3. Planet formation

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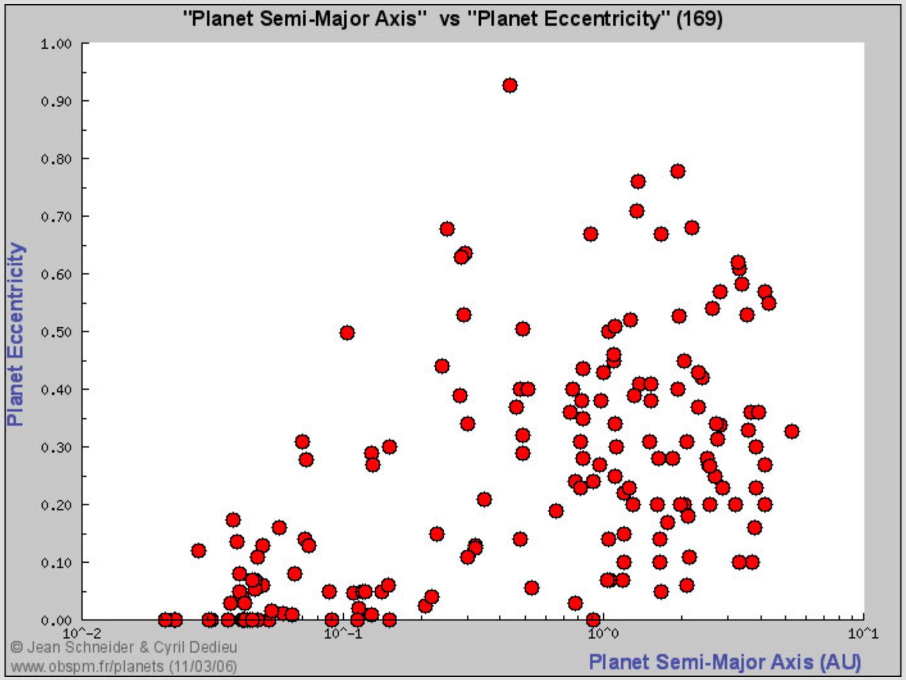
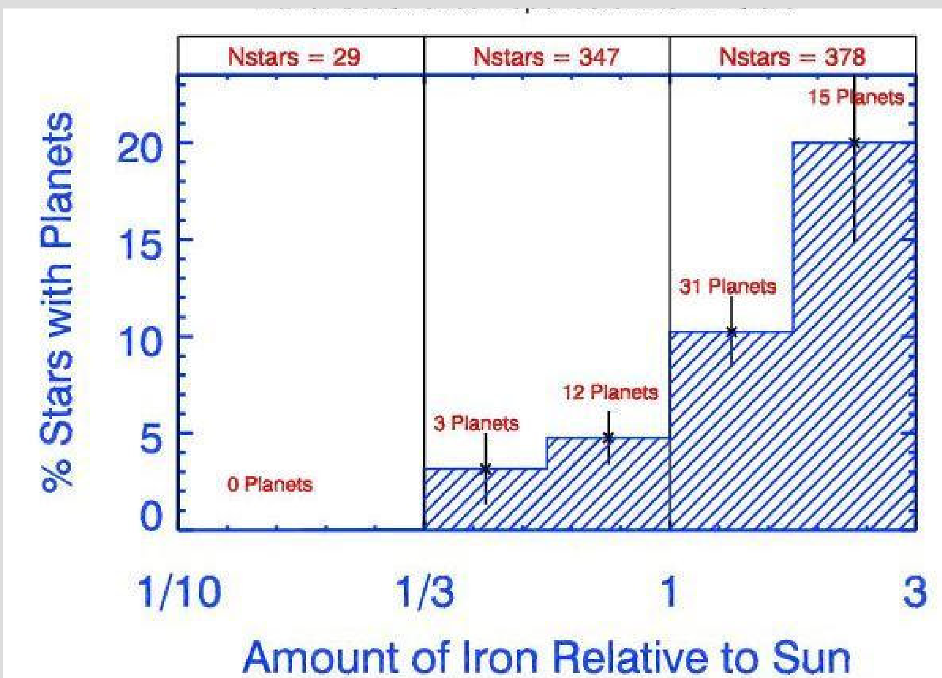
Properties of planetary systems

- all giant planets in the solar system have a > 5 AU while extrasolar giant planets have semi-major axes as small as a = 0.02 AU
- planetary orbital angular momentum is close to direction of Sun's spin angular momentum (within 7^o)
- 3 of 4 terrestrial planets and 3 of 4 giant planets have obliquities (angle between spin and orbital angular momentum) < 30^o
- interplanetary space is virtually empty, except for the asteroid belt and the Kuiper belt
- planets account for < 0.2% of mass of solar system but > 98% of angular momentum

Properties of planetary systems

- orbits of major planets in solar system are nearly circular ($e_{Mercury}$=0.206, e_{Pluto}=0.250); orbits of extrasolar planets are *not* (e_{median}=0.28)
- probability of finding a planet is proportional to mass of metals in the star





Properties of planetary systems

- planets suffer no close encounters and are spaced fairly regularly (Bode's law: a_n=0.4 + 0.3$\times$2^n)

planet	semimajor axis (AU)	n	a_n (AU)
Mercury	0.39	$-\infty$	0.4
Venus	0.72	0	0.7
Earth	1.00	1	1.0
Mars	1.52	2	1.6
asteroids	2.77 (Ceres)	3	2.8
Jupiter	5.20	4	5.2
Saturn	9.56	5	10.0
Uranus	19.29	6	19.6
Neptune	30.27	7	38.8
Pluto	39.68	8	77.2

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*predicted

+exceptions

Properties of planetary systems

- Oort cloud:
  - ~10^{12} comets of 1 km or larger
  - radii >10^4 AU
  - approximately spherical
  - source of long-period comets (P > 200 yr) and short-period comets (200 yr > P > 20 yr)
- Kuiper belt
  - ~10^9 comets
  - radii > 35 AU
  - flattened disk
  - source of Jupiter-family comets (P < 20 yr)

Properties of planetary systems

- most planets have satellites

planet	number	M_{max}/M_{planet}
Earth	1	0.012
Mars	2	1.7×10^{-8}
Jupiter	61	7.8×10^{-5}
Saturn	31	2.4×10^{-4}
Uranus	27	4.1×10^{-5}
Neptune	13	2.1×10^{-4}
Pluto	3	0.15

Properties of planetary systems

- solid planetary and satellite surfaces are heavily cratered; cratering rate must have been far greater in first 10^9 yr of solar system history than it is now ("late heavy bombardment")
- age of solar system is $4.56 \pm 0.02 \times 10^9$ yr
- "terrestrial" planets (Mercury, Venus, Earth, Mars) are composed of rocky, refractory (high condensation temperature) material
- "giant" planets (Jupiter, Saturn) composed mostly of H and He but are enriched in metals and appear to have rock-ice core of 10-20 Earth masses
- "intermediate" or "ice" planets (Uranus and Neptune) also have cores but are only 5-20% H and He (*not* "terrestrial")
- gas disks around young stars dissipate in 10^6 – 10^7 yr

What is a planet?

Version 1:

- main-sequence stars burn hydrogen ($M>0.08\ M_\odot=80\ M_{Jupiter}$)
- brown dwarfs have masses too low to burn hydrogen but large enough to burn deuterium ($80\ M_{Jupiter}<M<13\ M_{Jupiter}$)
- planets have masses $< 13\ M_{Jupiter}$
- Good points: mass is easy to measure; maximum mass of close companions to stars is around 15 $M_{Jupiter}$ (brown-dwarf desert)
- Bad points: deuterium burning has no fundamental relation to the formation or properties of a planet

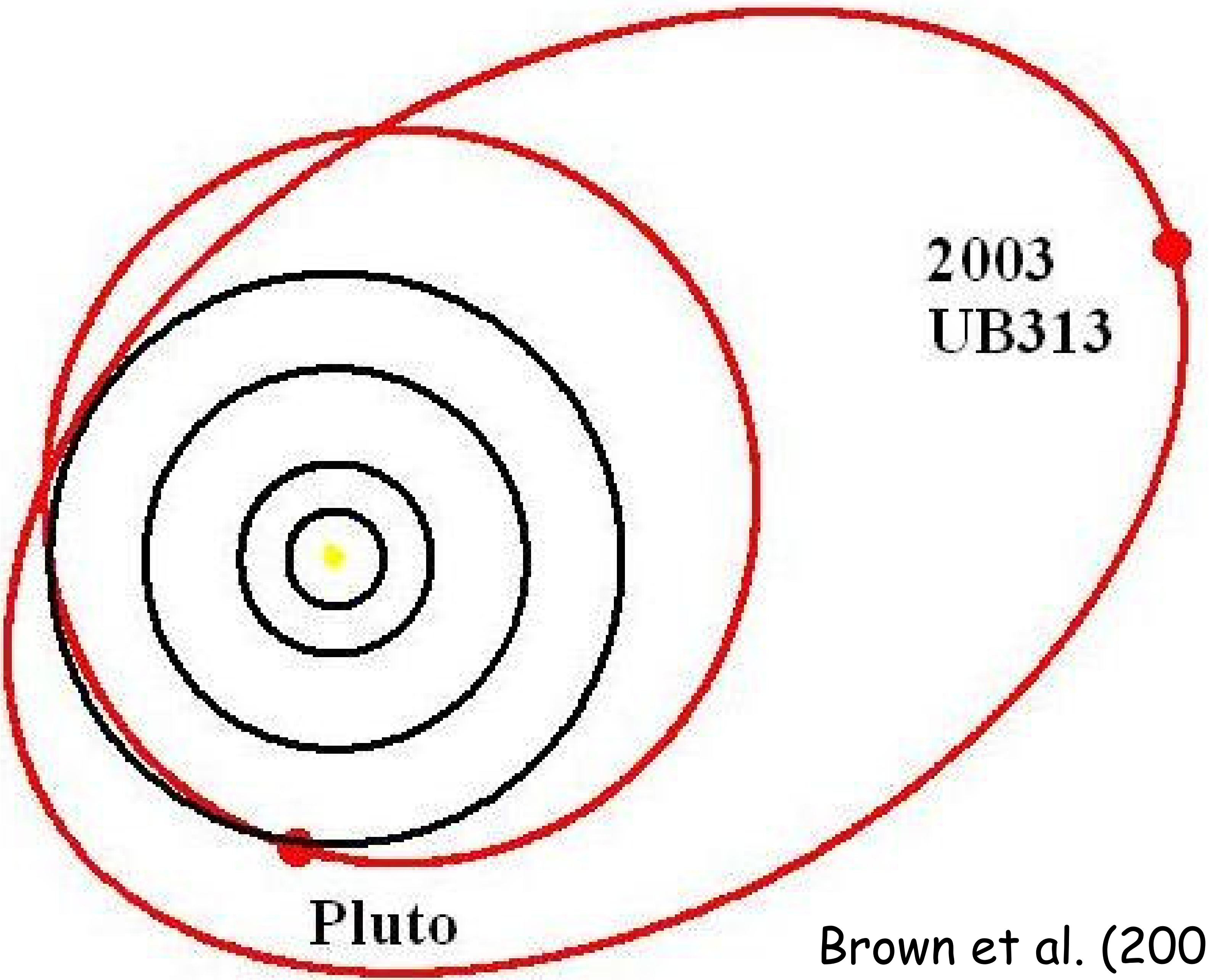
What is a planet?

