

# Simulations of Small Scale Structure of Cold Dark Matter with **GRACOS**

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Alexandria Library

April 6, 2006

# Outline

- N-body simulation code *GRACOS*
  - Parallel computing
  - Load balancing techniques
  - Adaptive P3M algorithm
  - Multiple timestepping (in progress)
- Large Scale Structure
  - LCDM universe simulations
- Small Scale structure
  - Caustics
  - Comparison with analytic models
  - Implications on direct detection experiments

# P3M N-body algorithm

- Gravitational Force

Total grav. force = Particle-Mesh(PM) + Particle-Particle(PP)  
Long Range Force                      Short Range Force

PM: Long Range component using FFT

PP: Short range component, traditionally  
using direct summation over close pairs of particles  
Direct summation is extremely time consuming

- Equation of motion

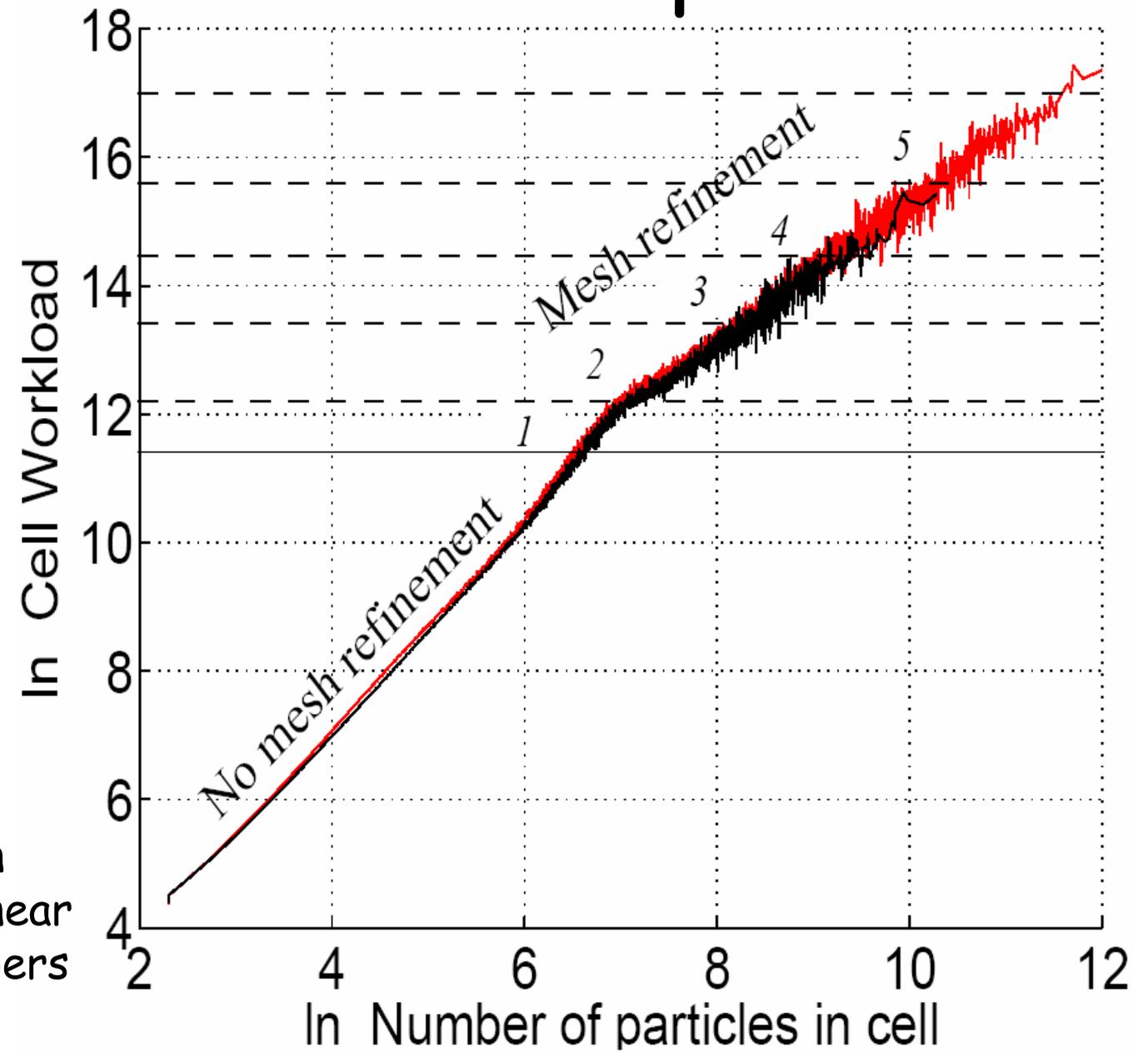
in code units

$$\frac{d^2 \mathbf{x}}{d\tau^2} + \frac{1}{a} \frac{da}{d\tau} \frac{d\mathbf{x}}{d\tau} = -\nabla_{\mathbf{x}} \phi$$

$$\frac{d\tilde{\mathbf{x}}}{d\tilde{t}} = \tilde{\mathbf{v}}, \quad \frac{d\tilde{\mathbf{v}}}{d\tilde{t}} = \tilde{\mathbf{g}}$$

