1. The stability of the solar system

Frontiers of Astronomy Workshop/School
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Stability of the solar system

The problem:

A point mass is surrounded by N > 1 much smaller masses on nearly circular, nearly coplanar orbits. Is the configuration stable over very long times (up to 10^{10} orbits)?

Some of the issues:

- what is the fate of the Earth?
- why are there so few planets in the solar system?
- where do comets and meteors come from?
- what do extrasolar planetary systems look like?
- can we calibrate the geological timescale over the last 50 Myr?
- how do dynamical systems behave over very long times?

Stability of the solar system

Newton:

"blind fate could never make all the Planets move one and the same way in Orbs concentric, some inconsiderable irregularities excepted, which could have arisen from the mutual Actions of Planets upon one another, and which will be apt to increase, until this System wants a reformation"

"In the system of planets, Newton sees careful planning on the part of Providence that has ensured long-term stability; but the stability is not completely permanent…Providence must intervene to prevent gravitational collapse, and will thereby demonstrate a continuing concern for the welfare of mankind. In short, Providence has…a regular servicing contract with the solar system." – Hoskins (1985)

Stability of the solar system

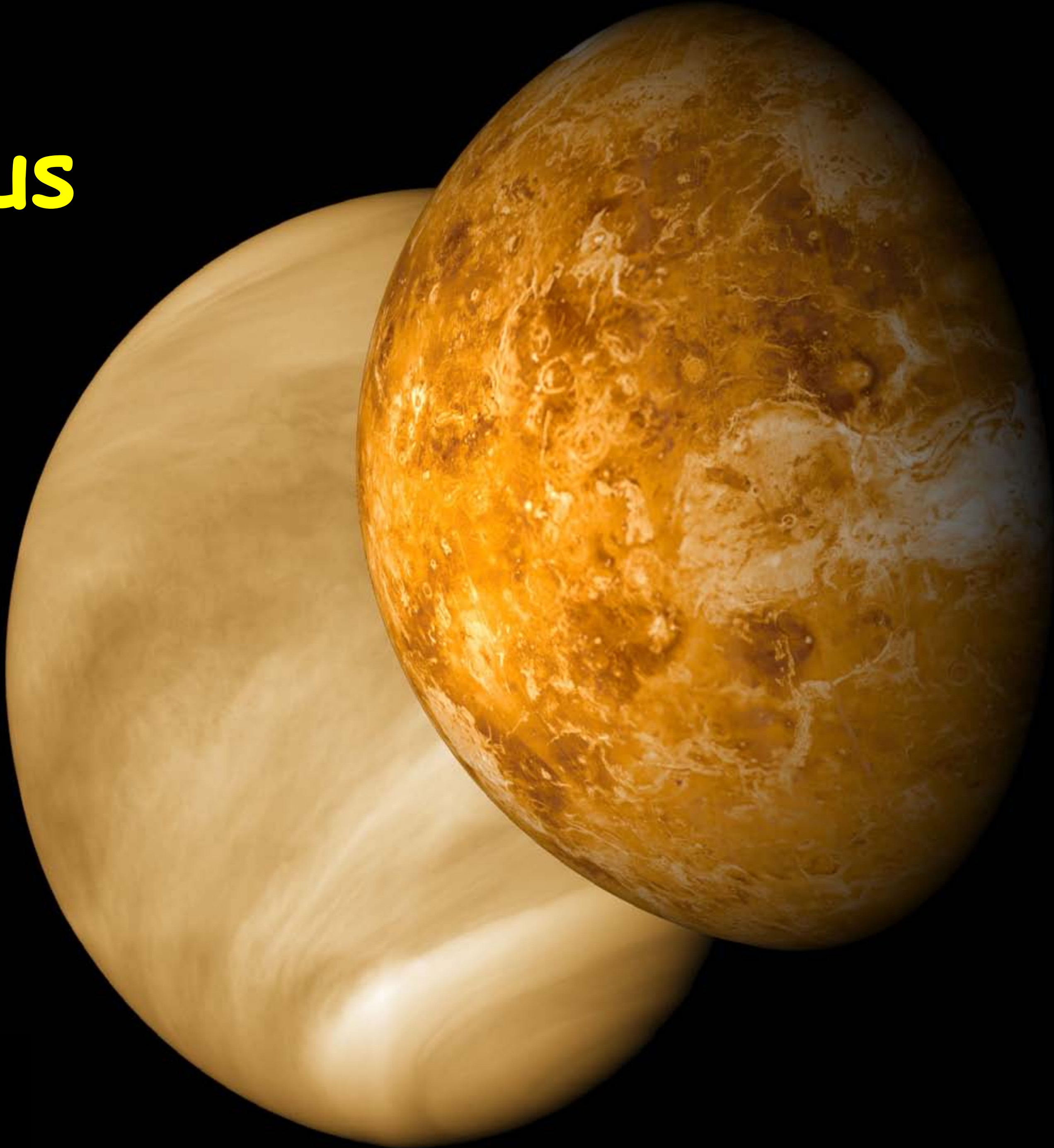
Laplace:

"An intelligence knowing, at a given instant of time, all forces acting in nature, as well as the momentary positions of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world and those of the smallest atoms in one single formula, provided it were sufficiently powerful to subject all data to analysis. To it, nothing would be uncertain; both future and past would be present before its eyes."

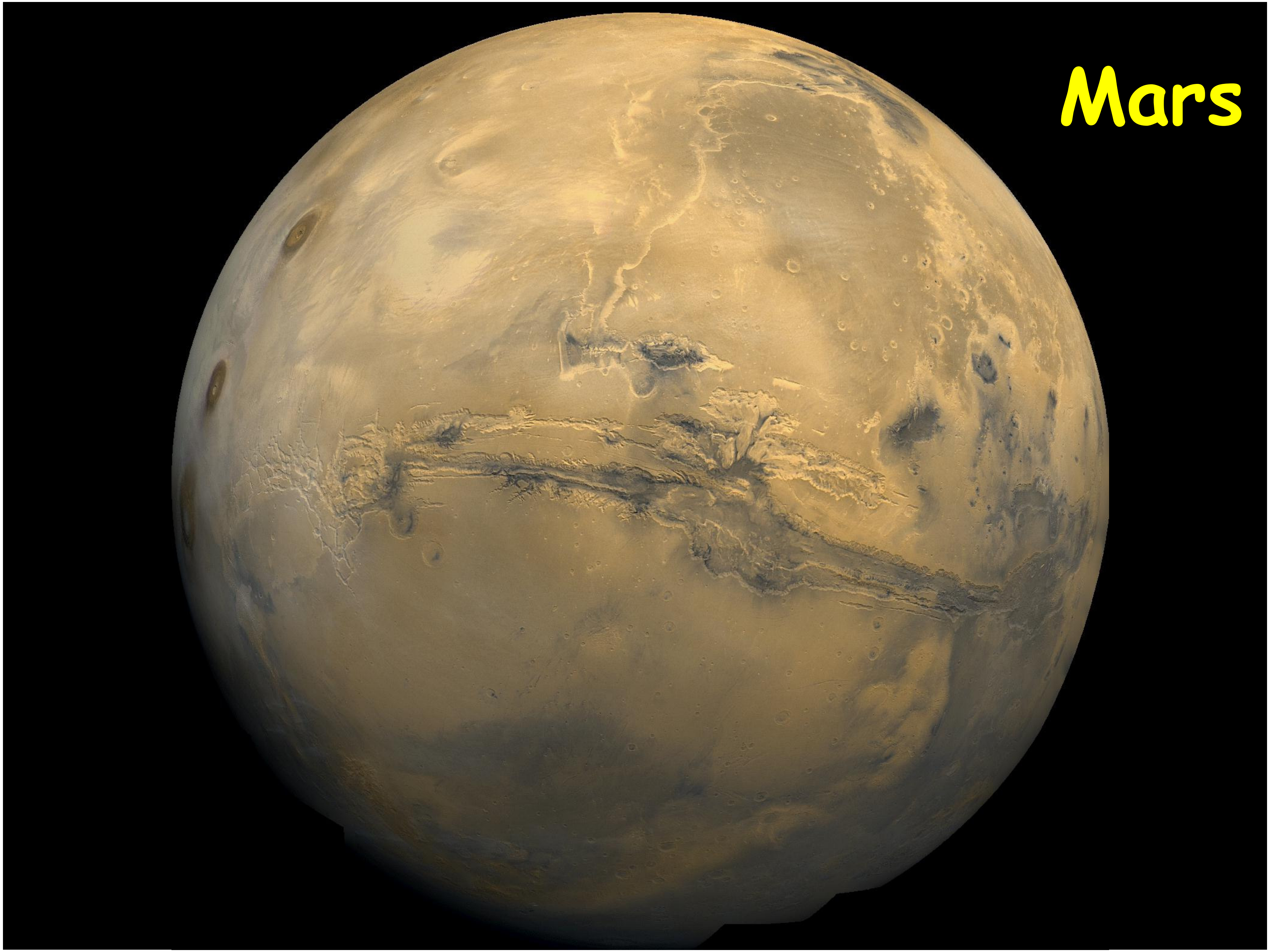
$\Rightarrow$ **Laplacian determinism, clockwork universe, etc.**