Version 2:

- planets are objects similar to the planets in our own solar system
- Bad points: is a Jupiter-mass object at a=0.02 AU a planet? is Pluto a planet? Is our solar system special?

Version 3:

- anything formed in a disk around a star is a planet
- Bad points: figuring out how something is formed is really hard, and what do we call them until we do?



Brown et al. (2005)

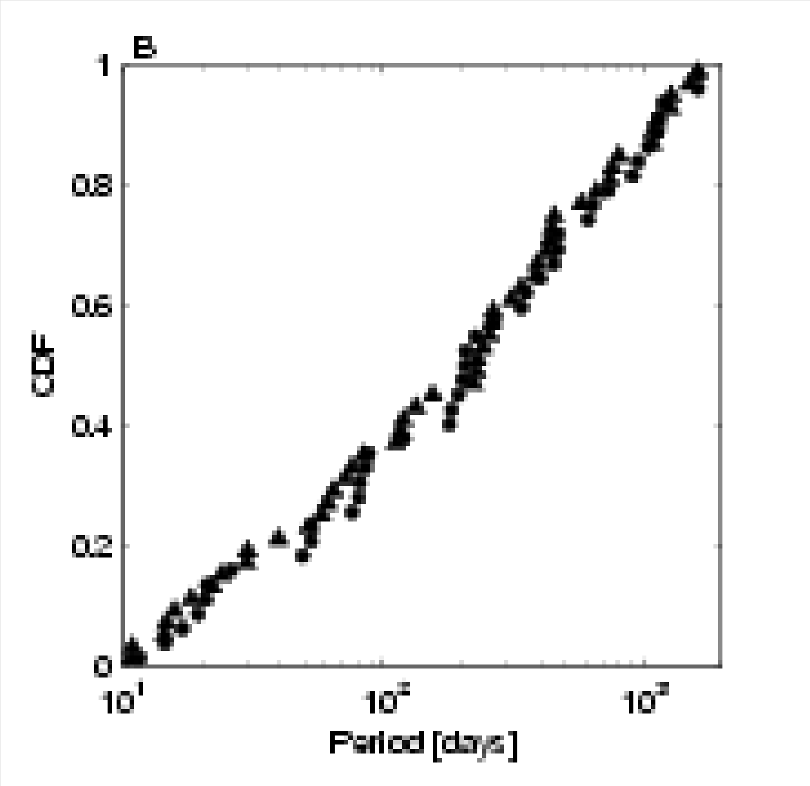
The encounter hypothesis

- Close encounter with a passing star rips material off the Sun that spreads into a long filament and condenses into planets (Buffon 1745, Jeans 1928, Jeffreys 1929)
- Problems: **Prove!**
  - very rare event: needs impact parameter < 2 $R_\odot$ so only happens to 1 in 10^8 stars
  - specific angular momentum of order $(GM_\odot R_\odot)^{1/2}$ not $(GM_\odot a_J)^{1/2}$; factor 30 too small (Russell 1935) (not a problem for some extrasolar planets!)
  - 1 Jupiter mass of material requires digging to R ~ 0.1 $R_\odot$ where temperature ~5×10^5 K and resulting blob will have positive energy, and cooling time ~ 10^{10} sec. Blob expands adiabatically and disperses (Spitzer 1939)
  - where did Jupiter's deuterium come from?

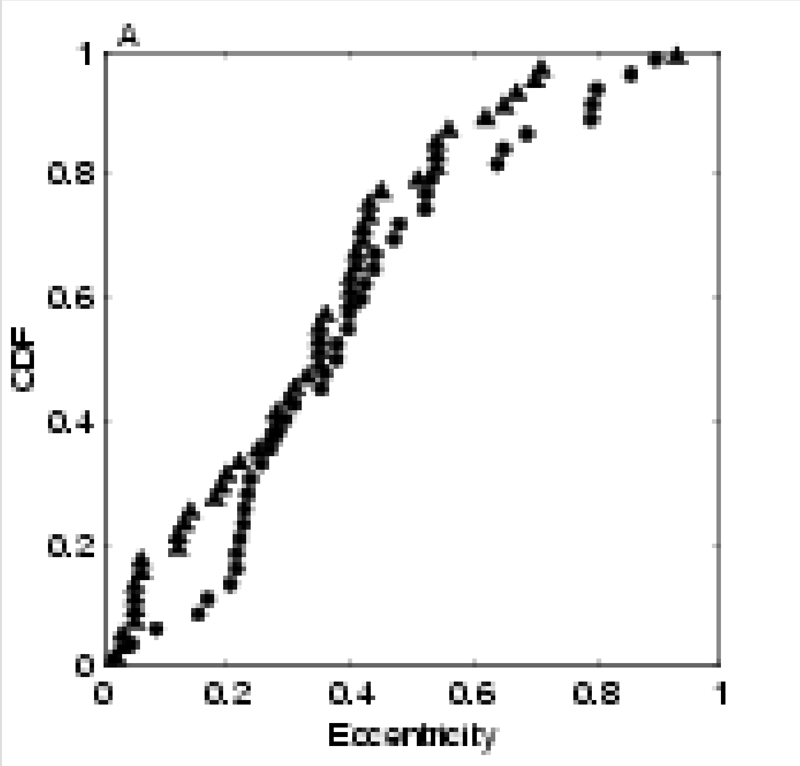
The brown-dwarf hypothesis

- extrasolar "planets" are simply very low-mass stars that form from collapse of multiple condensations in protostellar clouds
- distribution of eccentricities and periods of extrasolar planets very similar to distributions for binary stars

Cumulative distribution functions in period and eccentricity for extrasolar planets and low-mass companions of spectroscopic binaries



period



eccentricity

from Zucker & Mazeh (2001)

The brown dwarf hypothesis

- extrasolar “planets” are simply very low-mass stars that form from collapse of multiple condensations in protostellar clouds
- distribution of eccentricities and periods of extrasolar planets very similar to distributions for binary stars
- but:
  - why is there a brown-dwarf desert?
  - how did planets in solar system get onto circular, coplanar orbits?
  - how do you make planets with solid cores, or terrestrial planets?

The nebular hypothesis

- the Sun and planets formed together out of a rotating cloud of gas (the "solar nebula")
- gravitational instabilities in the gas disk condense into planets (Kant 1755)
- Good points: variations might work to form Jupiter, Saturn, extrasolar gas giants
- Bad points: how do you make Uranus, Neptune, terrestrial planets?

The planetesimal (Safronov) hypothesis

- forming Sun is surrounded by a gas disk (like nebular hypothesis)
- planets form by multi-stage process:

1. as the disk cools, rock and ice grains condense out and settle to the midplane of the disk – chemistry and gas drag are dominant processes
2. small solid bodies grow from the thin dust layer to form km-sized bodies ("planetesimals") - gas drag, gravity and chemical bonding are dominant processes
3. planetesimals collide and grow – gravitational scattering and solar gravity are dominant processes. "Molecular chaos" applies and evolution is described by statistical mechanics

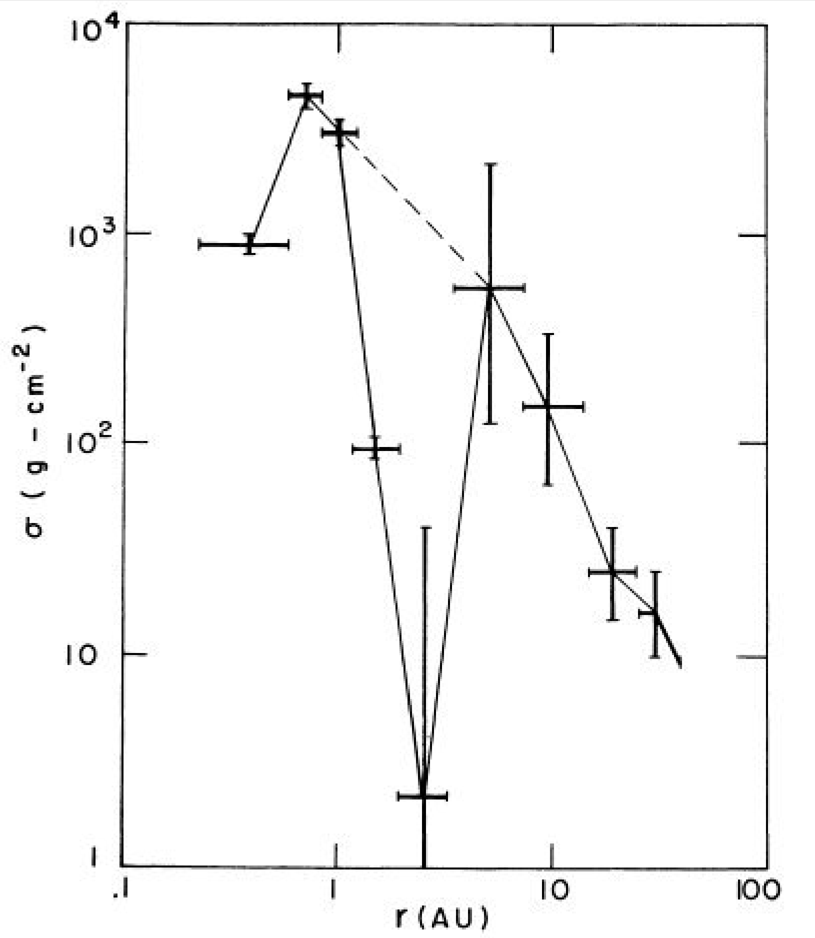
The planetesimal (Safronov) hypothesis

planets form by multi-stage process:

1. rock and ice grains condense out and settle
2. formation of km-sized planetesimals
3. planetesimals collide and grow
4. a few planetesimals grow large enough to dominate evolution. Orbits become regular or weakly chaotic and are described by celestial mechanics rather than statistical mechanics ("planetary embryos")
5. on much slower timescales, planetary embryos collide and grow into "planetary cores"
6. cores of intermediate and giant planets accrete gas envelopes

requires growth by 45 orders of magnitude in mass through ~6 different physical processes!

Minimum solar nebula



- add volatile elements to each planet to augment them to solar composition
- spread each planet into an annulus reaching halfway to the next planet
- smooth the resulting surface density:

$$\Sigma(r) \approx 3 \times 10^3 \text{ g cm}^{-2} (1 \text{ AU}/r)^{1.5}$$

Prove!