Leapfrog integrator

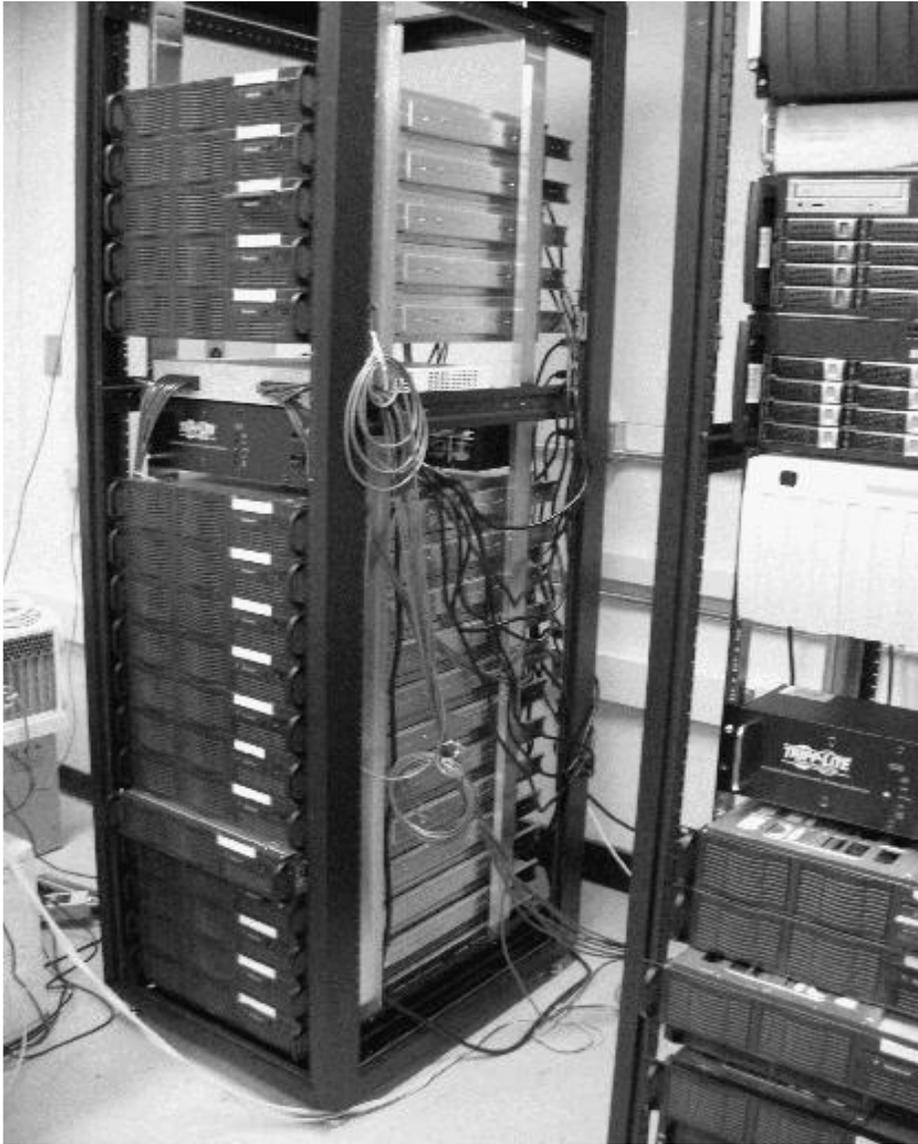
# Optimization of PP computation



FFT based solution

The direct summation  
Workload becomes linear  
At high particle numbers

# Beowulf Clusters of Computers



Parallelization: the problem of separation of task among the processors/nodes

- distribution of the data
  - distribution of computation
  - distribution of storage
- Must all be close to uniform!

*MIT beowulf cluster: assembly and installation by:  
O. Scwarz, A. Shirokov, E. Bertschinger, S. Hughes*

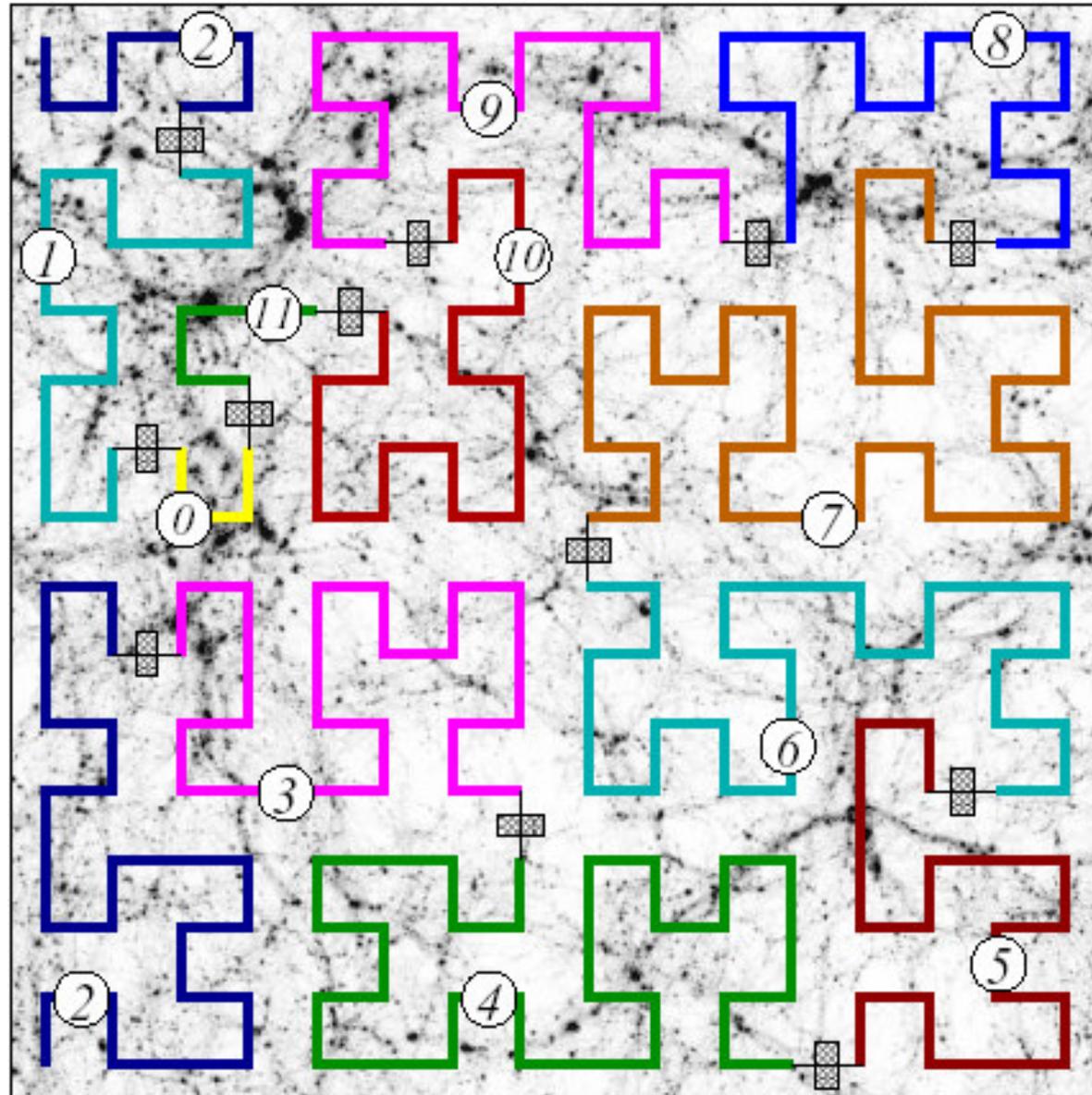
# Hilbert Curve Domain decomposition

[Pilkington & Baden (1996)] Pilkington, J. & Baden, S. 1996, IEEE Trans. Par. Dist. Systems, 7, 288