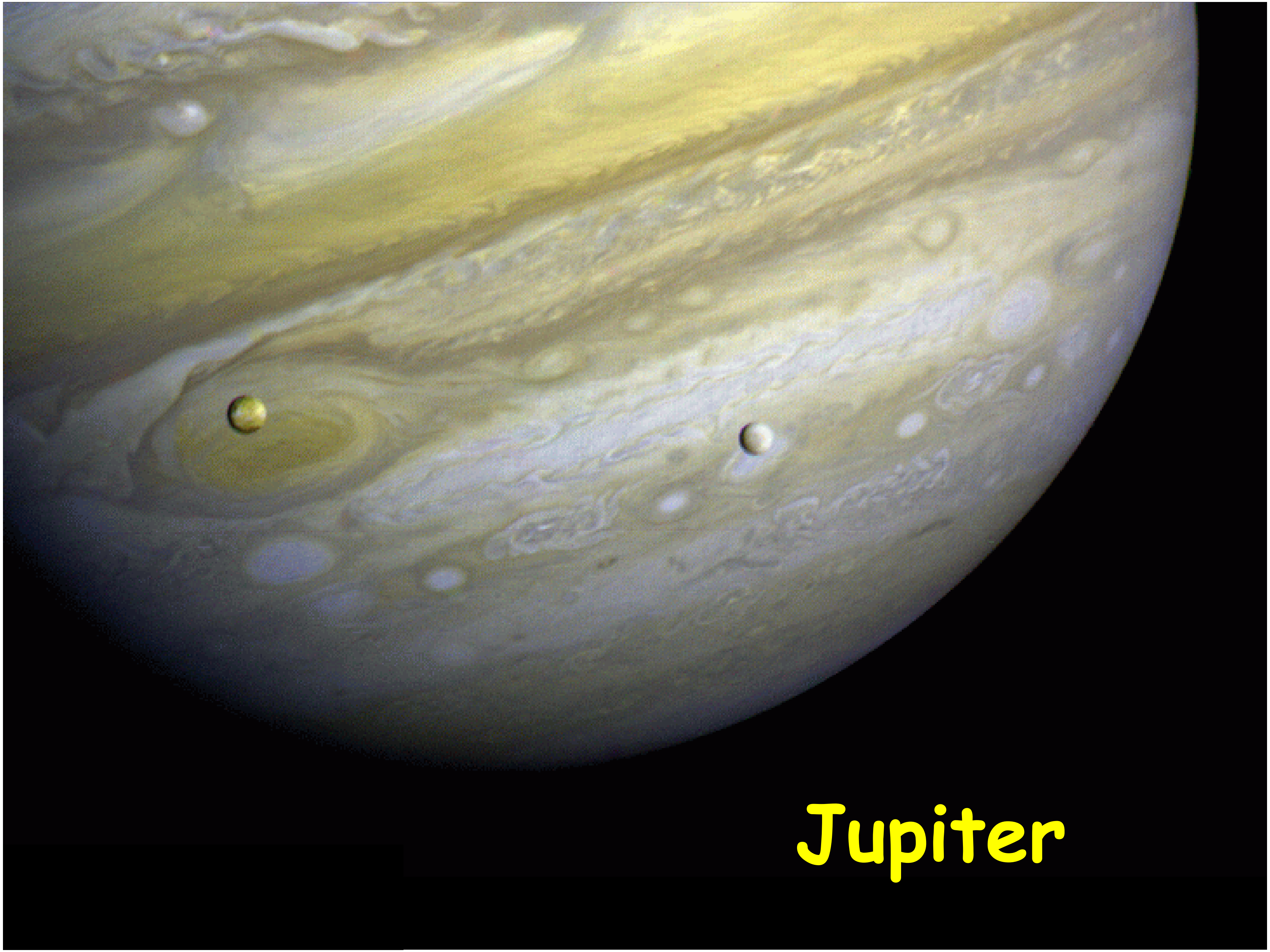
Mercury

Venus

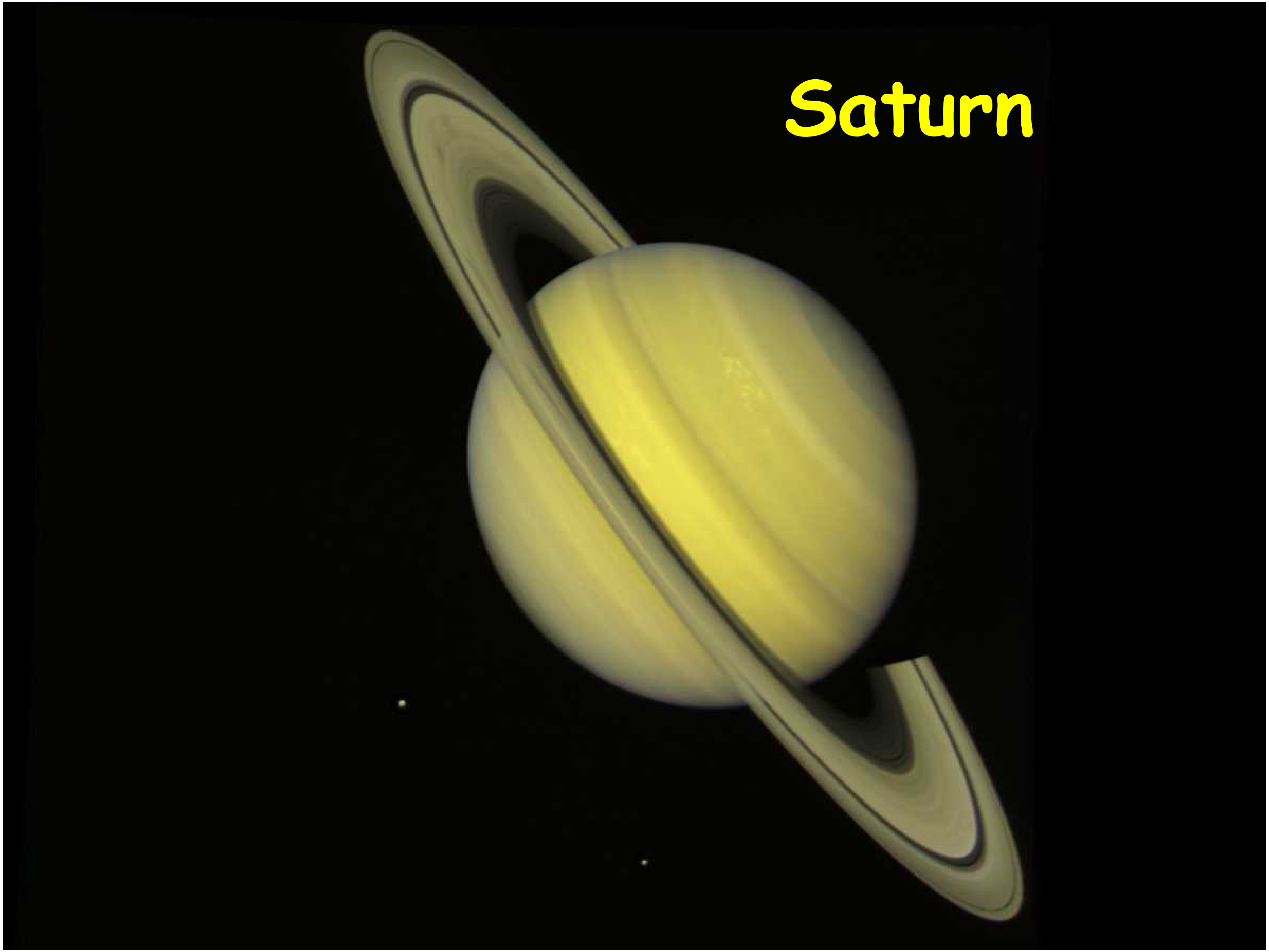


Earth

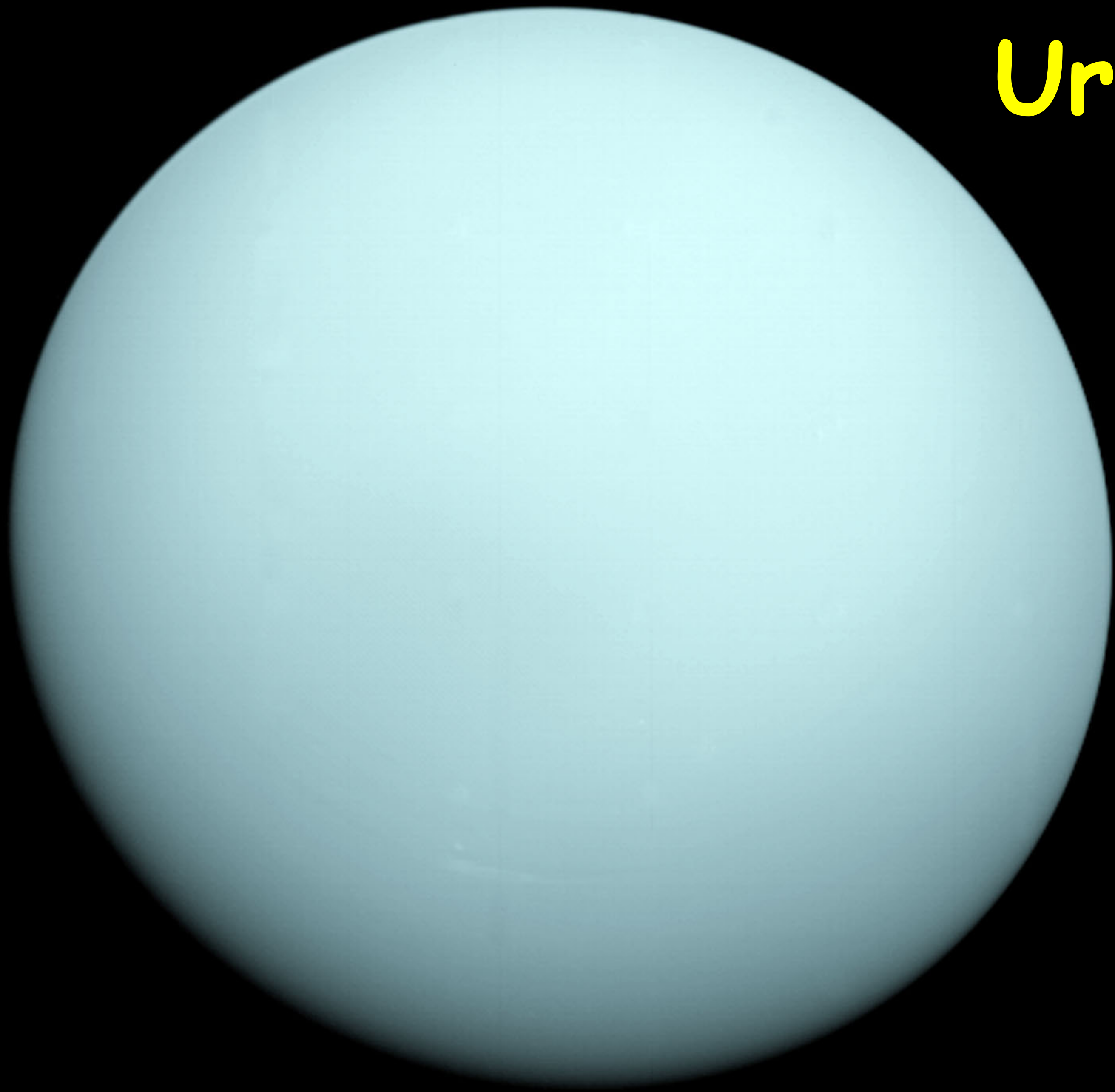

Mars

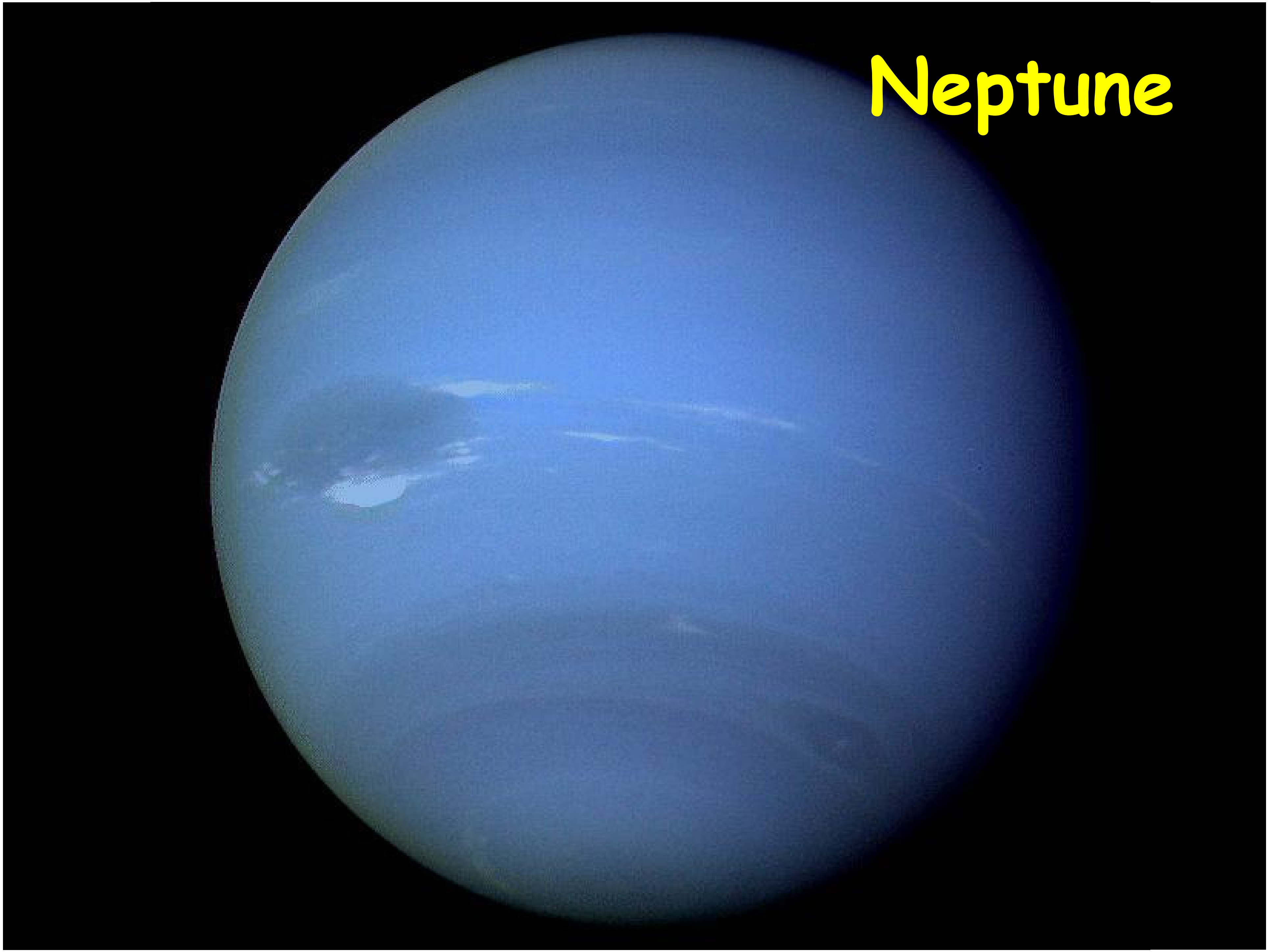

Jupiter

Saturn



Uranus



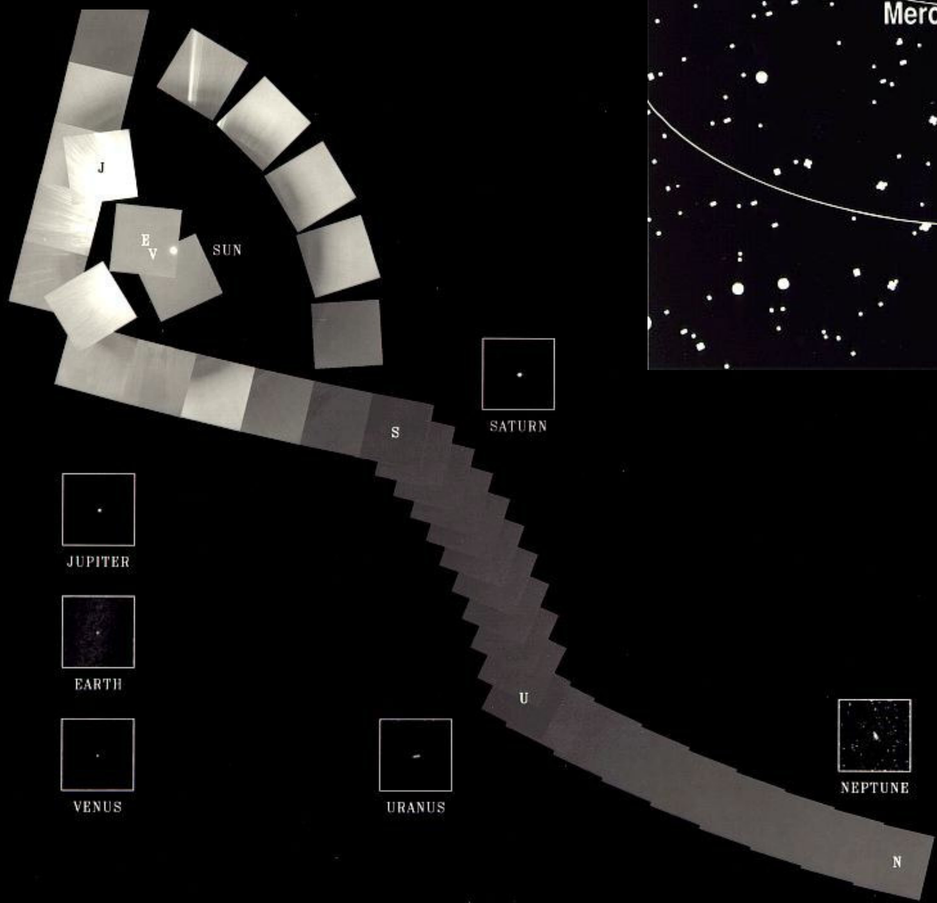

Neptune

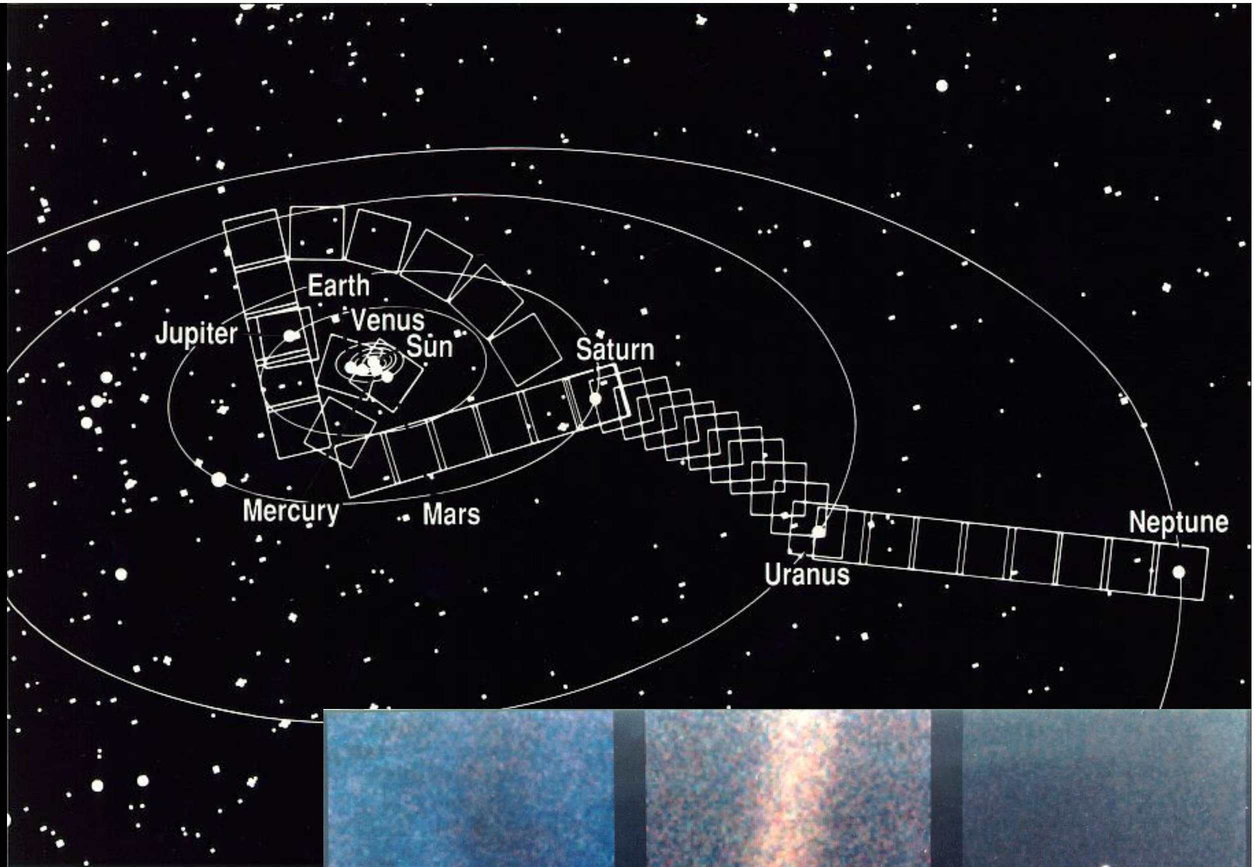
Pluto and Charon

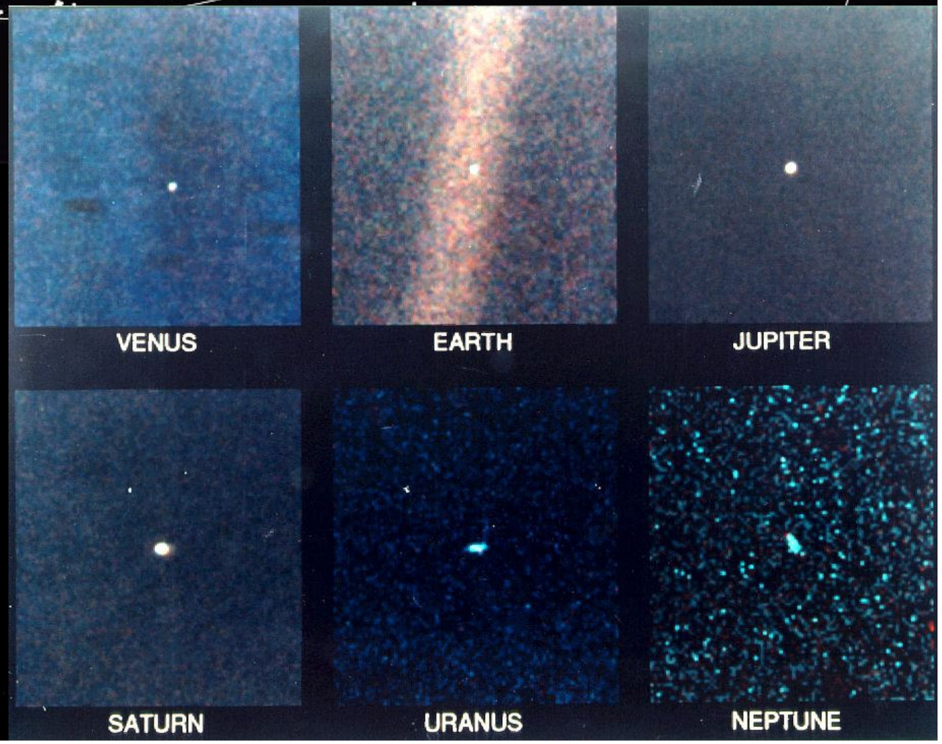
asteroids


comets


Voyager 1,
February 1990







Stability of the solar system

The problem:

A point mass is surrounded by N much smaller masses on nearly circular, nearly coplanar orbits. Is the configuration stable over very long times (up to 10^{10} orbits)?

How can we solve this?

- many famous mathematicians and physicists have attempted to find solutions, with limited success (Newton, Laplace, Lagrange, Gauss, Poincaré, Kolmogorov, Arnold, Moser, etc.)