Minimum solar nebula

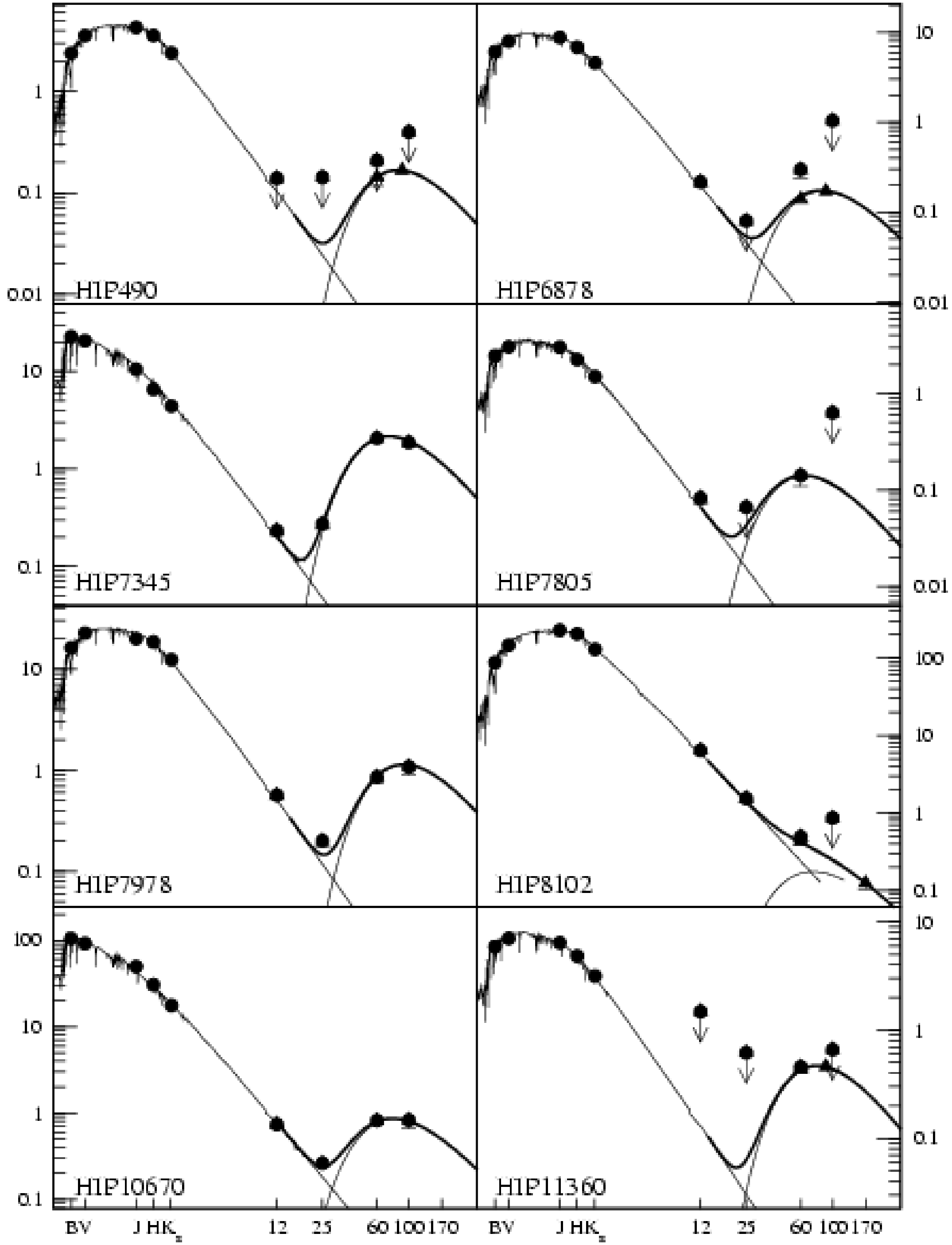
$$\Sigma(r) \approx 3 \times 10^3 \text{ g cm}^{-2} (1 \text{ AU}/r)^{1.5}$$

- assume 0.5% metals and divide into $r = 0.1\ \mu$ dust particles with density $\rho = 3$ g cm^{-3}
- geometric optical depth is

Prove!

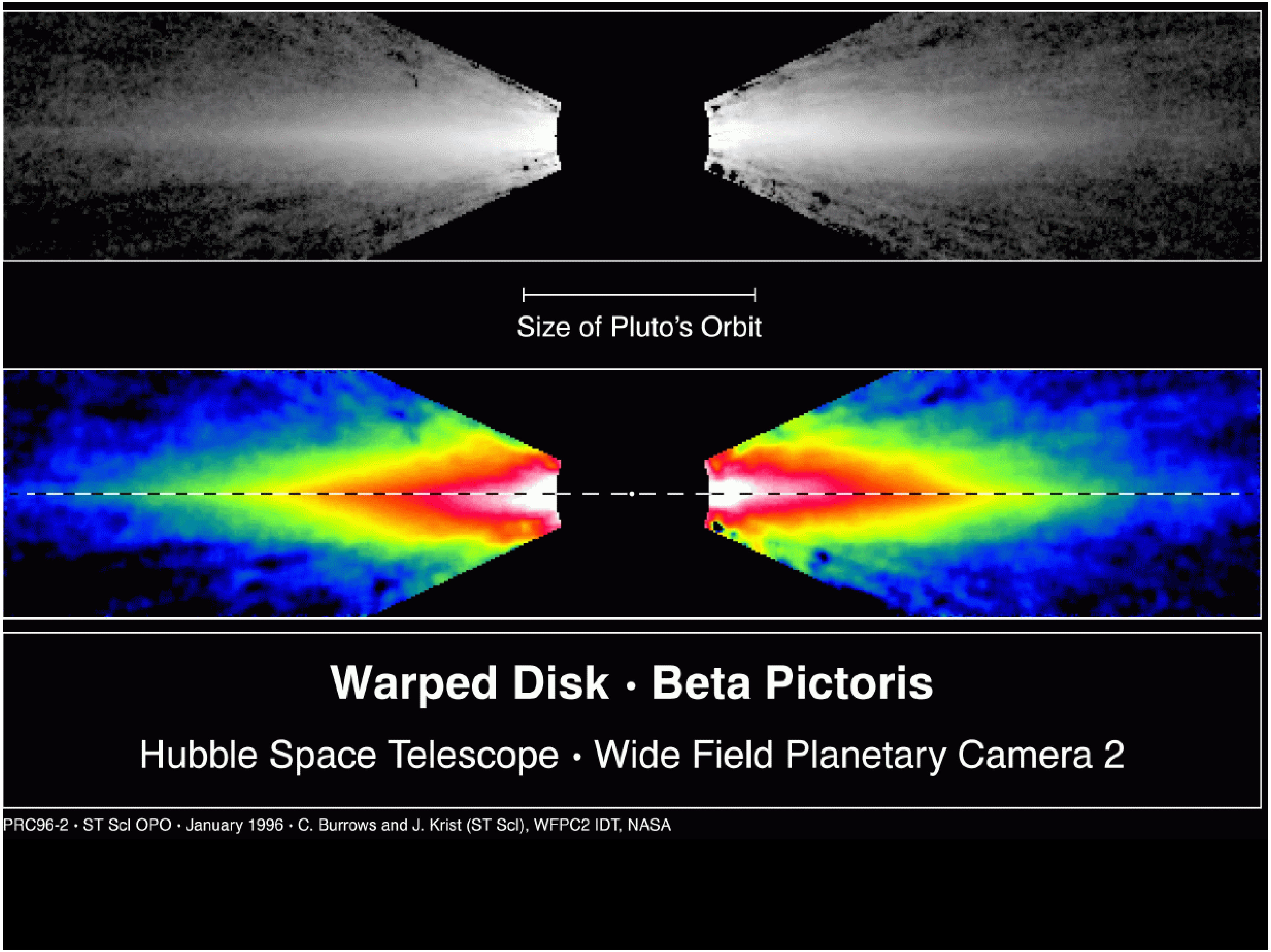
$$\tau \approx 4 \times 10^5 \ (1 \text{ AU}/r)^{1.5}$$

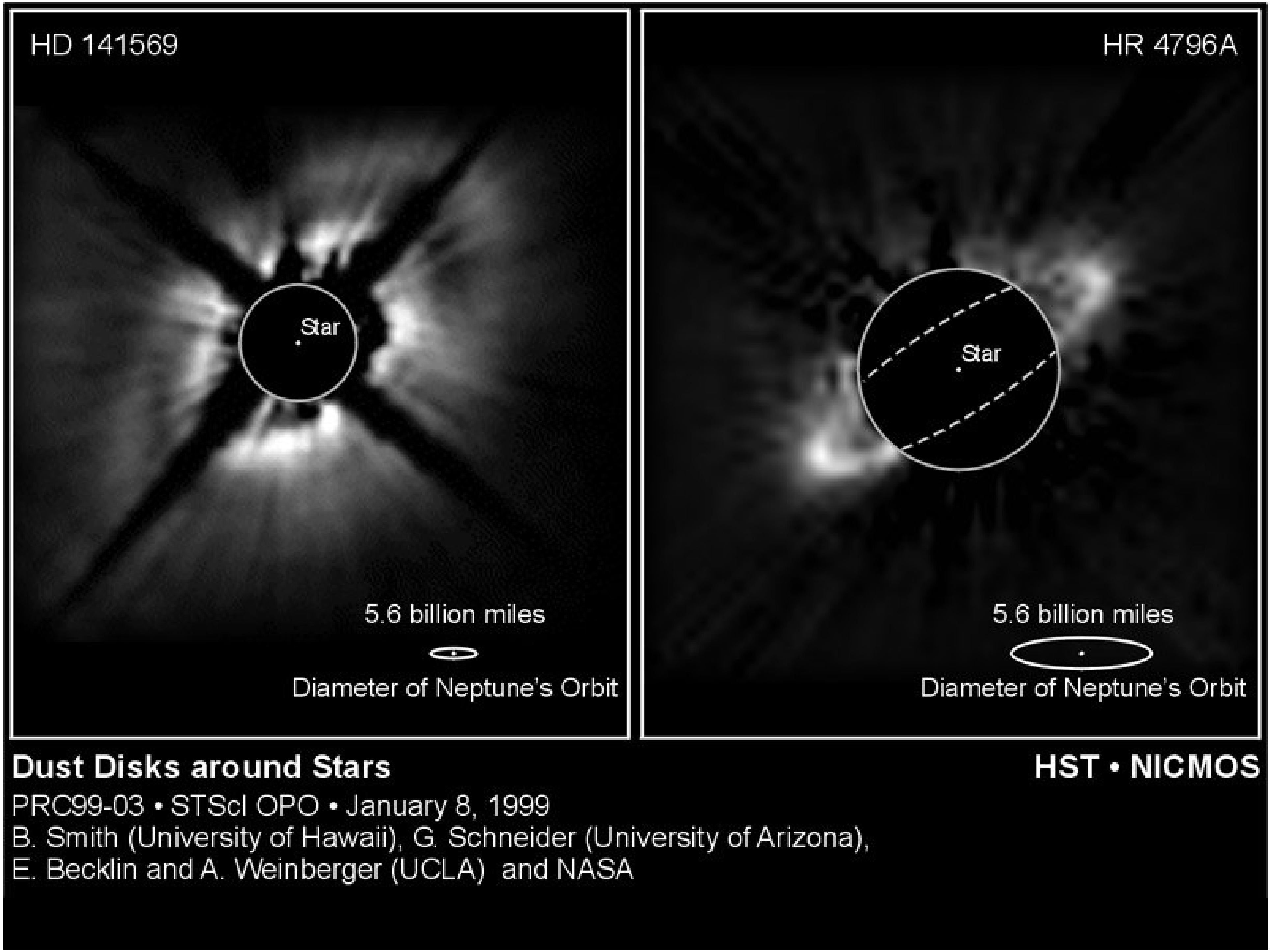
i.e. *disk is opaque to very large distances*