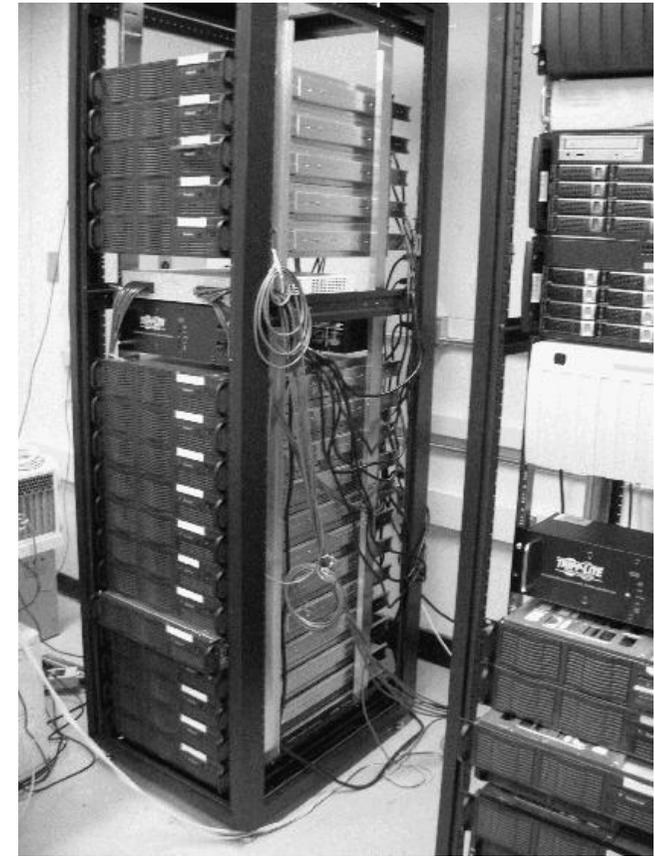
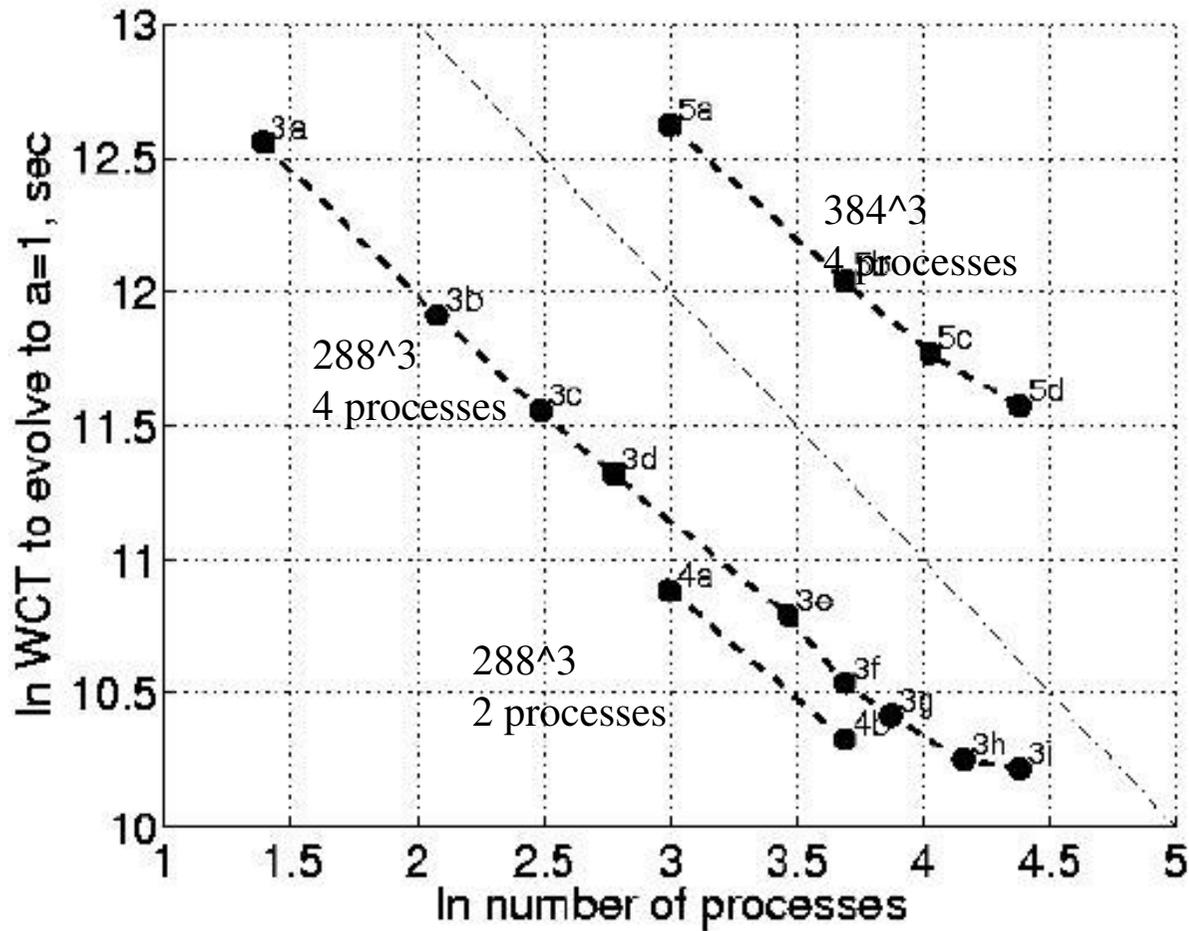
The solution:  
(independently  
adopted  
by gadget2)

Introduce a curve  
And the partitions

Partitions are  
moving  
along the curve to  
adjust for change  
in particle  
distribution