- only feasible approach is numerical solution of equations of motion by computer, but:
  - needs sophisticated algorithms to avoid buildup of errors
  - needs $\sim 10^{12}$ timesteps, and difficult to parallelize
  - small computers are as good as big ones

Units

1 astronomical unit (AU) = mean Earth-Sun distance = 1.496×10^{13} cm

1 solar mass = $M_\odot = 2 \times 10^{33}$ gm

1 Jupiter mass ($M_{Jupiter}$) = 0.001 $M_\odot$ = 318 Earth masses

The equations of motion

$$\frac{d^2\mathbf{x}_i}{dt^2} = -G\sum_{j=1}^{N} \frac{m_j}{|\mathbf{x}_i-\mathbf{x}_j|^3}(\mathbf{x}_i - \mathbf{x}_j) + \mathbf{a}_{GR} + \mathbf{a}_S$$

Here a_{GR} is correction due to general relativity ($<10^{-8}$) and a_S is correction due to satellites ($< 10^{-7}$)

Masses m_j determined from spacecraft tracking to better than $10^{-9} M_\odot$

Initial conditions determined from radar ranging, spacecraft tracking, and telescope observations. Fractional accuracy better than 10^{-7}

The equations of motion

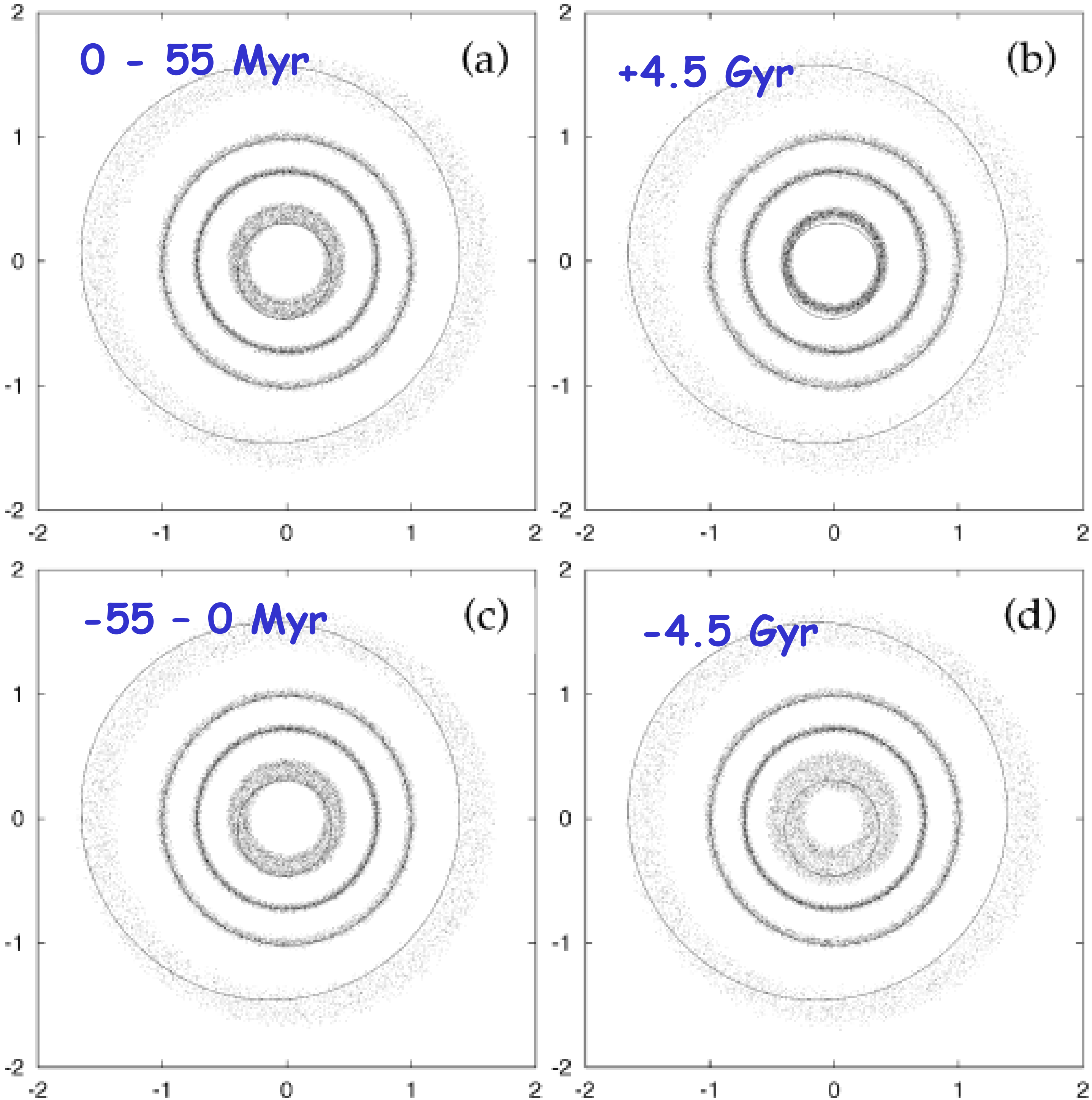
Principal uncertainties:

- asteroids (fractional effect < 10^{-9})
- solar quadrupole moment (fractional effect < 10^{-10}, even for Mercury)
- mass loss from Sun through radiation and solar wind, and drag of solar wind on planetary magnetospheres (<10^{-14})
- Galactic tidal forces (fractional effect <10^{-13})
- passing stars (closest passage about 500 AU)

To a very good approximation, the solar system is an isolated dynamical system described by a known set of equations, with known initial conditions

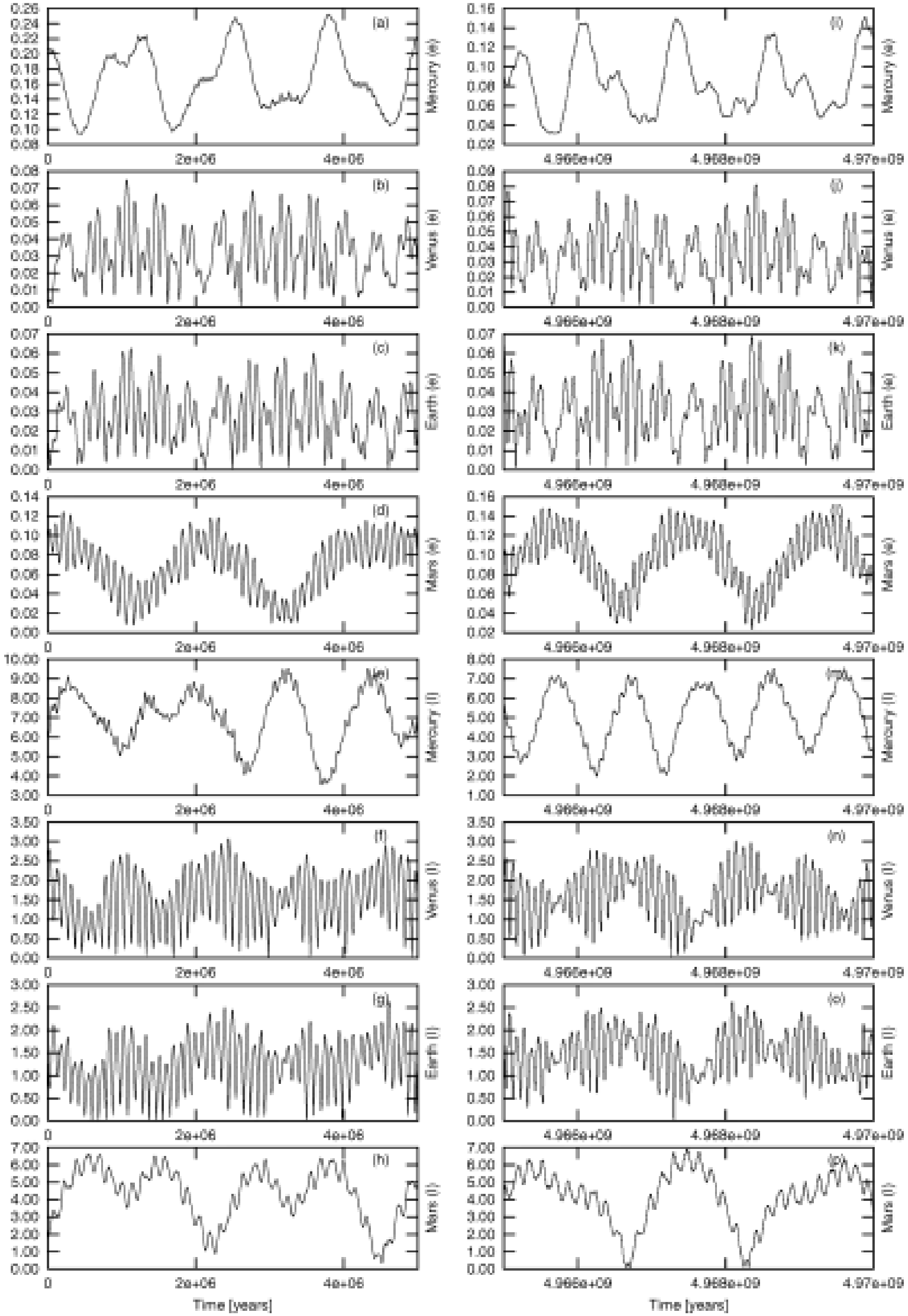
All we have to do is integrate them for ~10^{10} orbits (4.5×10^9 yr backwards to formation, or 7×10^9 yr forwards to red-giant stage)