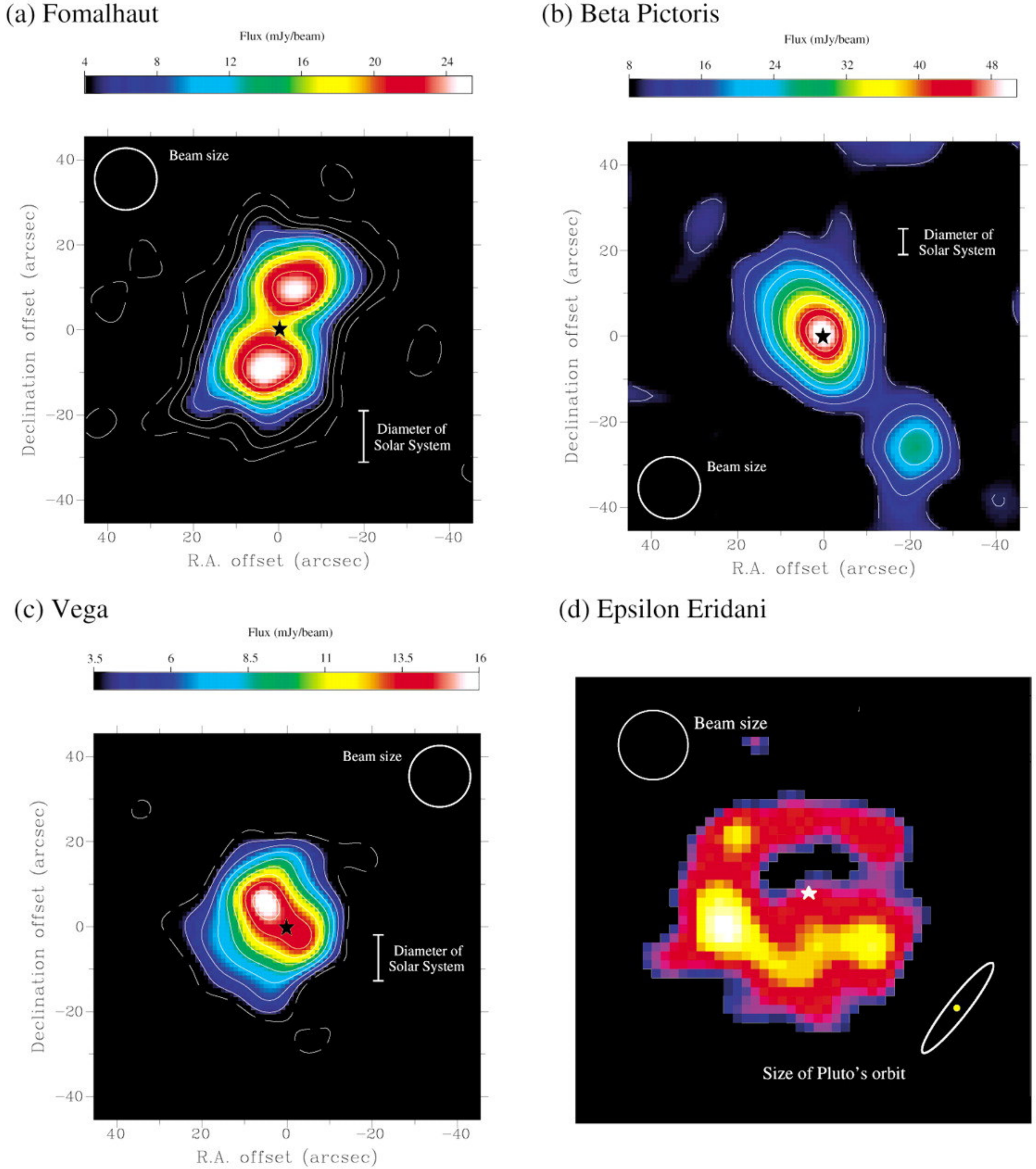


the "Vega phenomenon"

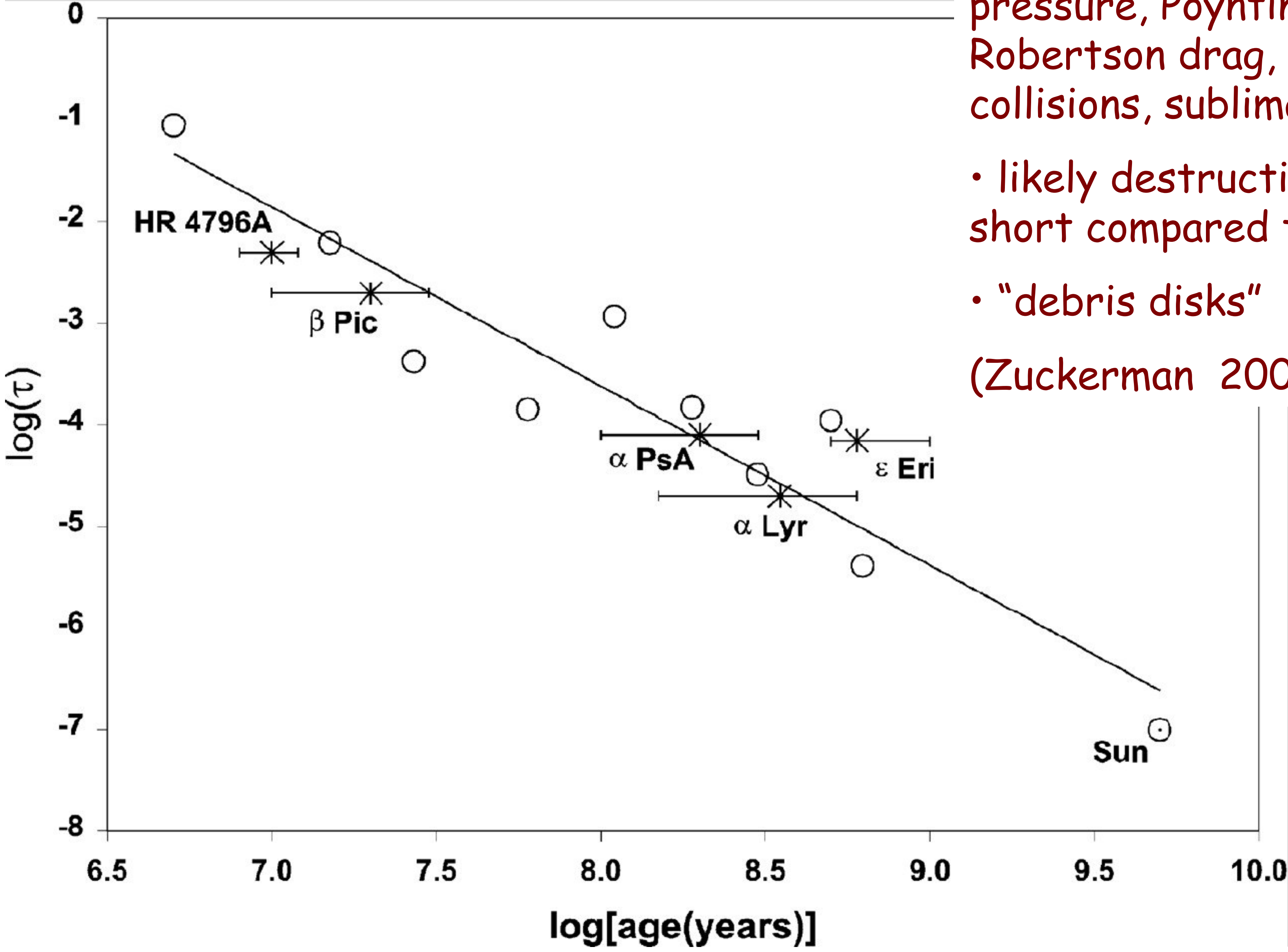
(Zuckerman & Song 2003)


Size of Pluto's Orbit
Warped Disk • Beta Pictoris
Hubble Space Telescope • Wide Field Planetary Camera 2
PRC96-2 • ST ScI OPO • January 1996 • C. Burrows and J. Krist (ST ScI), WFPC2 IDT, NASA


HD 141569
Star
5.6 billion miles
Diameter of Neptune's Orbit
HR 4796A
Star
5.6 billion miles
Diameter of Neptune's Orbit
Dust Disks around Stars
HST • NICMOS
PRC99-03 • STScI OPO • January 8, 1999
B. Smith (University of Hawaii), G. Schneider (University of Arizona),
E. Becklin and A. Weinberger (UCLA) and NASA



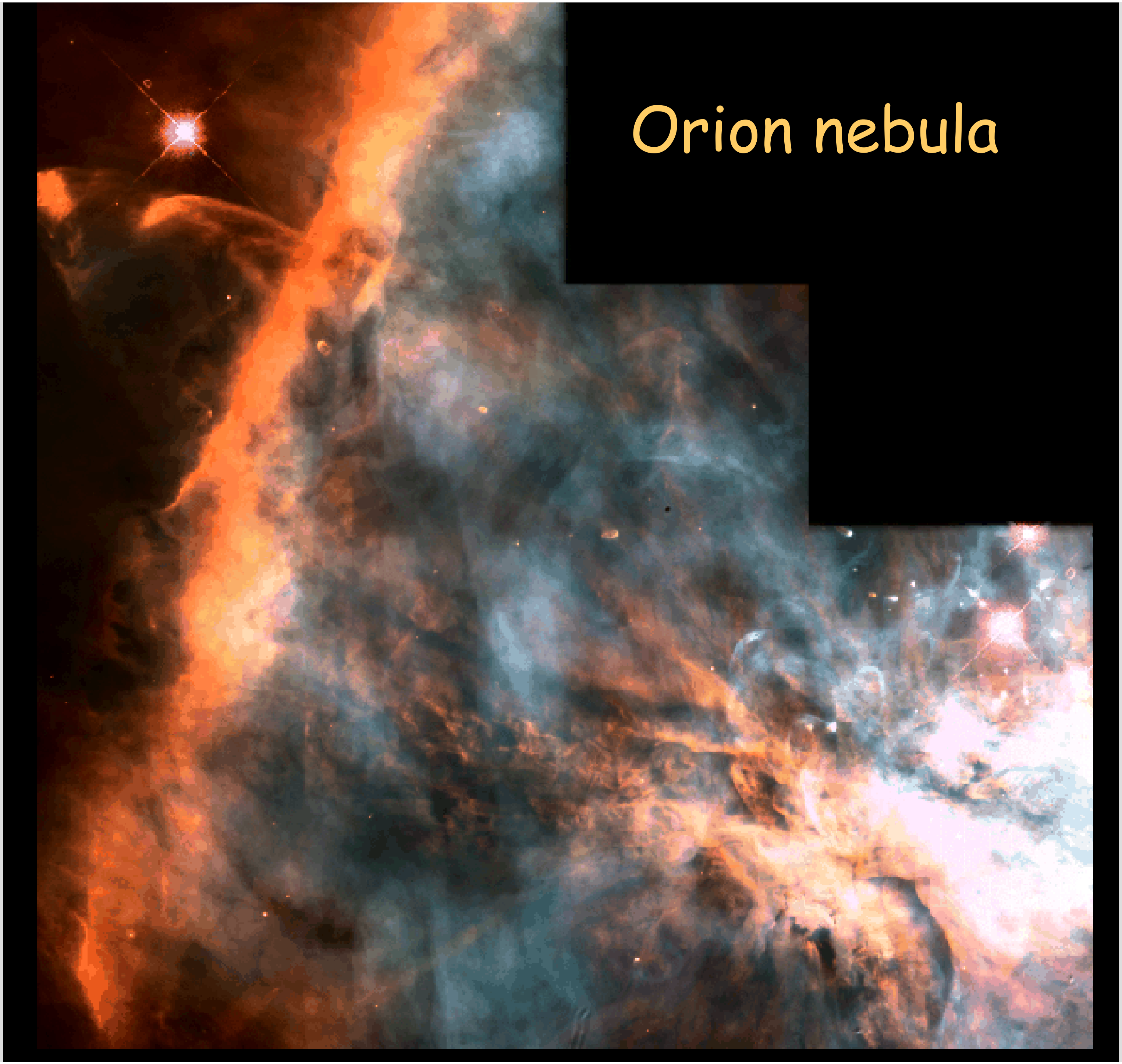
dust emission at 850 μ from SCUBA on JCMT. From Zuckerman (2001)