# Scalability



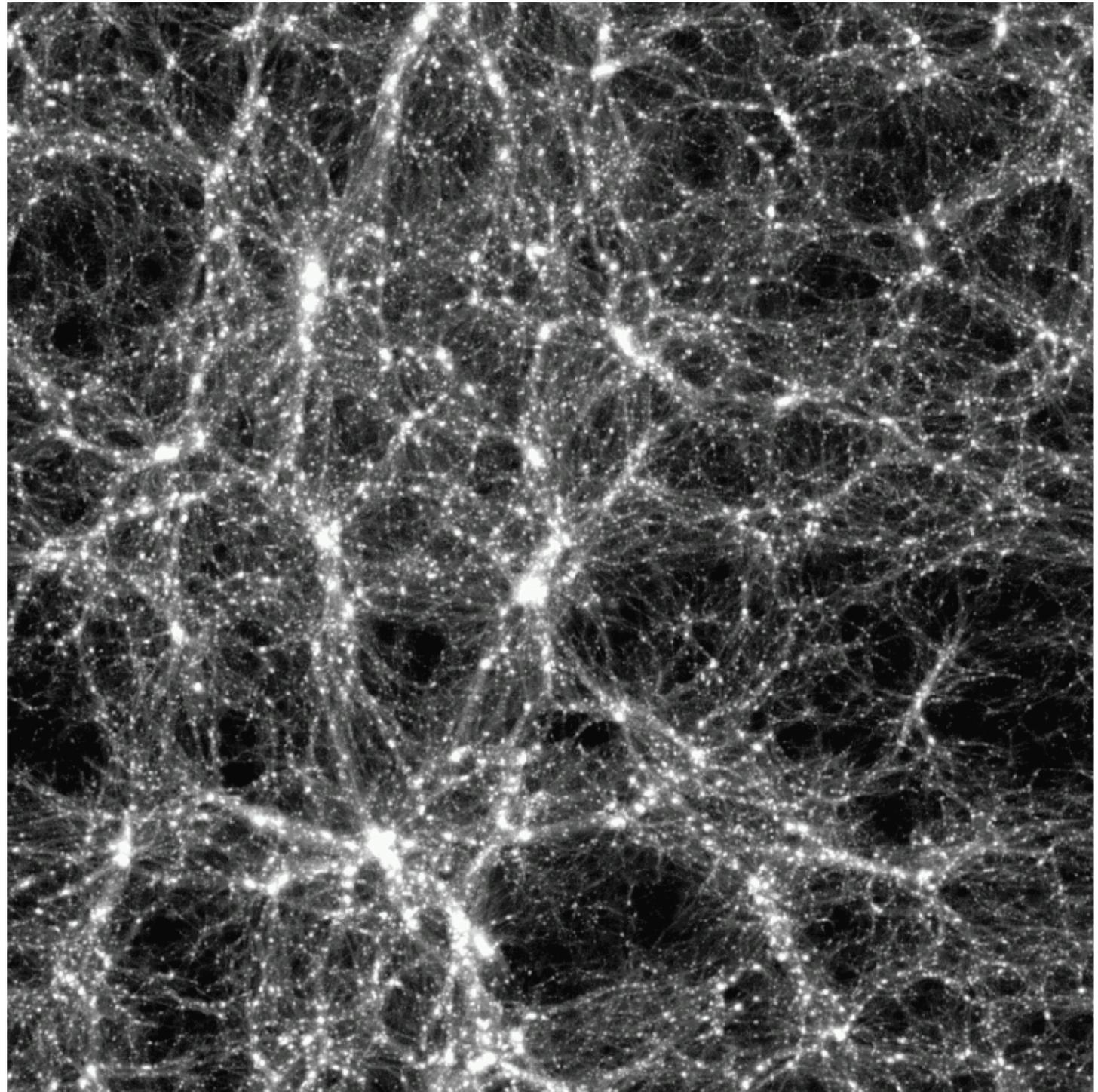
The maximum optimal number of processes increases with the problem size

Large Scale  
Structure

$\Lambda$ CDM  
universe

GRACOS  
(GRAVitational  
COSmology)  
Code

$z = 0$   
 $L = 200\text{Mpc}$



# P3M simulation

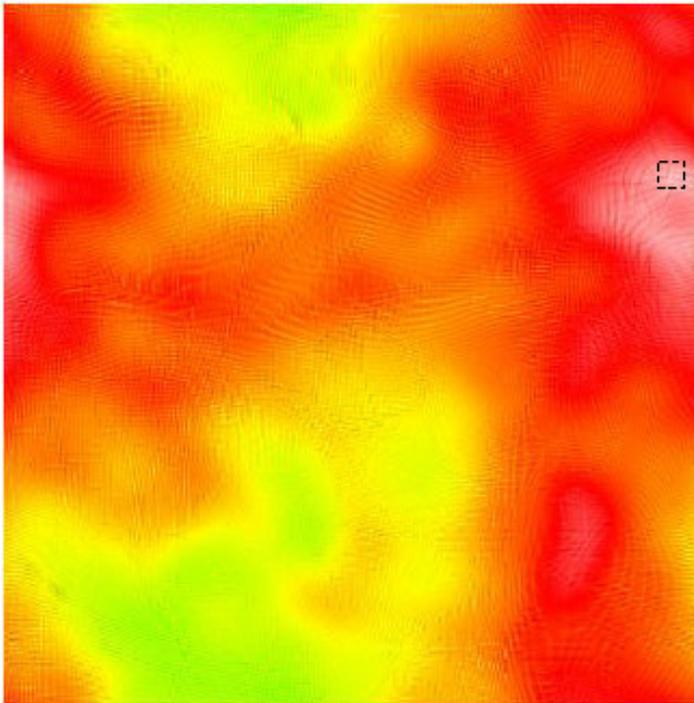
800<sup>3</sup> particles

Initial conditions  $n=-3$

$$P(\mathbf{k}) = P_0 k^n \left[ \frac{D_+(a)}{D_+(1)} \right] \exp(-k^2 l_s^2)$$

$$z_i = 350$$

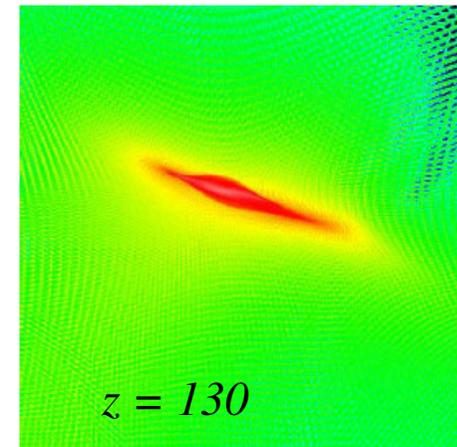
particle weighs  $3.1 \times 10^{-12} M_\odot$



