a model where bulge formation through disc instability is switched off. The vertical dashed line corresponds to the limit above which our morphological-type determination is robust (see [Section 3](#)).



Galaxy properties as a function of environment



Examples of questions one can address using semi-analytic models

- 1) What are the abundances of galaxies with different masses/luminosities at different redshifts?
- 2) How are galaxies with different masses/luminosities clustered at different redshifts?
- 3) How do the properties of galaxies differ in clusters compared to the “field”?
- 4) What are masses/luminosities/spatial distribution of the progenitors of present-day galaxies?
- 5) Where are galaxies identified at high redshifts located now ; we cannot infer whether one galaxy population will evolve to resemble another one observationally – the models PREDICT this causal connection.