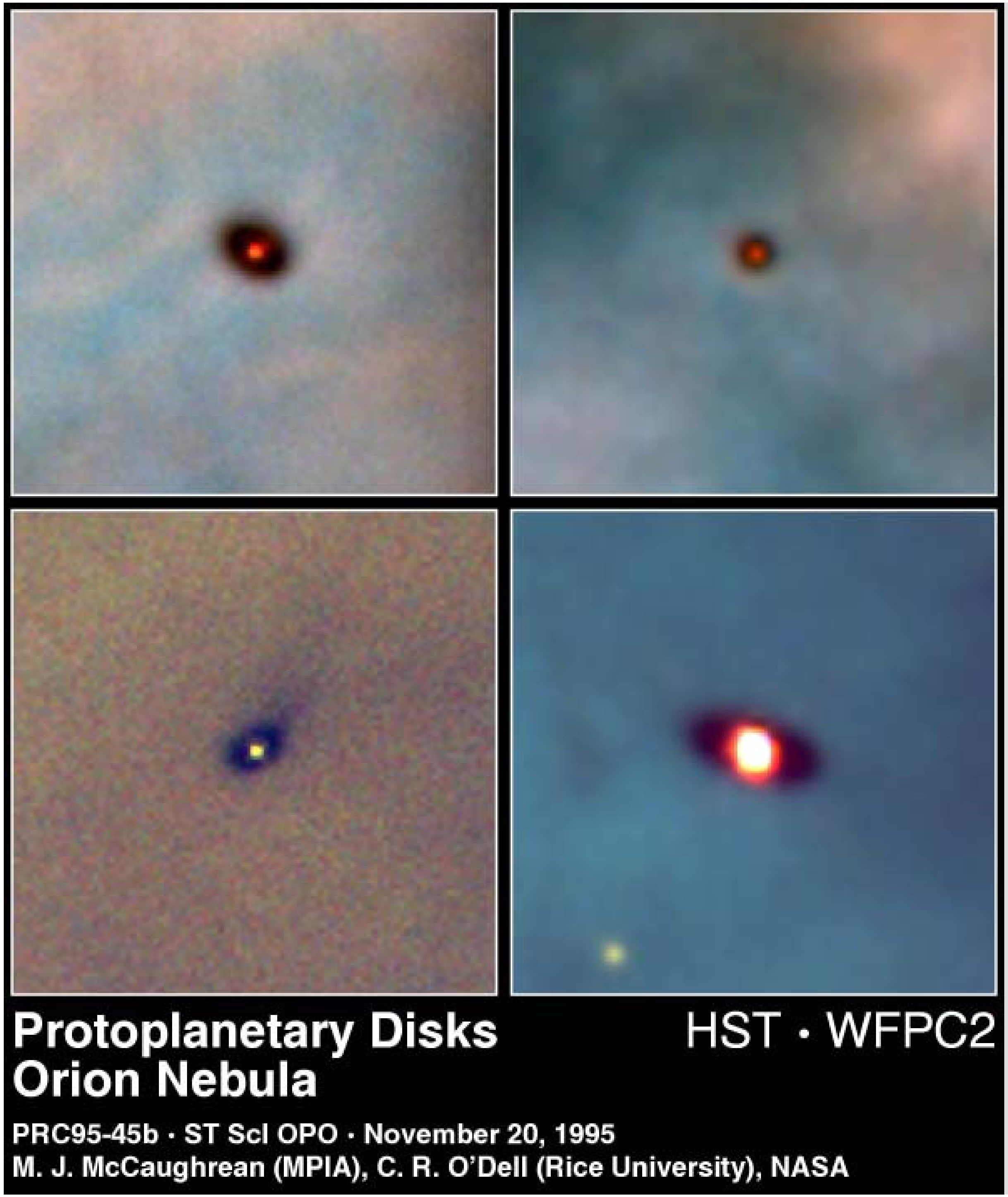


- destruction mechanisms include radiation pressure, Poynting-Robertson drag, collisions, sublimation
- likely destruction times short compared to age
- "debris disks"

(Zuckerman 2001)

Orion nebula





PROtoPLanetarY DiskS
= "proplyds"

Minimum solar nebula

$$\Sigma(R) \approx 3 \times 10^3 \text{ g cm}^{-2} \left(\frac{1\text{ AU}}{R}\right)^{1.5}.$$

Assume vertical distribution is isothermal,

$$\rho(z) \propto \exp\left(-\frac{\Phi}{kT}\right), \qquad \text{where} \qquad \Phi = -\frac{\mu m_p G M_\odot}{(R^2+z^2)^{1/2}} \simeq \text{const} + \frac{\mu m_p G M_\odot}{2R^3} z^2$$

where $\mu = 2$ for hydrogen molecules. Thus

$$\rho(z, R) = \rho_0(R) \exp\left[-\frac{z^2}{2h^2(R)}\right],$$

where

$$h^2 = \frac{kTR^3}{\mu m_p G M_\odot} \qquad \text{or} \qquad h = 7.20 \times 10^{11}\text{ cm} \left(\frac{T}{500\text{ K}}\right)^{1/2} \left(\frac{R}{1\text{ AU}}\right)^{3/2} \left(\frac{2}{\mu}\right)^{1/2}.$$

Prove!

Minimum solar nebula

$$\frac{h}{R} = 0.048 \left(\frac{T}{500\,\mathrm{K}}\right)^{1/2} \left(\frac{R}{1\,\mathrm{AU}}\right)^{1/2} \left(\frac{2}{\mu}\right)^{1/2}$$

$$\rho_0(R) = 1.7 \times 10^{-9}\mathrm{g\ cm^{-3}} \left(\frac{500\,\mathrm{K}}{T}\right)^{1/2} \left(\frac{1\,\mathrm{AU}}{R}\right)^{3}.$$

Sound speed : $c = \left(\frac{\gamma k T}{\mu m_p}\right)^{1/2} = 1.70\,\mathrm{km\,s^{-1}} \left(\frac{T}{500\,\mathrm{K}}\right)^{1/2} \left(\frac{2}{\mu}\right)^{1/2}.$

Angular speed: $\Omega = \left(\frac{GM_\odot}{R^3}\right)^{1/2} = \frac{2\pi}{1\,\mathrm{yr}}.$

Mean free path: $\lambda \approx \frac{\mu m_p}{\pi r^2 \rho} \sim 10\,\mathrm{cm}.$ **Prove!**

Stability of the minimum solar nebula

Consider a disk with surface density Σ, angular speed Ω, and sound speed c, and examine a small patch of size L.

- mass is $M \sim \Sigma L^2$
- gravitational potential energy is $E_G \sim -GM^2/L \sim -G\Sigma^2 L^3$
- energy in rotational motion is $E_R \sim M(\Omega L)^2 \sim \Sigma\Omega^2 L^4$
- internal energy is $E_P \sim Mc^2 \sim \Sigma L^2 c^2$
- stable if $E_G + E_R + E_P > 0$ or

$$-G\Sigma^2 L^3 + \Sigma\Omega^2 L^4 + \Sigma L^2 c^2 > 0, \text{ or}$$

$$-G\Sigma L + \Omega^2 L^2 + c^2 > 0$$

for all L. The quadratic function on the left reaches its minimum at $L = G\sigma/2\Omega^2$, and this is positive if

$$2c\Omega/G\Sigma > 1.$$

Prove!

Accurate calculations show that gravitational stability requires that Toomre's parameter

$$Q = \frac{c\Omega}{\pi G\Sigma} > 1$$

The nebular hypothesis revisited

For standard parameters at 1 AU, Q= 170 **Prove!**

Minimum solar nebula is *very* stable!

This is a big problem for the nebular hypothesis. How to fix it:

- increment surface density by factor 10 above minimum solar nebula
- consider only formation of giant planets at 10 AU, where temperature is lower
- probably Q ~ 1.5 is sufficient for instability

Gravitational instability is just possible for extreme parameters; nebular hypothesis might work for Jupiter and Saturn and extrasolar gas giants, but not Uranus, Neptune, terrestrial planets

Formation of planetesimals

Dust condenses out of the cooling gaseous disk (iron, silicates, nickel in inner solar system; ammonia and ice in outer solar system)

Maximum growth rate of dust is $dr/dt \sim \alpha\, c\rho_g/\rho_p$ where c is sound speed, α is mass fraction of particulate material in gas phase, $\rho_g \sim 10^{-9}$ g/cm^3 is gas density, $\rho_p \sim 3$ g/cm^3 is particle density. Yields $dr/dt \sim 1$ cm/yr

Dust settles to the midplane of the disk through competition between gravitational force $-m\, d\Phi/dz = -m(GM/R^3)z = -m\Omega^2 z$, and gas drag force $F = -\pi r^2 \rho_g c v_z$, so equating these

$$v_z = \frac{dz}{dt} = -\frac{z}{\tau}$$

where

$$\tau \approx \frac{\rho_g c}{\Omega^2 r \rho_p} \approx \frac{\Sigma}{\Omega r \rho_p} \approx 100 \text{ yr} \left(\frac{1 \text{ cm}}{r}\right)$$

Prove!

Therefore particles grow to ~10 cm in ~10 yr before settling to the midplane of the disk

Formation of planetesimals

Gas disk is supported against solar gravity by pressure and centrifugal force:

$$R\Omega_g^2 = \frac{GM_\odot}{R^2} + \frac{1}{\rho_g}\frac{dp}{dR} \approx \frac{GM_\odot}{R^2} - \frac{c^2}{R} \qquad \text{or} \qquad \Omega_g \approx \Omega - \frac{c^2}{2\Omega R^2}.$$

Thus gas disk rotates about 0.2% *slower* than Keplerian, i.e. there is a "headwind" of about $5 \times 10^3\,\mathrm{cm\,s^{-1}}$ and a drag force per unit mass $\mathbf{F}/m = -X(\mathbf{v} - \mathbf{v}_g)$.

Small particles are entrained in the wind; large particles are unaffected by the wind; intermediate particles spiral inwards.

Formation of planetesimals

Small particles are entrained in the wind; large particles are unaffected by the wind; intermediate particles spiral inwards at a rate

$$\frac{1}{R}\frac{dR}{dt} \simeq -\frac{2X\Omega(\Omega - \Omega_g)}{X^2 + \Omega^2}.$$

Prove!

Maximum inspiral rate at ($X = \Omega$) equals headwind speed. Inspiral time $\sim 100\,\mathrm{yr}$ for $r = 30\,\mathrm{cm}$ particles (!)