Goal is quantitative accuracy ($\Delta\varphi \ll 1$ radian) over 10^8 yr and qualitative accuracy over 10^{10} yr



innermost four planets

Ito & Tanikawa (2002)



Ito & Tanikawa (2002)

Pluto's peculiar orbit

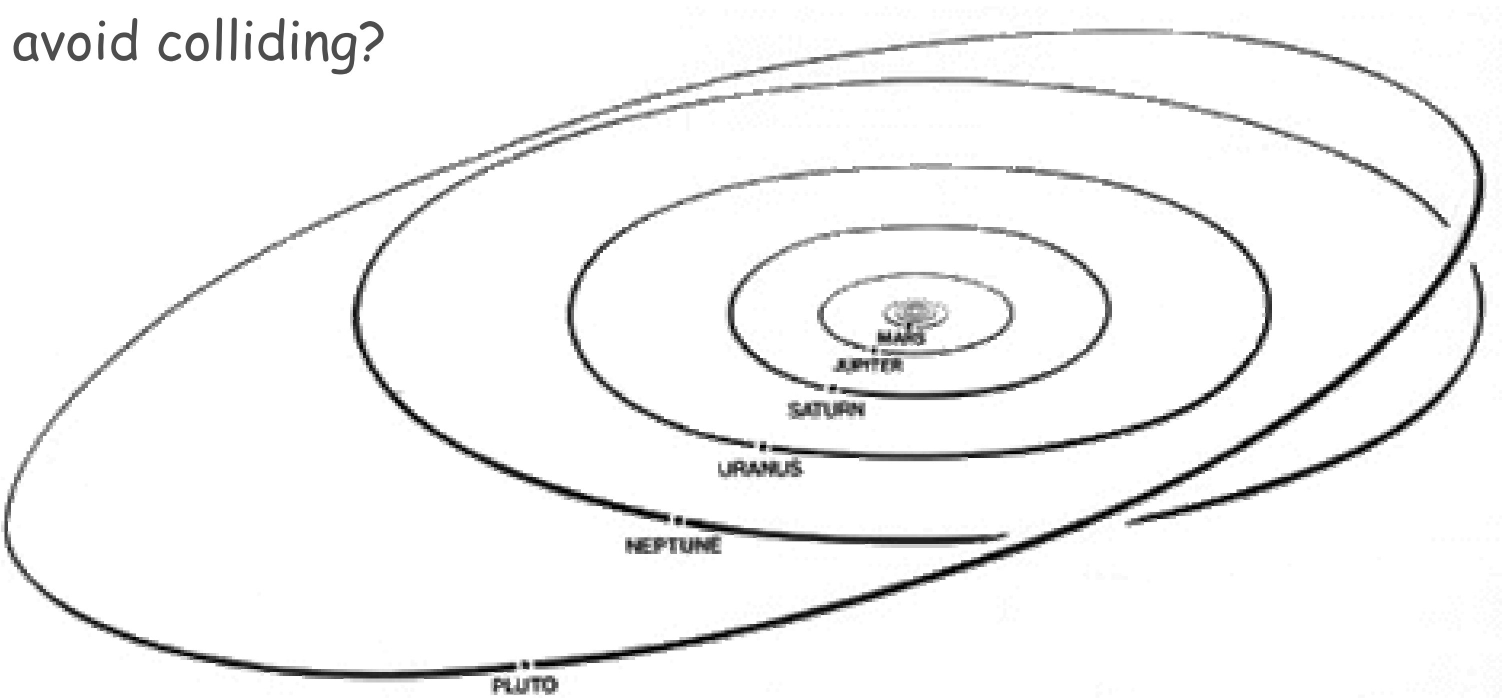
Pluto has:

•the highest eccentricity of any planet ($e = 0.250$)

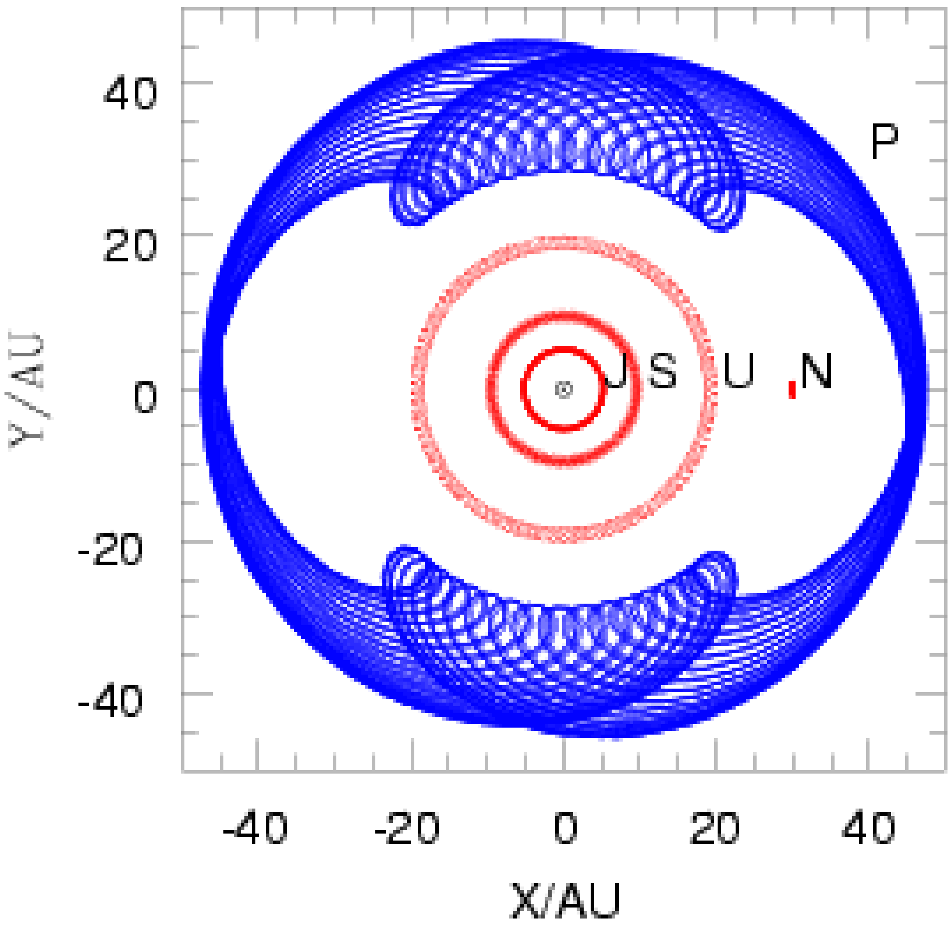
•the highest inclination of any planet ($i = 17^o$)

•perihelion distance $q = a(1 - e) = 29.6$ AU that is smaller than Neptune's semimajor axis ($a = 30.1$ AU)

How do they avoid colliding?



Pluto's peculiar orbit



Orbital period of Pluto = 247.7 y

Orbital period of Neptune = 164.8 y

247.7/164.8 = 1.50 = 3/2

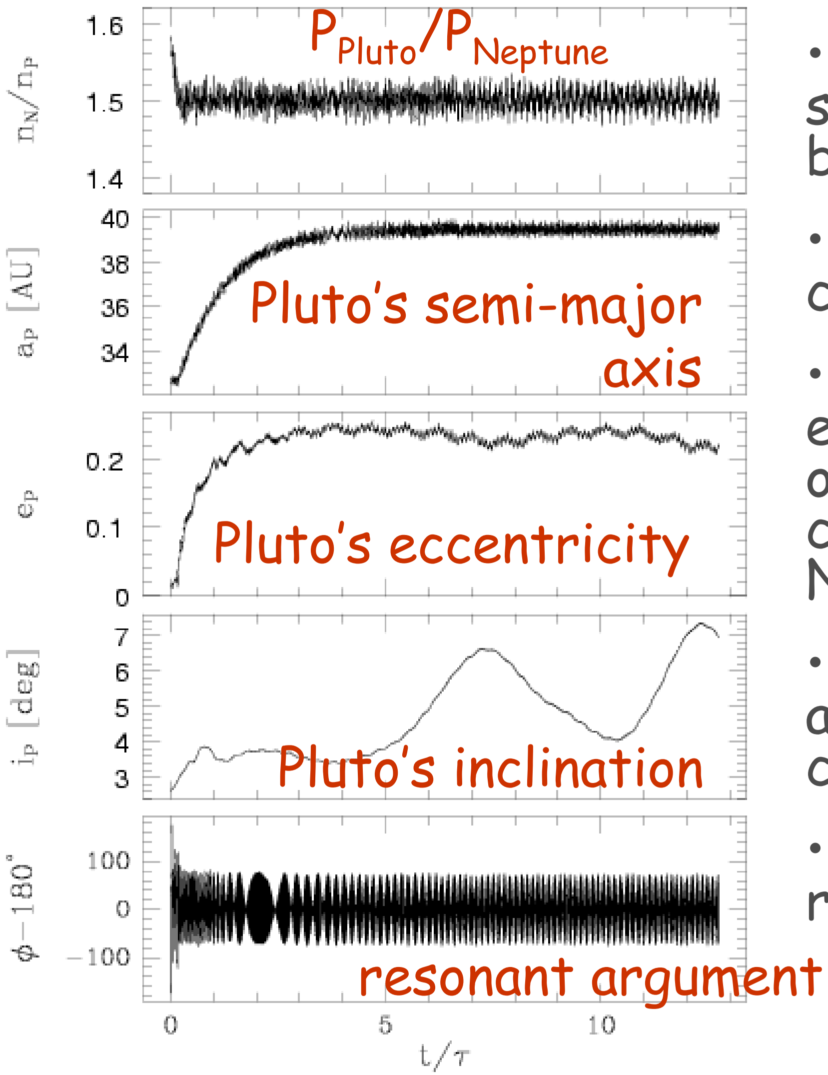
Resonance ensures that when Pluto is at perihelion it is approximately 90° away from Neptune