$$k_c \approx 1.7 \times 10^6 \text{ Mpc}^{-1}$$

*The First Caustic to Form*

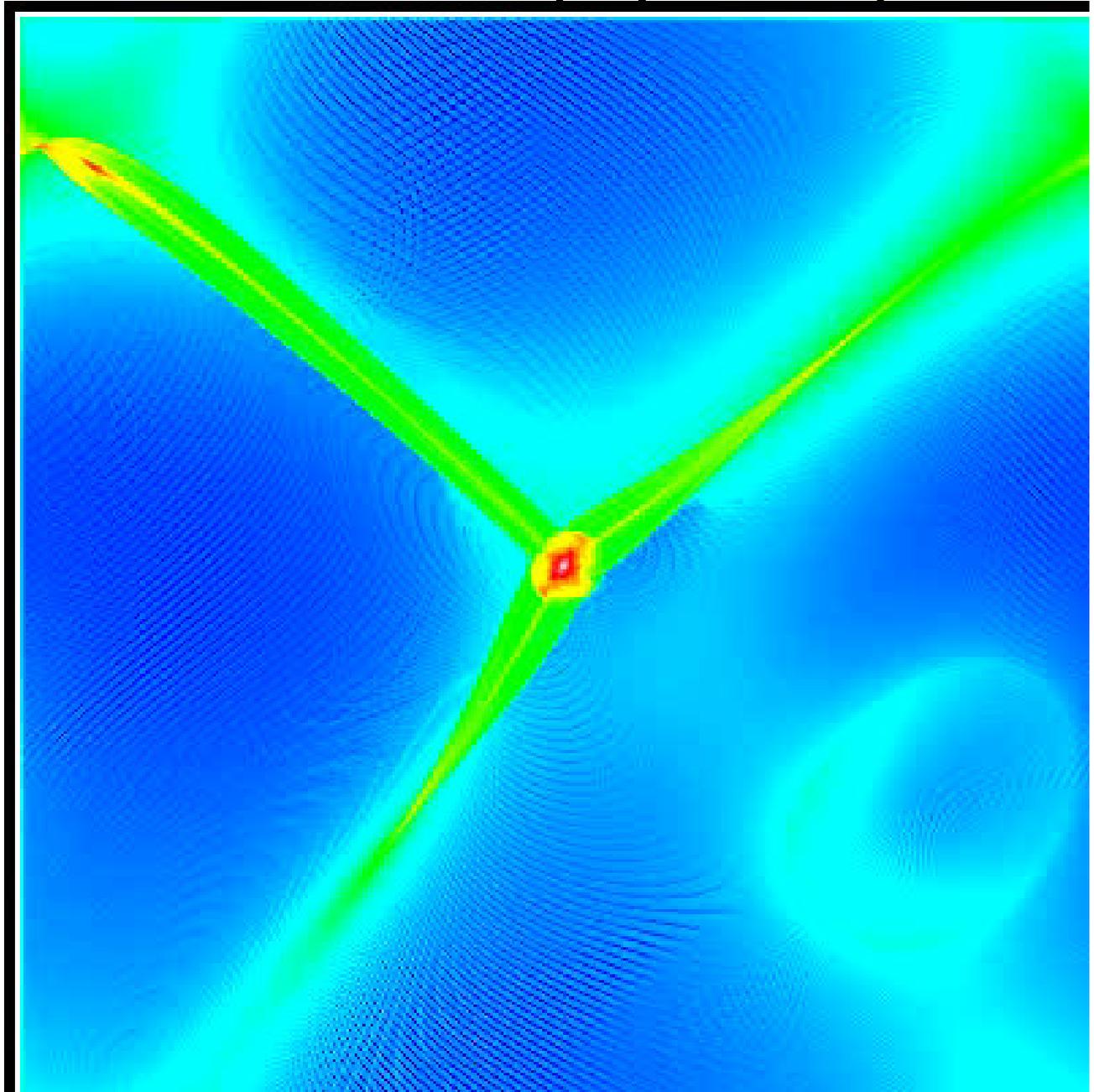
$z \lesssim 350$   
 $L = 20 \text{ pc}$



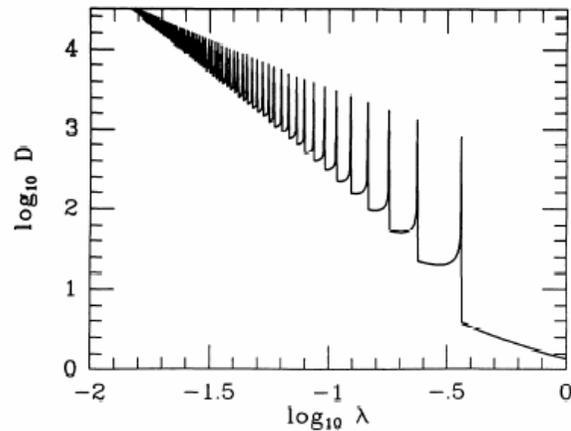
# PM 512<sup>3</sup> particle simulation: physical space

4 smoothing lengths  
per sidelength in ICs

- Many shells, - caustics
- No implied symmetry
- Inner caustic survives

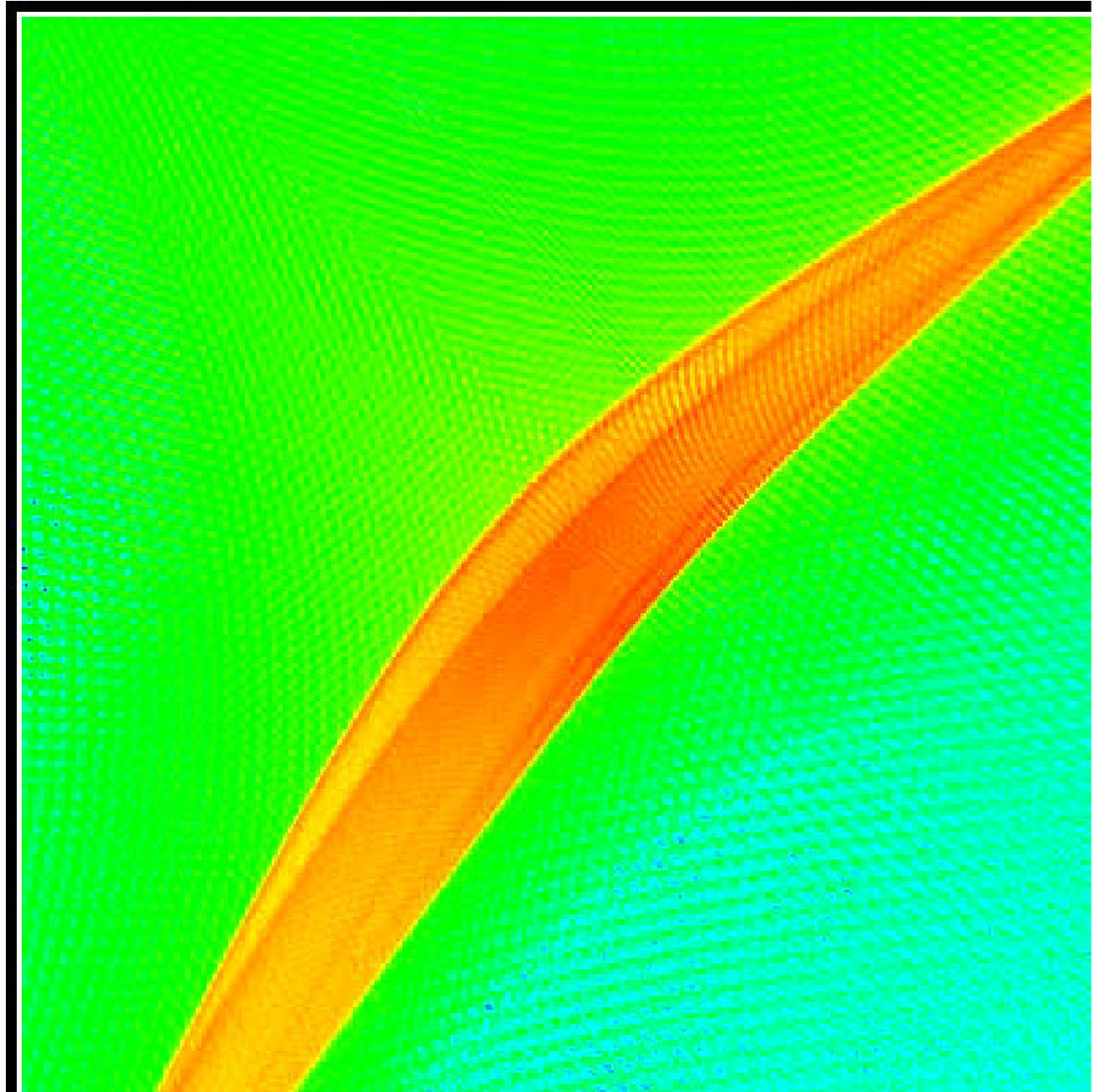
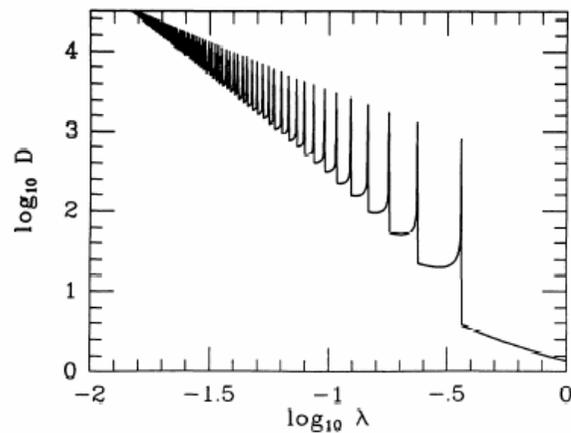