"Planetesimals" must form in $< 100\,\mathrm{yr}$.

Formation of planetesimals

There are two competing mechanisms for jumping the meter-size hurdle:

1. **Gravitational instability (the Goldreich-Ward mechanism):**

- as solids settle to the midplane of the gas disk the particulate disk becomes gravitationally unstable when Toomre's parameter

$$Q=c\,\Omega/\pi G\Sigma < 1$$

- Here c, Σ are velocity dispersion and surface density of particles. If solid mass fraction is 0.5%, Σ=15 g cm^{-2} at 1 AU which requires c < 15 cm s^{-1} or thickness $h = c/\Omega < 800$ km

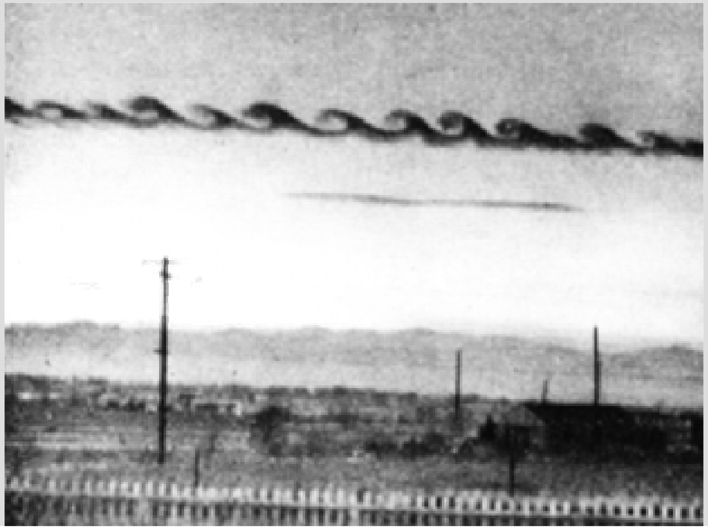
Prove!

- For Q << 1 all wavelengths < $\lambda_c = 4\pi^2 G\Sigma/\Omega^2 = 1 \times 10^9$ cm are unstable. Maximum unstable mass is $M_c = \pi\Sigma(\lambda/2)^2 = 10^{19}$ gm, corresponding to radius of 10 km

Formation of planetesimals

the Goldreich-Ward mechanism, continued:

- gas disk rotates slower than Keplerian by about 0.2%. This leads to strong shear at the surface of the particulate disk
- shear induces Kelvin-Helmholtz instability which leads to turbulent velocities of order $v - v_g \sim c^2/\Omega R \sim 5 \times 10^3$ cm s^{-1} which gives Q > 100 and suppresses gravitational instability
- possibly K-H instability can be suppressed if solid/gas surface density ratio enhanced by factors of 2-10 (Youdin & Shu 2002)



Formation of planetesimals

2. Sticky collisions

- particle velocities are turbulent ($v \sim 5 \times 10^3$ cm s^{-1}) but collisions lead to sticking
- characteristic growth time $r\rho_p / \Sigma_p \Omega \sim 3$ yr **Prove!**
- but:
  - rocks don't stick when they collide!
  - icy bodies fracture at these high speeds
  - largest inclusions in meteorites are a few cm

3. Other instabilities?

- Youdin & Goodman (2005)

Formation of planets

- once the meter hurdle is jumped, gas drag becomes unimportant
- further growth occurs through collisions.

What is the collision cross-section between a test particle and a body of mass m and radius r? Without gravity,

$$\sigma=\pi r^2$$

With gravity,

Prove!

$$\sigma=\pi r^2(1+\Theta) \quad \Theta = 2Gm/rv^2 = v_{escape}^2/v^2$$

here Θ is the Safronov number. The cross-section is enhanced by $1+\Theta$ through gravitational focusing.

When $\Theta \gg 1$ growth is very fast because

- gravitational focusing enhances the cross-sections
- collision debris doesn't have to stick

Formation of planets

Rate of mass growth is

$$dm/dt = \pi\, r^2\, \rho\, v(1+\Theta)$$

But $\rho \sim \Sigma/h$ where h is disk thickness, and $h \sim v/\Omega$:

$$dm/dt \sim \pi\, r^2 \Sigma\Omega\, (1+\Theta)$$

and since $m=4\pi\rho_p\, r^3/3$,

$$dr/dt \sim \Sigma\Omega/\rho_p\, (1+\Theta)$$

Prove!

1. Orderly growth:

All growing planetesimals have similar mass and velocity dispersion. Then we expect $\Theta \sim 1$ since near-misses are about as common as collisions

For minimum solar nebula

$$dr/dt \sim 20\ \text{cm/yr}\ (1\ \text{AU}/R)^3$$

Prove!

Needs 10^7 yr to form Earth, $>10^9$ yr to form Jupiter, even longer for Uranus and Neptune

Formation of planets

2. Runaway growth

A few bodies grow much faster than the others. Then

$dr/dt \sim \Sigma\Omega/\rho_p (1+\Theta) \sim (\Sigma\Omega/\rho_p)(1+2Gm/rv^2)$

so for the most massive particles

$dr/dt \sim \Sigma\Omega\, Gr^2/v^2$

so growth of massive bodies runs away (formally, they reach infinite mass in finite time)

Needs 10^7 yr to form Jupiter, longer for Uranus and Neptune

Planet migration

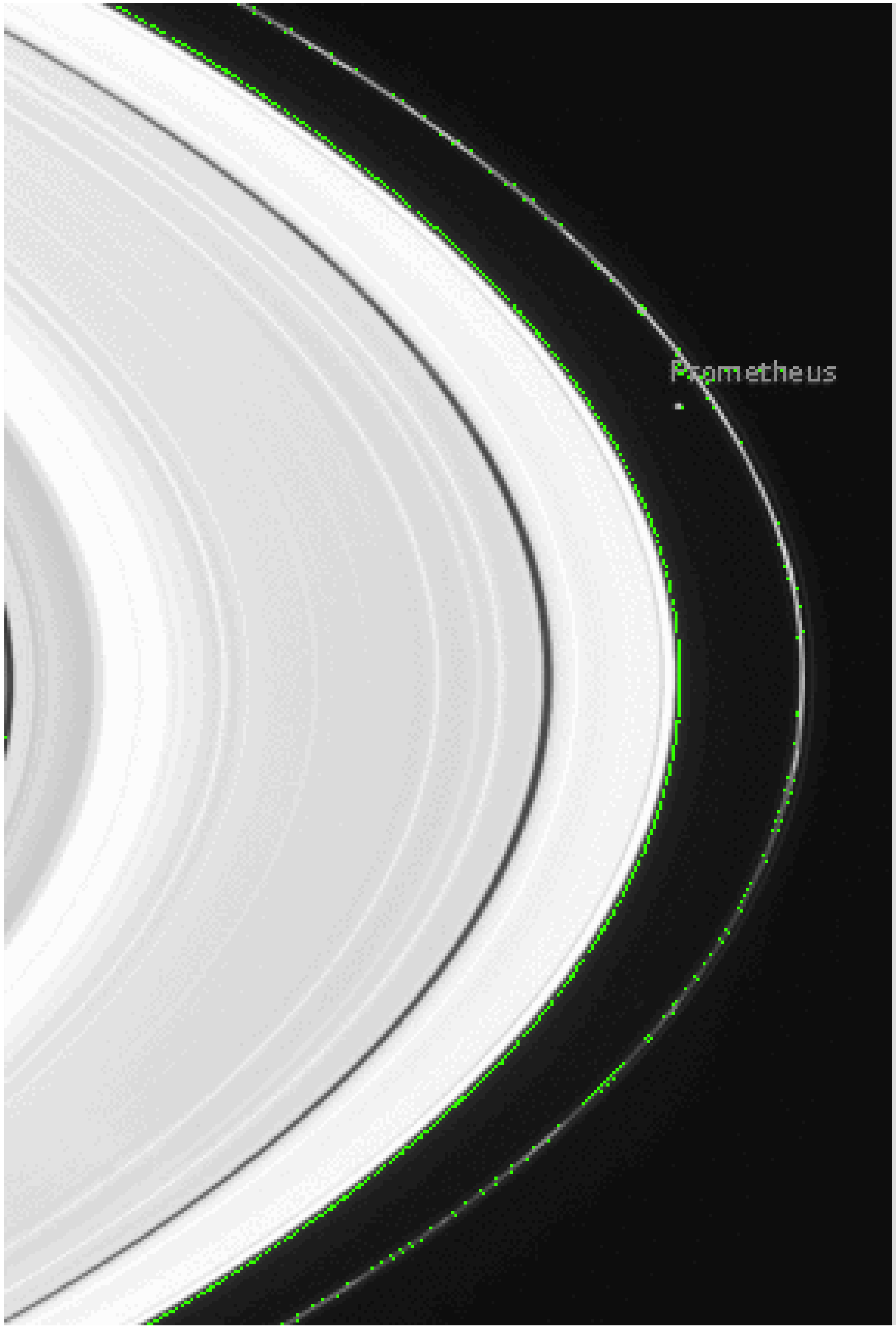
Temperature in disk $\propto 1/r^{1/2}$. At $r < 0.1$ AU no elements condense so planetesimals cannot form. So why are there planets there?

Gravitational interactions between a planet and the surrounding gas disk leads to repulsive torques between them.

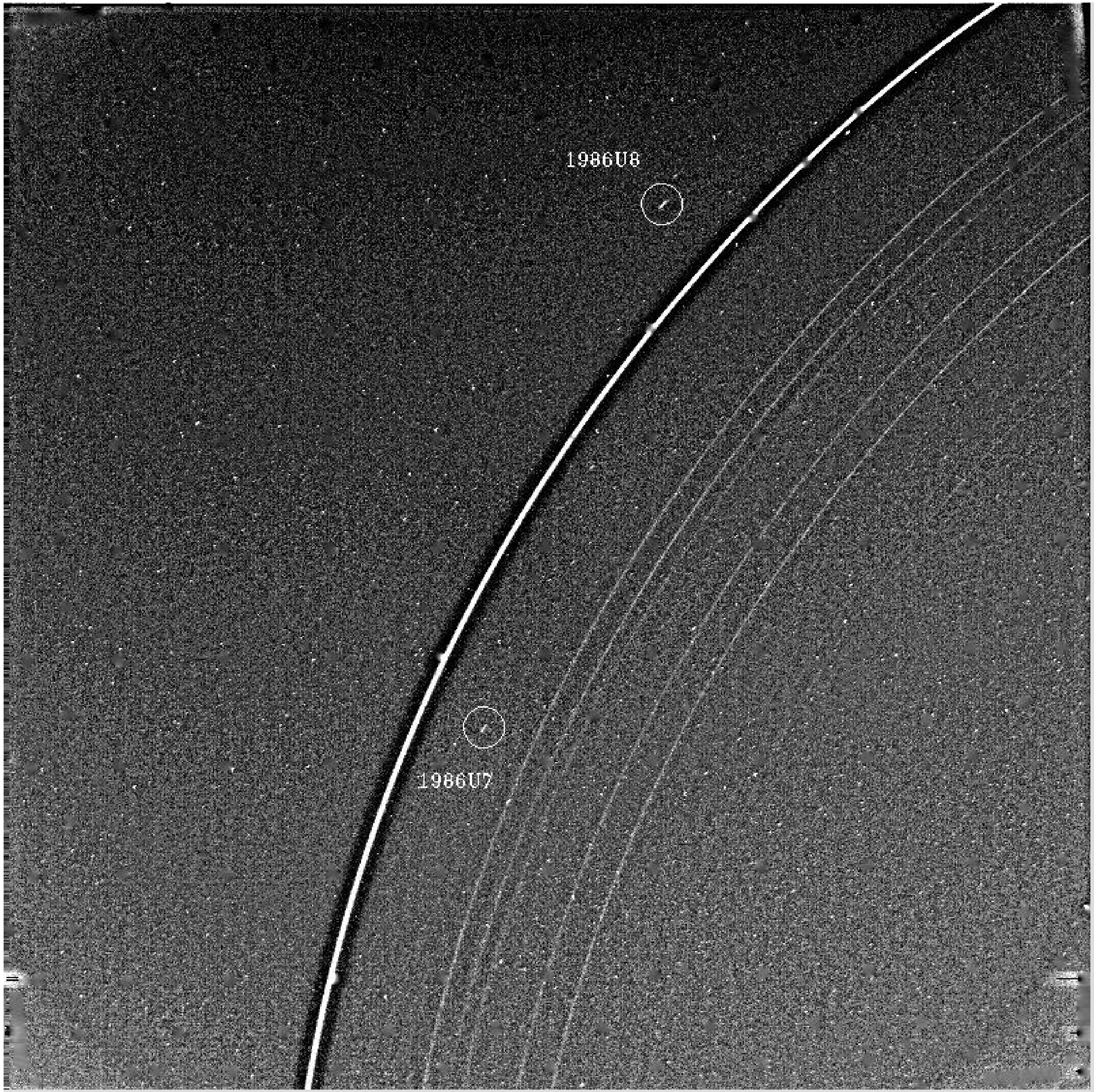
The torque depends only on the surface density of the disk, not viscosity, pressure, self-gravity, etc.