Resonant argument:

Φ= 3×(longitude of Pluto) – 2×(longitude of Neptune) – (perihelion of Pluto)

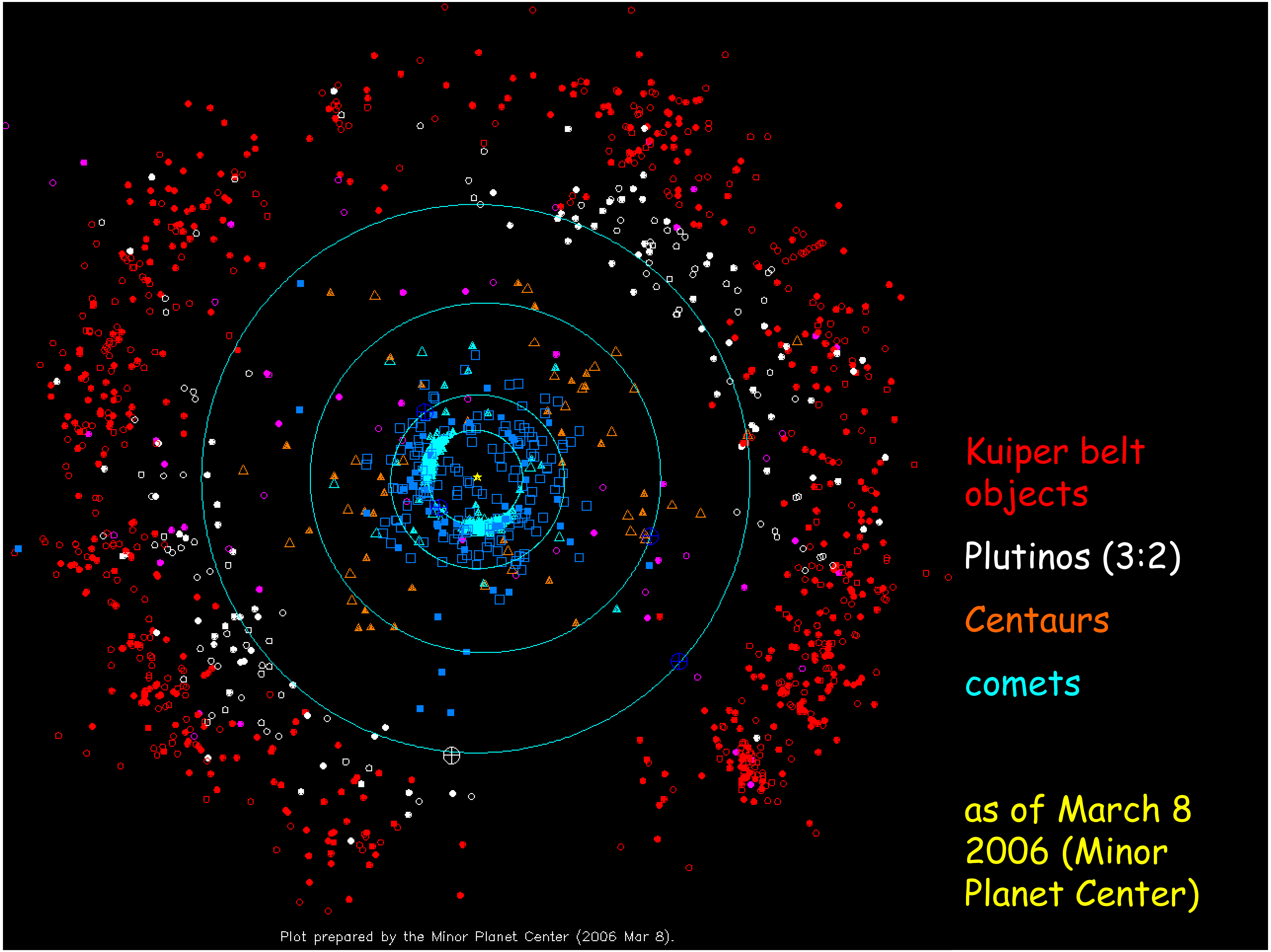
librates around π (Cohen & Hubbard 1965)

Pluto's peculiar orbit



- early in the history of the solar system there was debris left over between the planets
- ejection of this debris by Neptune caused its orbit to migrate outwards
- if Pluto were initially in a low-eccentricity, low-inclination orbit outside Neptune it is inevitably captured into 3:2 resonance with Neptune
- once Pluto is captured its eccentricity and inclination grow as Neptune continues to migrate outwards
- other objects may be captured in the resonance as well

Malhotra (1993)


Kuiper belt objects
Plutinos (3:2)
Centaurs
comets
as of March 8 2006 (Minor Planet Center)
Plot prepared by the Minor Planet Center (2006 Mar 8).

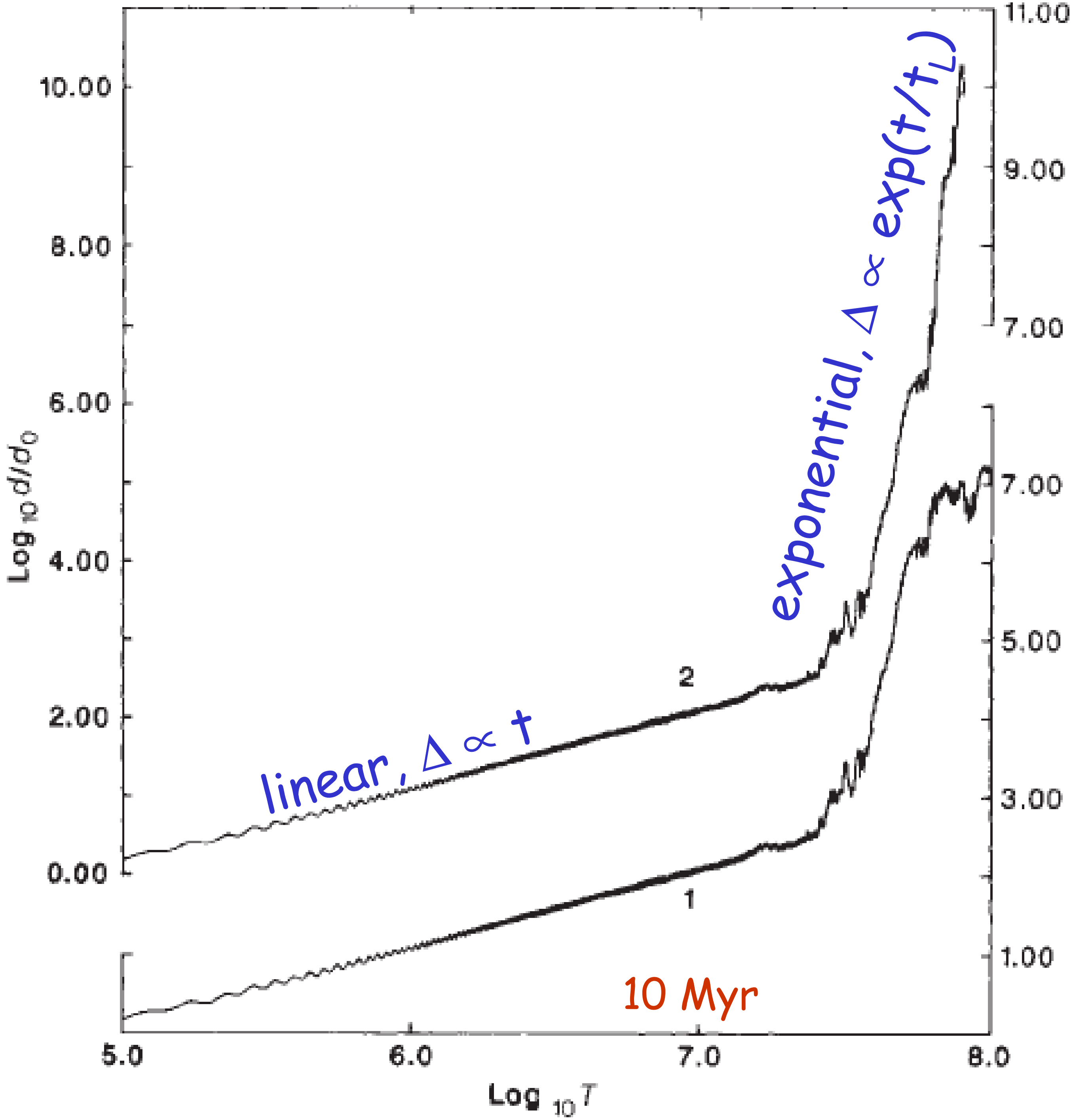
Two kinds of dynamical system

Regular

- highly predictable, "well-behaved"
- small differences grow linearly: $\Delta x, \Delta v \propto t$
- e.g. baseball, golf, simple pendulum, all problems in mechanics textbooks, planetary orbits on short timescales