*Bertschinger 1985*



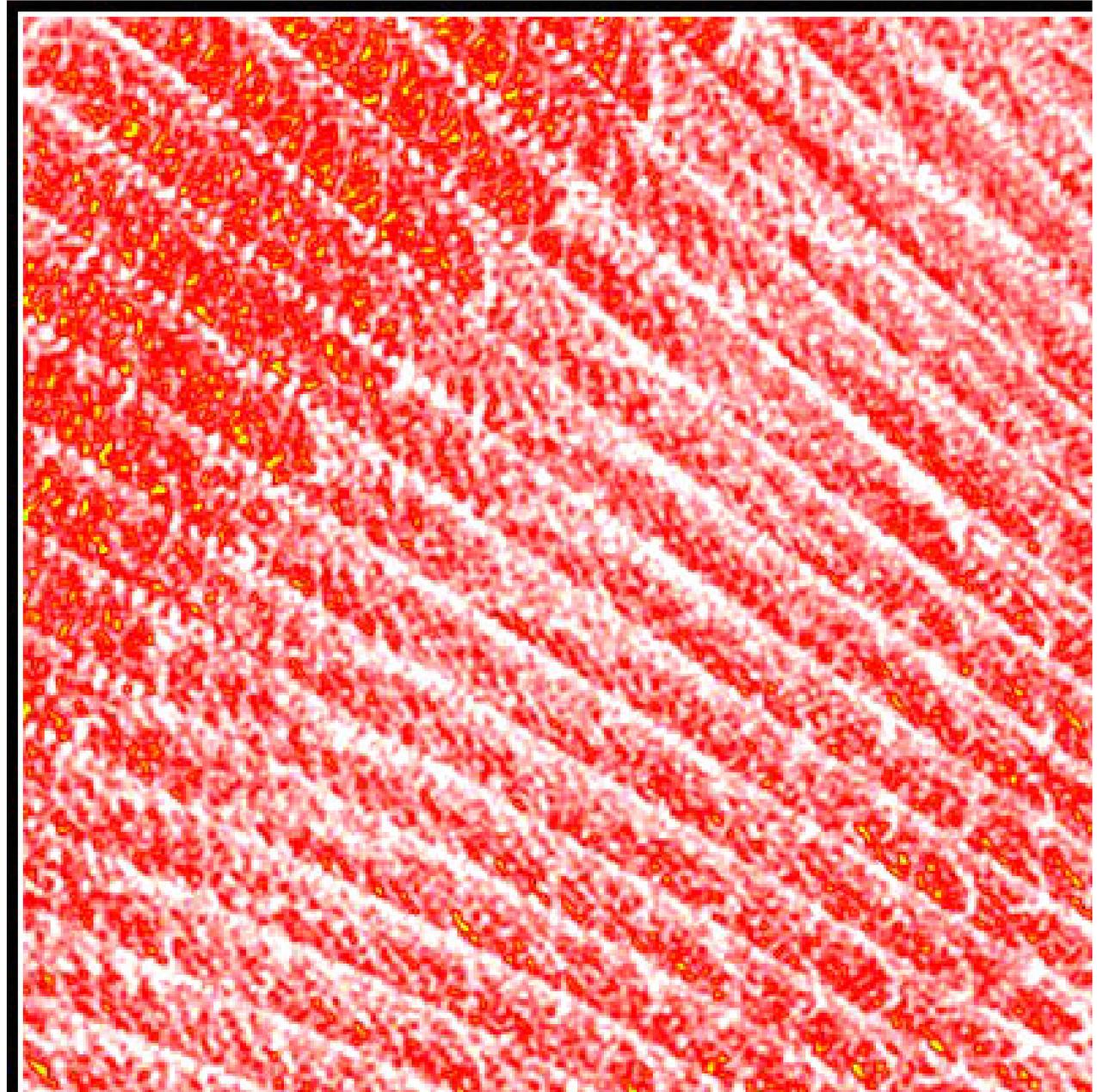
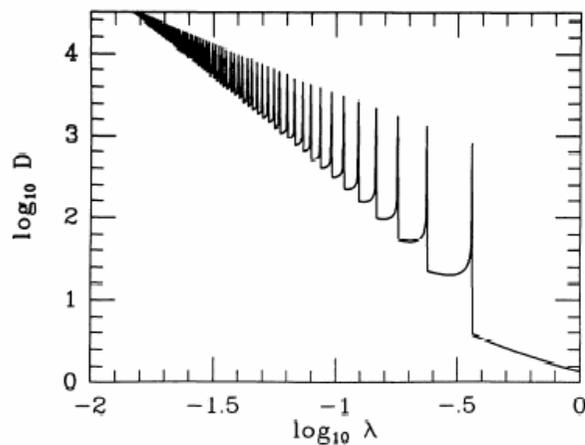
# PM $512^3$ simulation: physical space