Imbalance between inner and outer torques leads to:

- migration, usually inward
- gap formation



Repulsive torques can "shepherd" narrow rings and open gaps in wide rings



Cordelia and Ophelia at Uranus

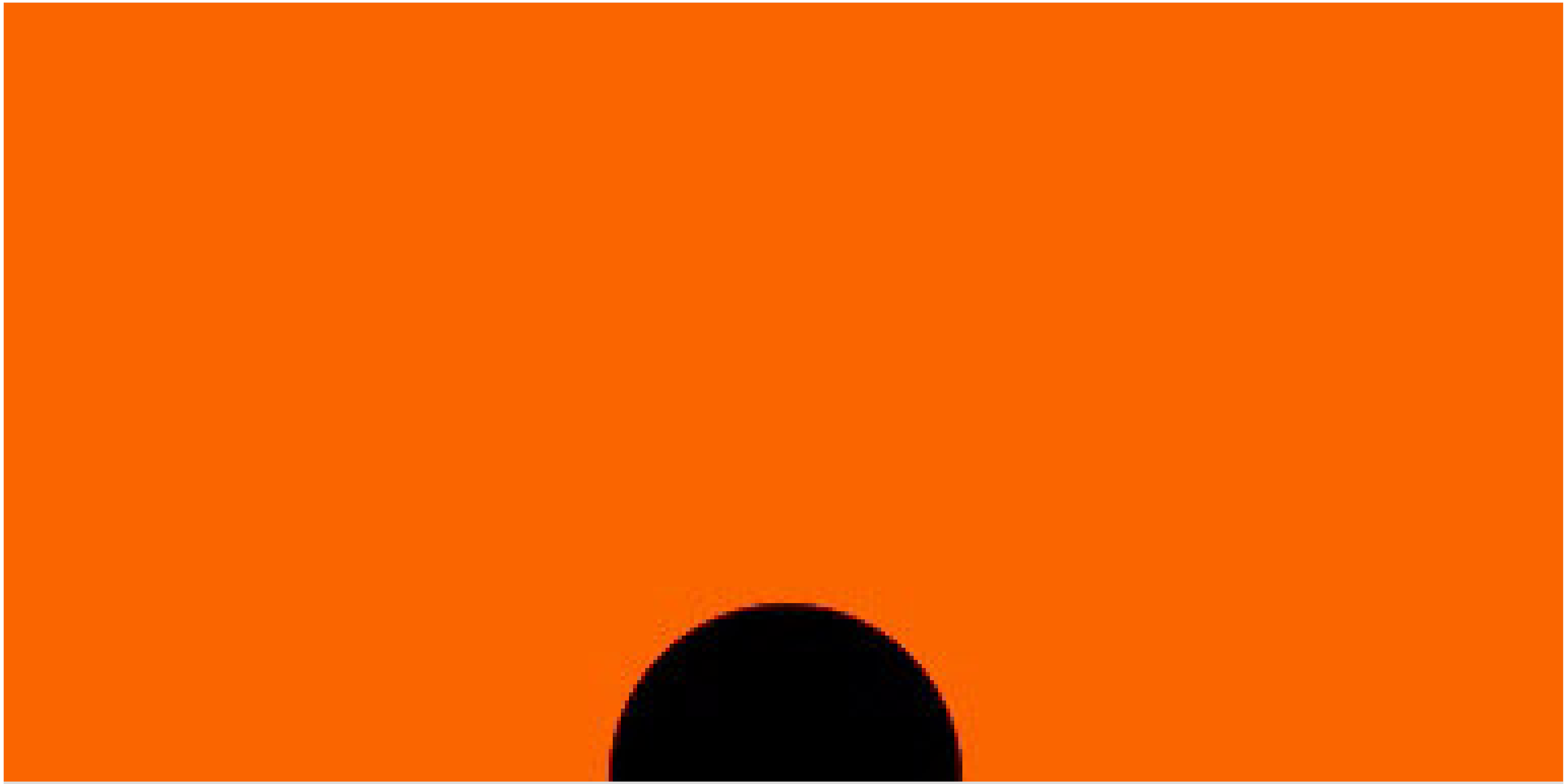
Types of migration

Type I: low mass planet only weakly perturbs the disk

- timescale of order Ω^{-1} $(\Sigma R^2/M_\odot)(M_p/M_\odot)$
- very rapid, ~ 10^4 years for Jupiter in minimum solar nebula
- usually inward

Type II: bigger planet opens a gap in the disk

- planet evolves with the disk on the disk's viscous evolution timescale (acts like a disk particle)
- probably ~ 10^3 - 10^5 yr timescale
- usually inward



from Masset (2002)

Migration

- migration from larger radii offers a plausible way to form giant planets at small radii, but:
  - why did the migration stop?
  - why are the planetary semimajor axes distributed over a wide range?
  - why did migration not occur in the solar system?
- outward migration by Uranus and Neptune helps to solve the timescale problem

Planet formation can be divided into two phases:

Phase 1

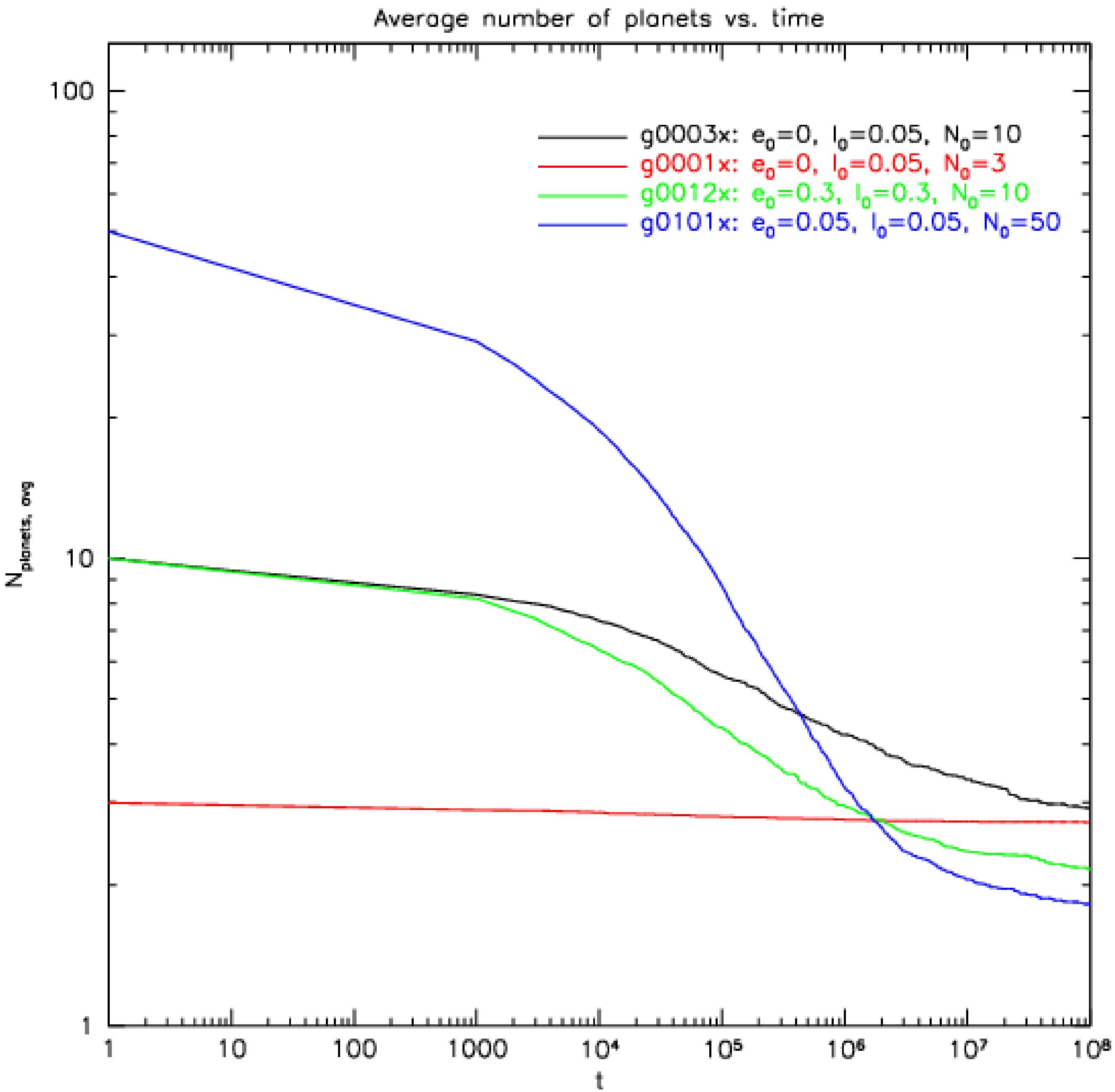
- protoplanetary gas disk → dust disk → planetesimals → planets
- solid bodies grow in mass by 45 orders of magnitude through at least 6 different processes
- lasts 0.01% of lifetime
- involves very complicated physics (gas, dust, turbulence, etc.)