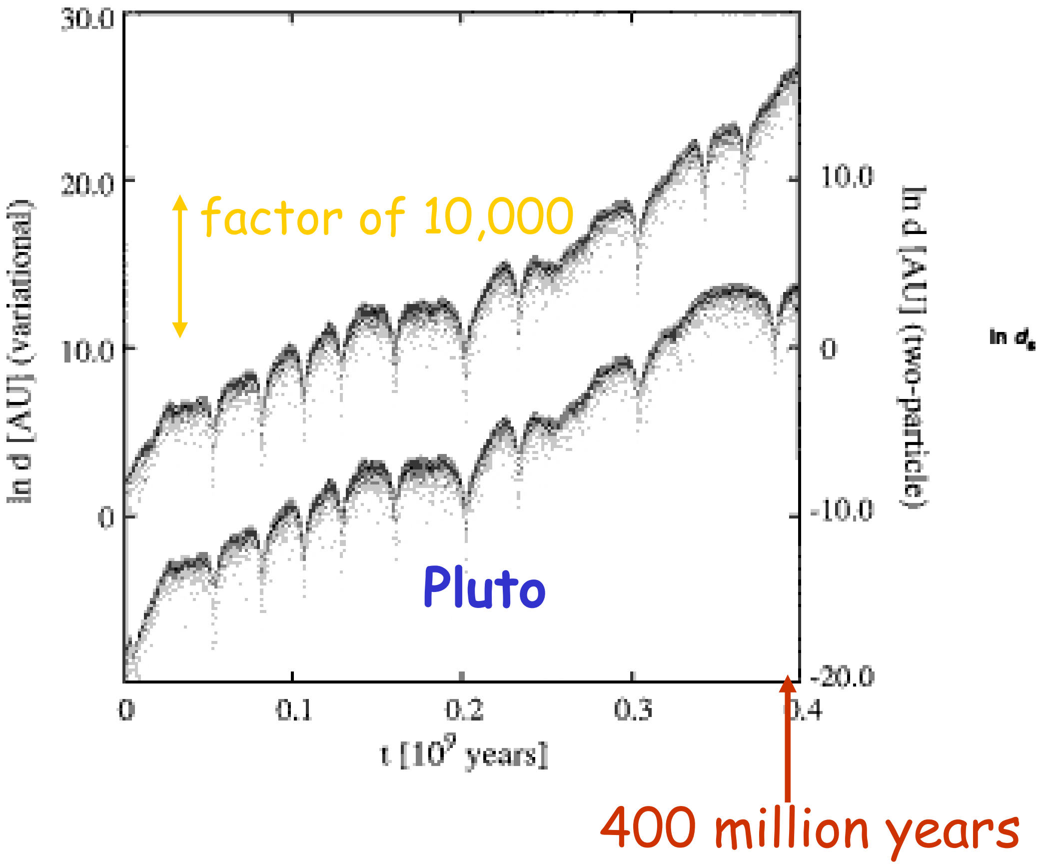
Chaotic

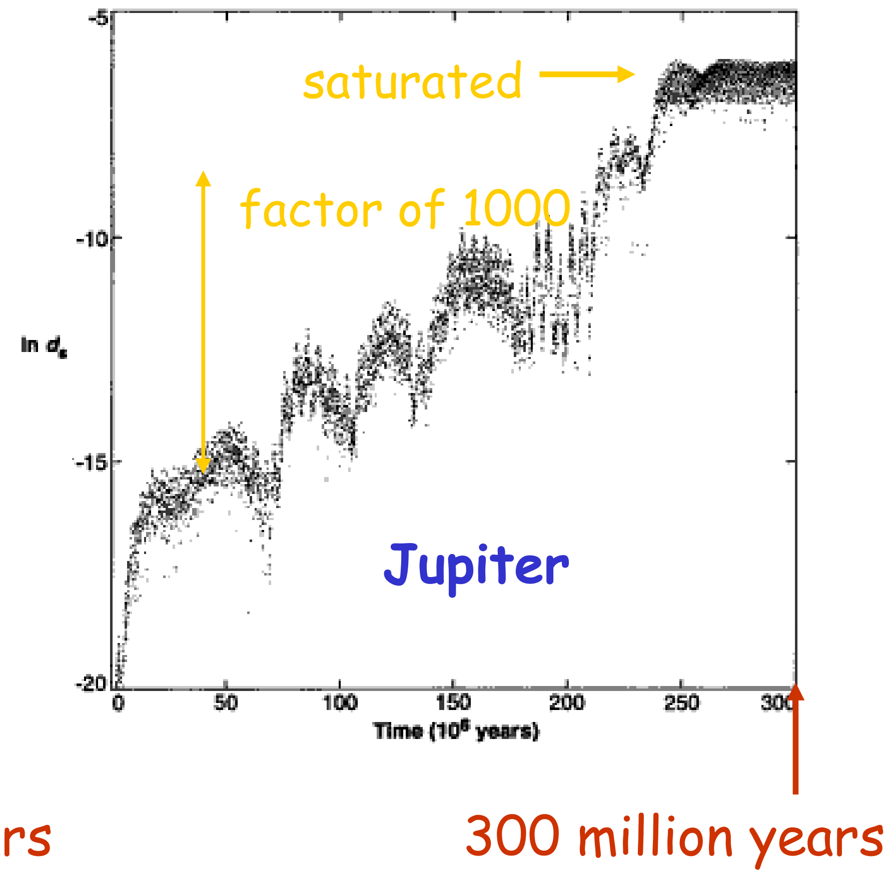
- difficult to predict, "erratic"
- small differences grow exponentially at large times: $\Delta x, \Delta v \propto \exp(t/t_L)$ where t_L is Liapunov time
- appears regular on timescales short compared to Liapunov time
- e.g. roulette, dice, pinball, weather, billiards, double pendulum



10.00
8.00
6.00
4.00
2.00
0.00
11.00
9.00
7.00
7.00
5.00
3.00
1.00
Log 10 d/d0
5.0
6.0
7.0
8.0
Log 10 T
2
1
linear, Δ ∝ t
exponential, Δ ∝ exp(t/tL)
10 Myr

Laskar (1989)



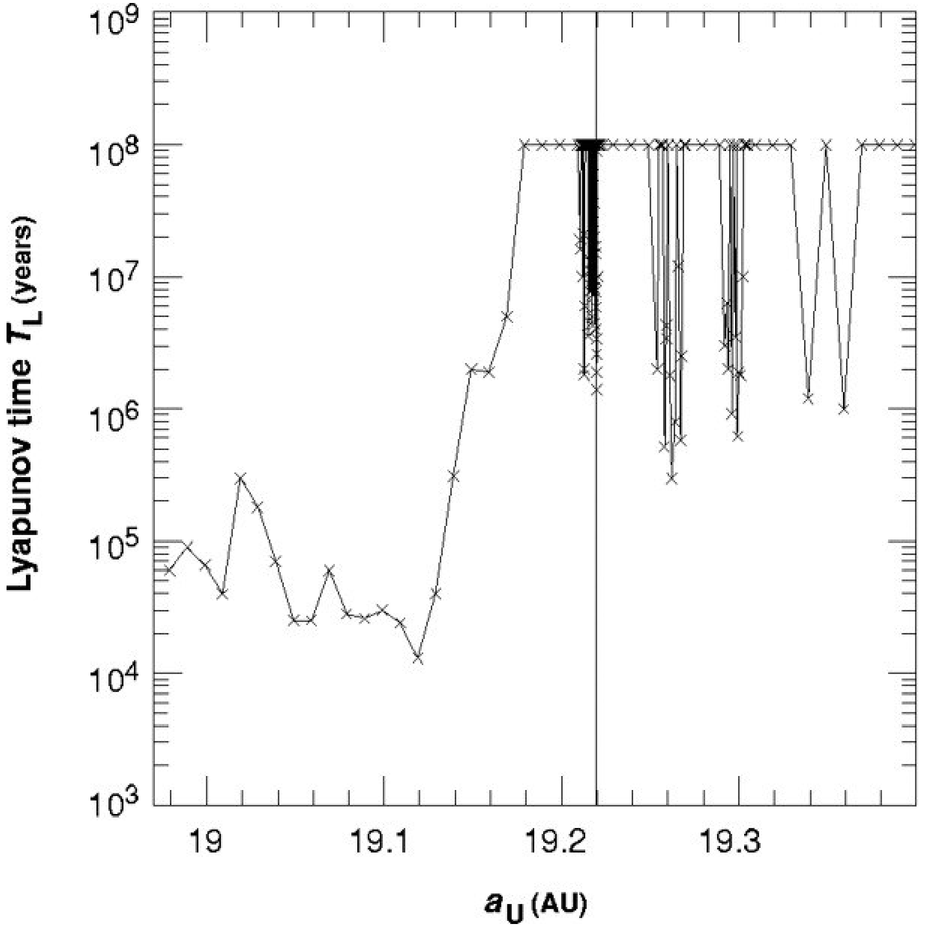


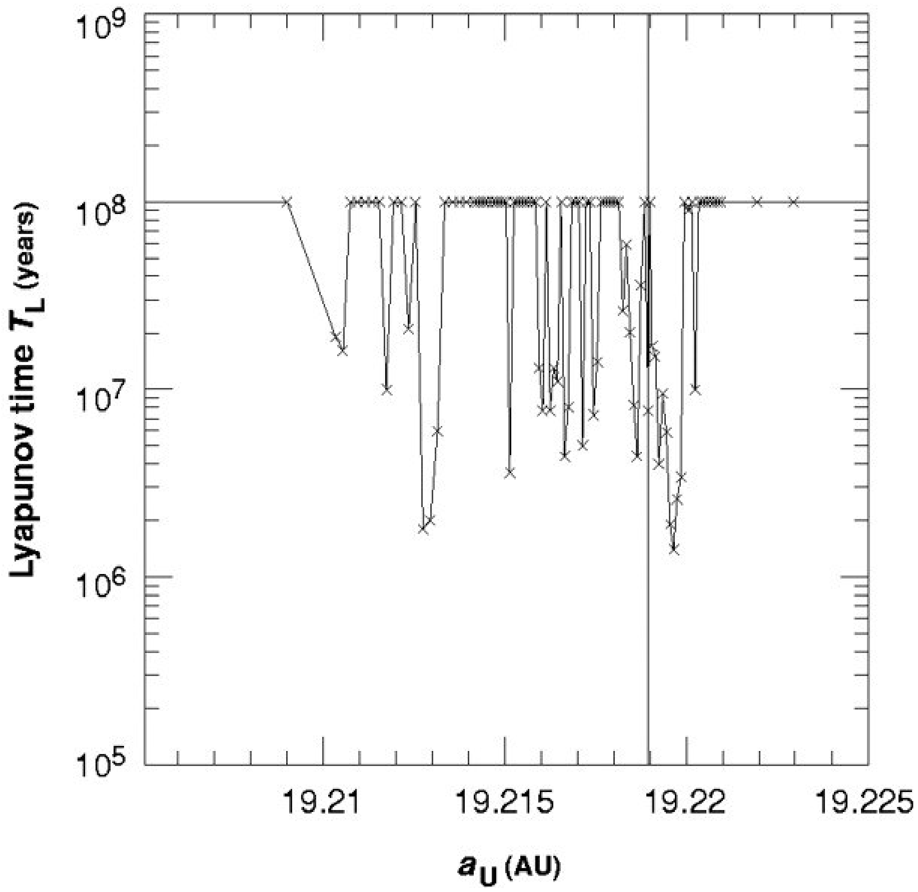
The orbit of every planet in the solar system is chaotic (Sussman & Wisdom 1988, 1992)

separation of adjacent orbits grows $\propto \exp(t / t_L)$ where Liapunov time t_L is 5-20 Myr $\Rightarrow$ factor of at least 10^{100} over lifetime of solar system