*Analytical model  
Bertschinger 1985*

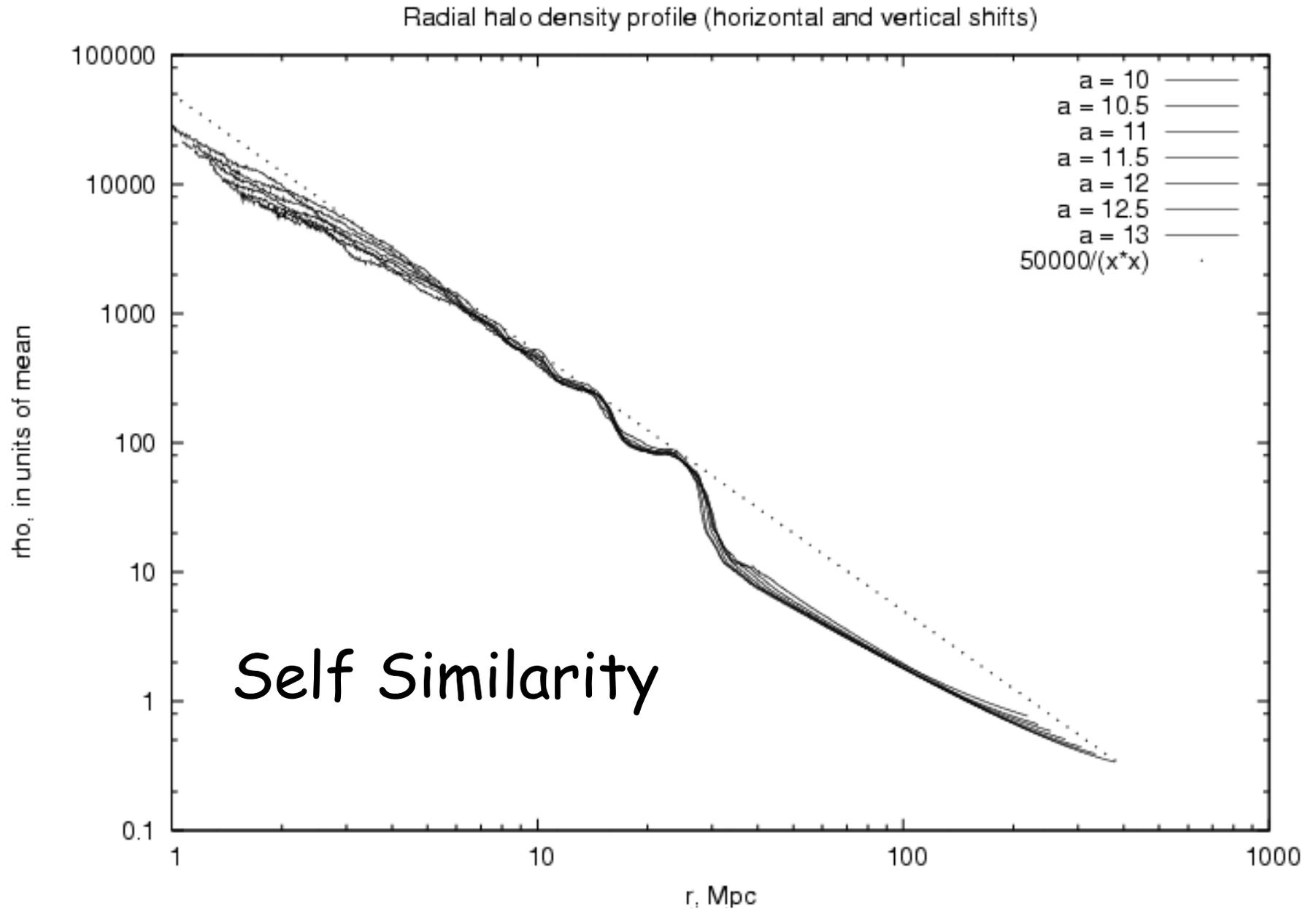


# PM 512<sup>3</sup> particle simulation

*Bertschinger 1985*

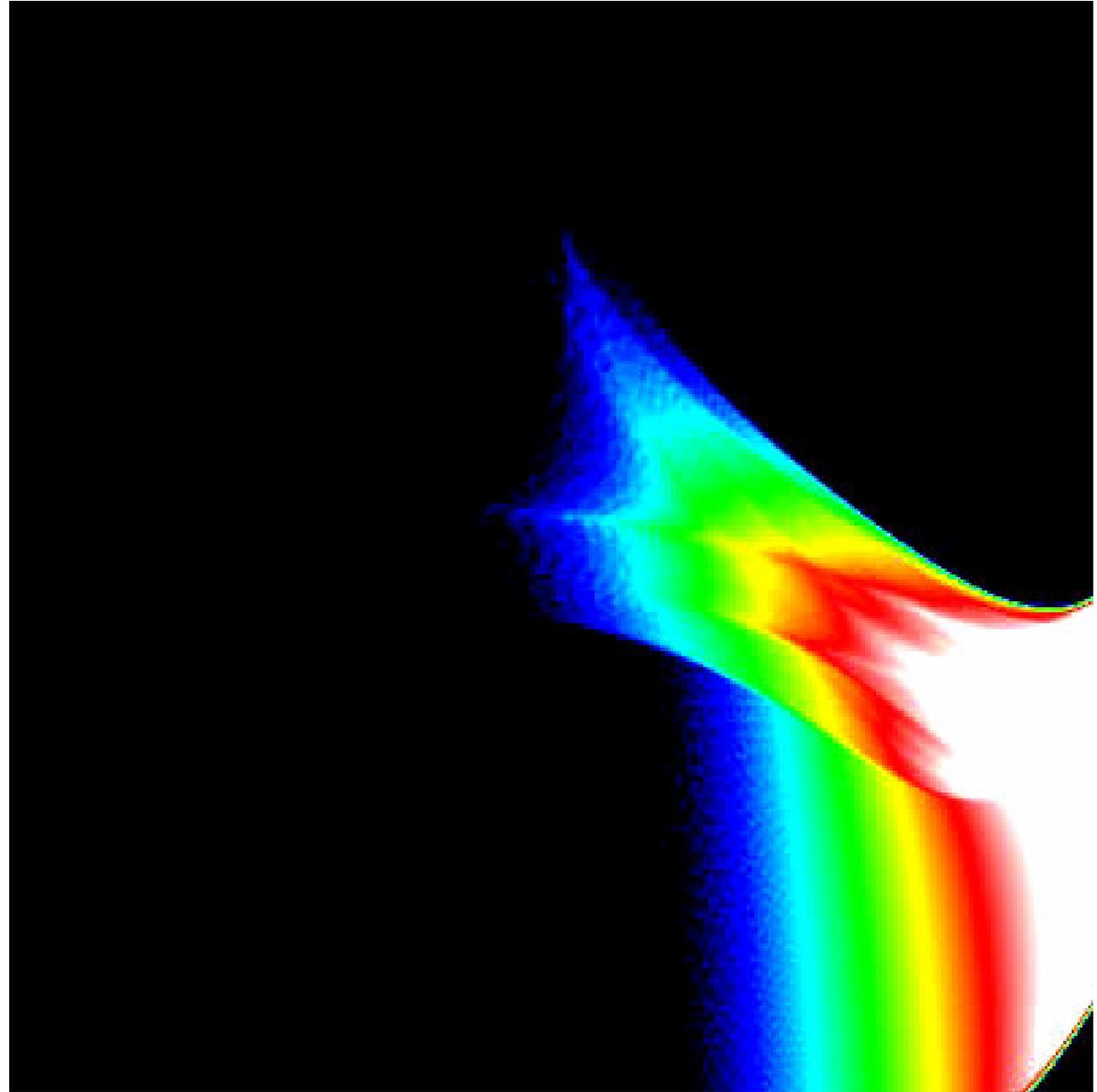
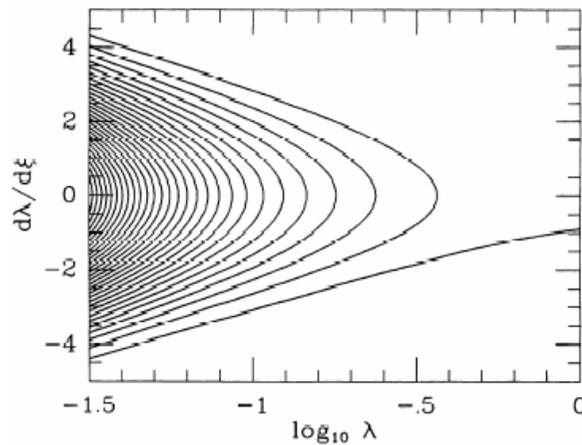