Phase 2

- subsequent dynamical evolution of planets due to gravity
- lasts 99.99% of lifetime
- involves very simple physics (only gravity)

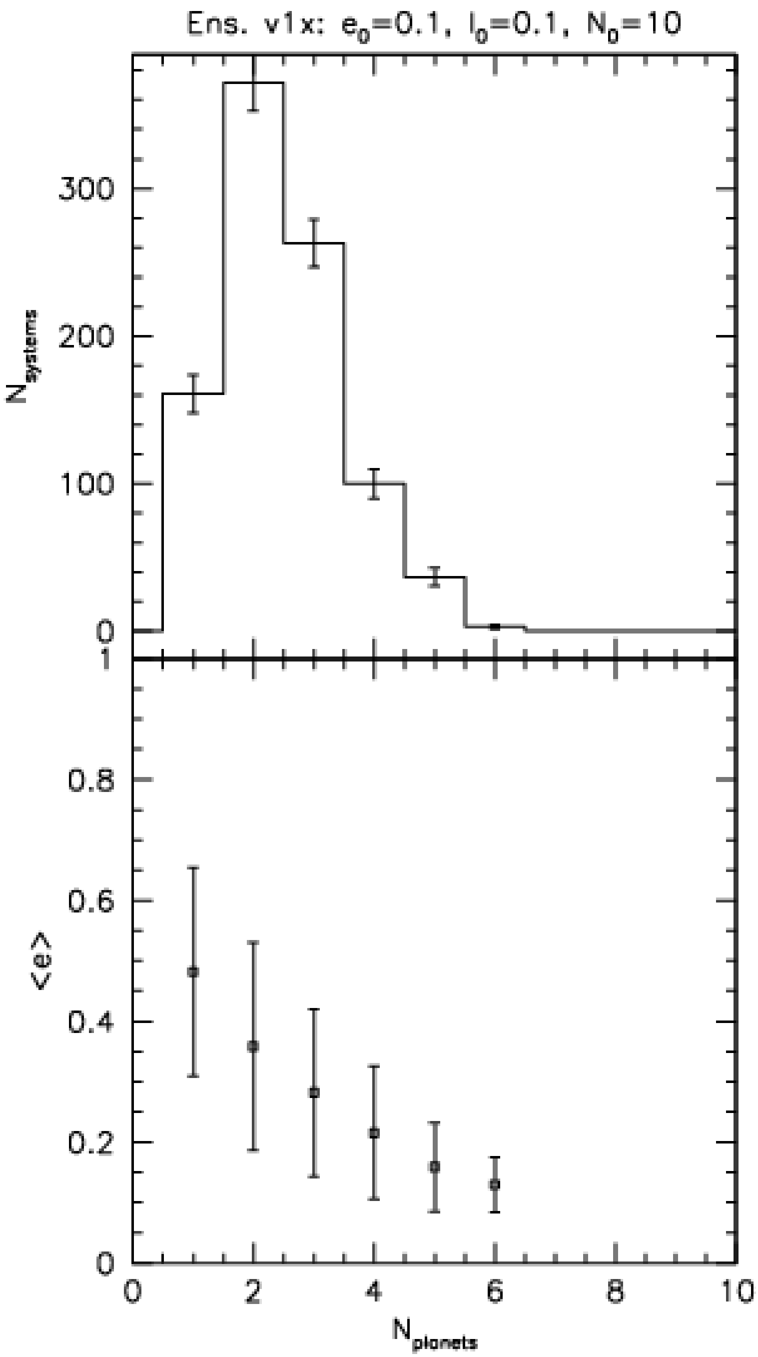
Modeling phase 2 (M. Juric, Ph.D. thesis)

- distribute N planets randomly between a=0.1 AU and 100 AU, uniform in log(a)
- choose masses randomly between 0.1 and 10 Jupiter masses, uniform in log(m)
- choose small eccentricities and inclinations from Maxwellian distribution with specified $\langle e^2 \rangle$, $\langle i^2 \rangle$
- follow for 100 Myr
- repeat 1000 times for each parameter set N, $\langle e^2 \rangle$, $\langle i^2 \rangle$



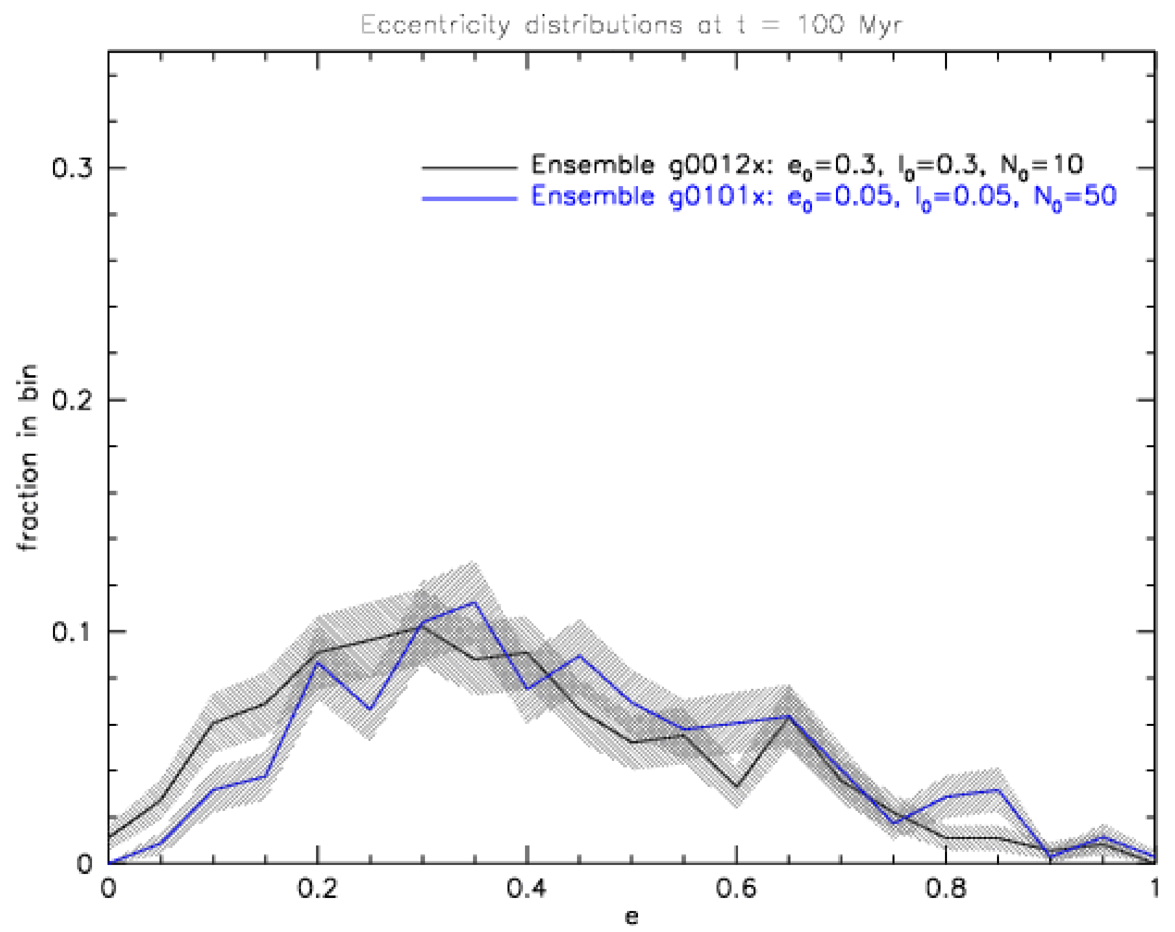
- many planets are ejected, collide, or fall into the central star
- most systems end up with an average of only 2-3 planets

(Juric 2006)



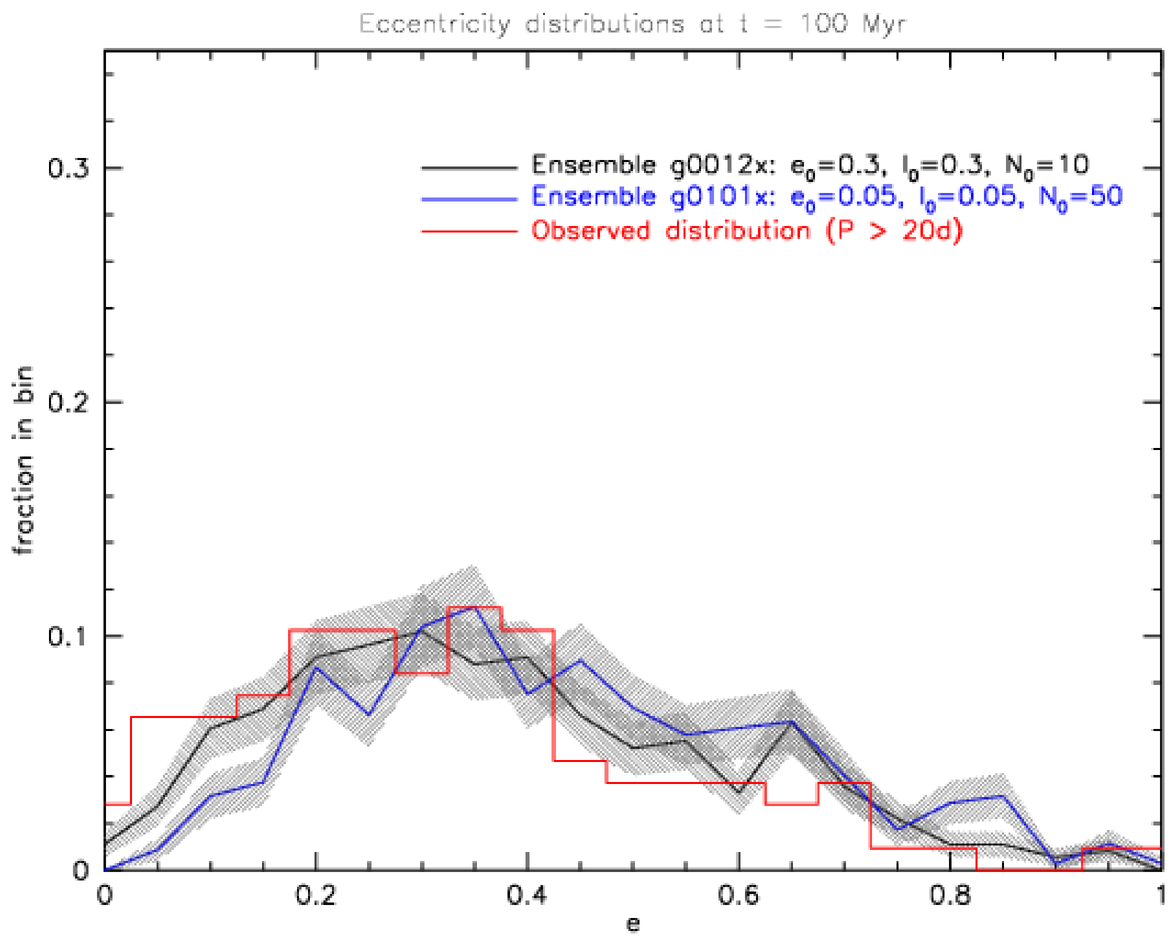
- mean eccentricity of surviving planets is correlated with number of surviving planets
- there are many high-eccentricity systems with 1 or 2 planets (the extrasolar planets?) and rare low-eccentricity systems with more planets (the solar system?)

(Juric 2006)



- a wide variety of systems converge to a common eccentricity distribution

(Juric 2006)



which matches the observed eccentricity distribution

(Juric 2006)

What I've left out

- origin of planetary rotation
- origin of planetary satellites
- origin of planetary atmospheres and oceans
- comets and Kuiper belt
- formation of gas giant envelopes