Causes of chaos

- chaos arises from overlap of resonances
- orbits with 3 degrees of freedom have three fundamental frequencies Ω_i. In spherical potentials, $\Omega_1=0$. In Kepler potentials $\Omega_1=\Omega_2=0$ so resonances are degenerate
- planetary perturbations lead to fine-structure splitting of resonances by amount $\sim O(\alpha)$ where $\alpha \sim m_{planet}/M_*$.
- two-body resonances have strength $\sim O(\alpha)$ and width $\sim O(\alpha)^{1/2}$.
- three-body resonances have strength $\sim O(\alpha^2)$ and width $\sim O(\alpha)$. Murray & Holman (1999) show that chaos in outer solar system arises from a 3-body resonance with critical argument Φ = 3×(longitude of Jupiter) - 5×(longitude of Saturn) - 7×(longitude of Uranus)
- small changes in initial conditions can eliminate or enhance chaos
- estimated lifetime 10^{18} yr

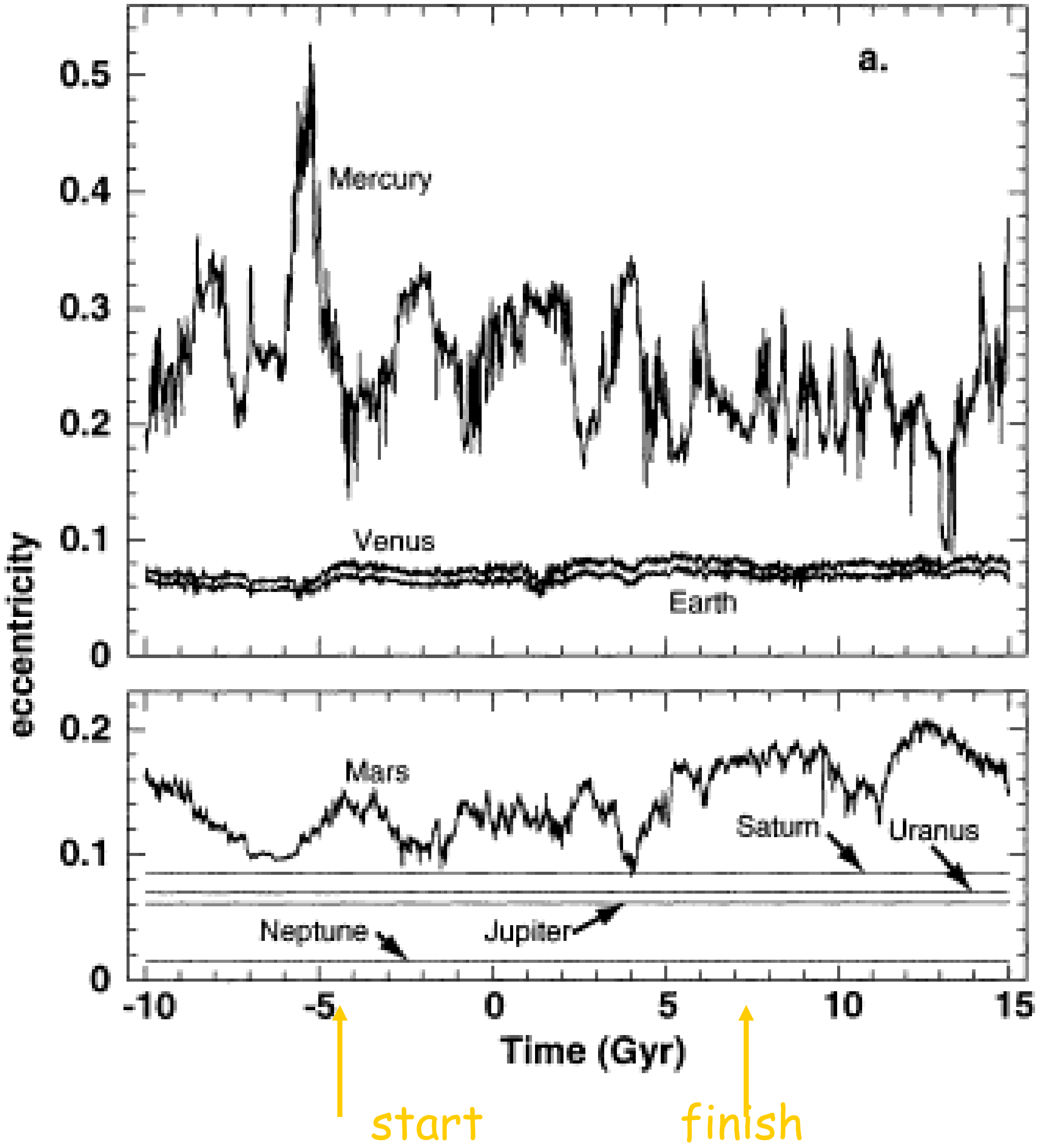




Murray & Holman (1999)

Consequences of chaos

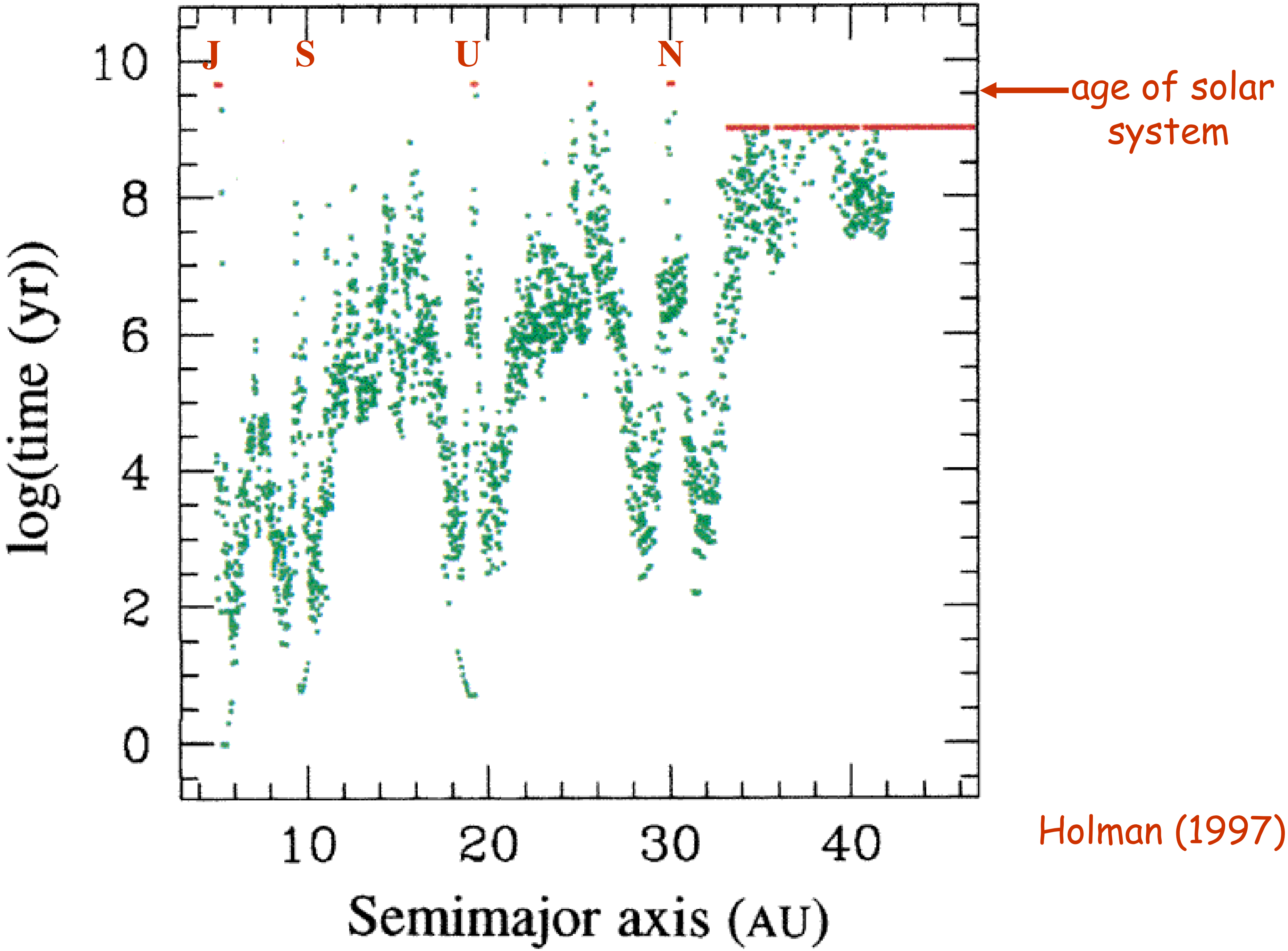
- orbits of all of the planets are chaotic with e-folding times for growth of small changes (Liapunov times) of 5-20 Myr (i.e. 200-1000 e-folds in lifetime of solar system
- positions (orbital phases) of planets are not predictable on timescales longer than 100 Myr
- the solar system is a poor example of a deterministic universe
- shapes of some orbits execute random walk on timescales of Gyr or longer


a.
Mercury
Venus
Earth
Mars
Saturn
Uranus
Neptune
Jupiter
eccentricity
0.5
0.4
0.3
0.2
0.1
0
-10
-5
0
5
10
15
Time (Gyr)
start
finish

Laskar (1994)

Consequences of chaos

- orbits of all of the planets are chaotic with e-folding times for growth of small changes (Liapunov times) of 5-20 Myr (i.e. 200-1000 e-folds in lifetime of solar system
- positions (orbital phases) of planets are not predictable on timescales longer than 100 Myr
- the solar system is a poor example of a deterministic universe
- shapes of some orbits execute random walk on timescales of Gyr or longer
- most chaotic systems with many degrees of freedom are unstable because chaotic regions in phase space are connected so trajectory wanders chaotically through large distances in phase space ("Arnold diffusion"). Thus solar system is unstable, although probably on *very* long timescales
- most likely ejection has *already happened* one or more times



J
S
U
N
10
8
6
4
2
0
log(time (yr))
10
20
30
40
Semimajor axis (AU)
age of solar system

Holman (1997)

Summary

- we can integrate the solar system for its lifetime
- the solar system is not boring on long timescales
- planet orbits are likely to be chaotic with e-folding times of 5-20 Myr
- the orbital phases of the planets are not predictable over timescales > 100 Myr
- "Is the solar system stable?" can only be answered statistically
- it is unlikely that any planets will be ejected or collide before the Sun dies
- most of the solar system is "full", and it is likely that planets have been lost from the solar system in the past