# PM 512<sup>3</sup> particle simulation: radial profile



# PM 512<sup>3</sup> particle simulation: phase space

Bertschinger 1985



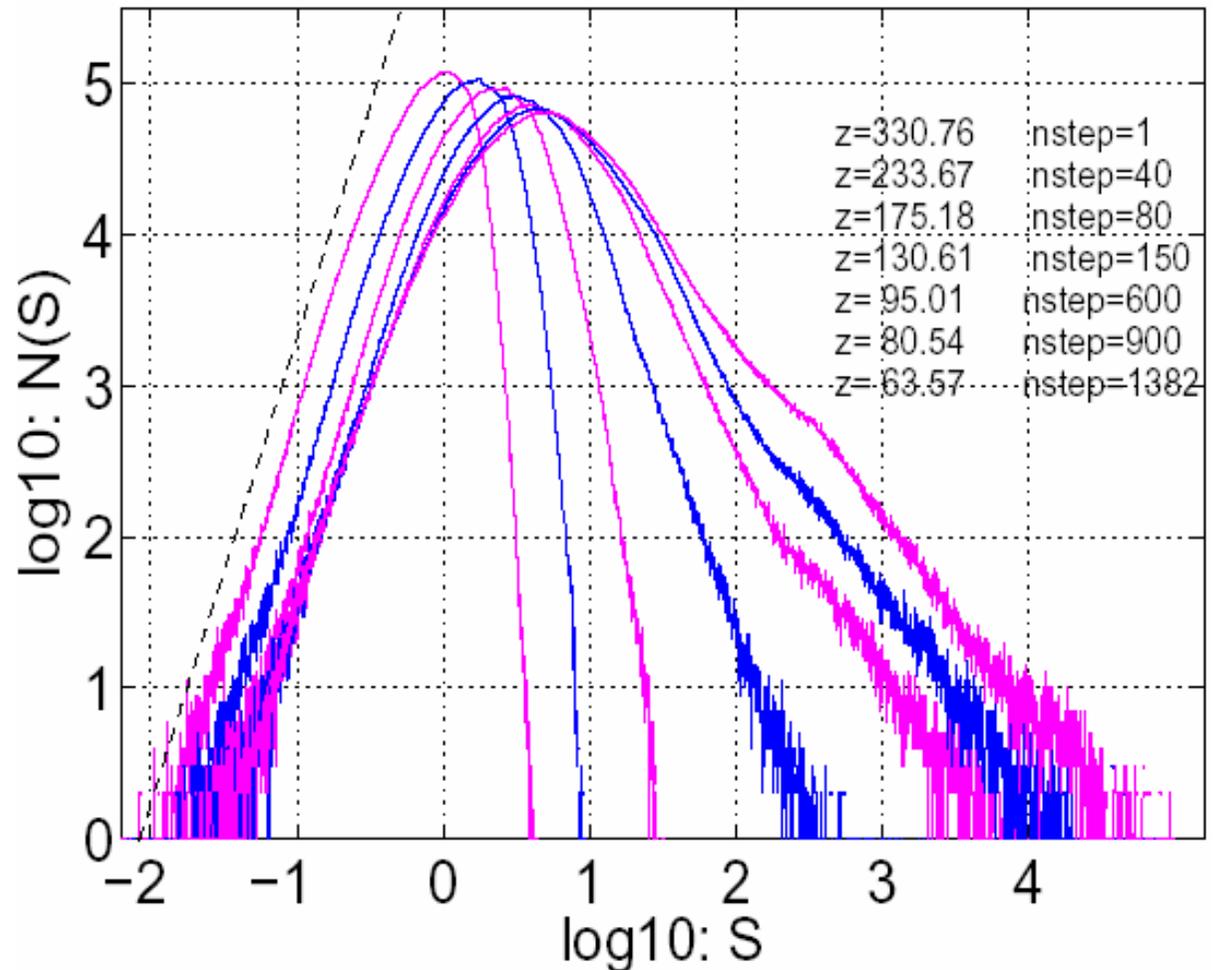
# Measuring the Detection Rate Distribution Function

$$f(v) = 4\pi v^2 f(\mathbf{v}) = \frac{4\pi v^2 e^{-v^2/2\sigma^2}}{(2\pi\sigma^2)^{3/2}}$$

$$N(\Gamma) \propto \begin{cases} \Gamma^3, & \Gamma \ll \Gamma_0 \\ \exp\left(-\frac{\Gamma^2}{2\sigma^2 n_\chi^2 \sigma_\chi^2}\right) & \Gamma \gg \Gamma_0 \end{cases}$$

$$N(\Gamma) \propto \Gamma^{-2.7 \pm 0.3}$$

Power law tail



Summary