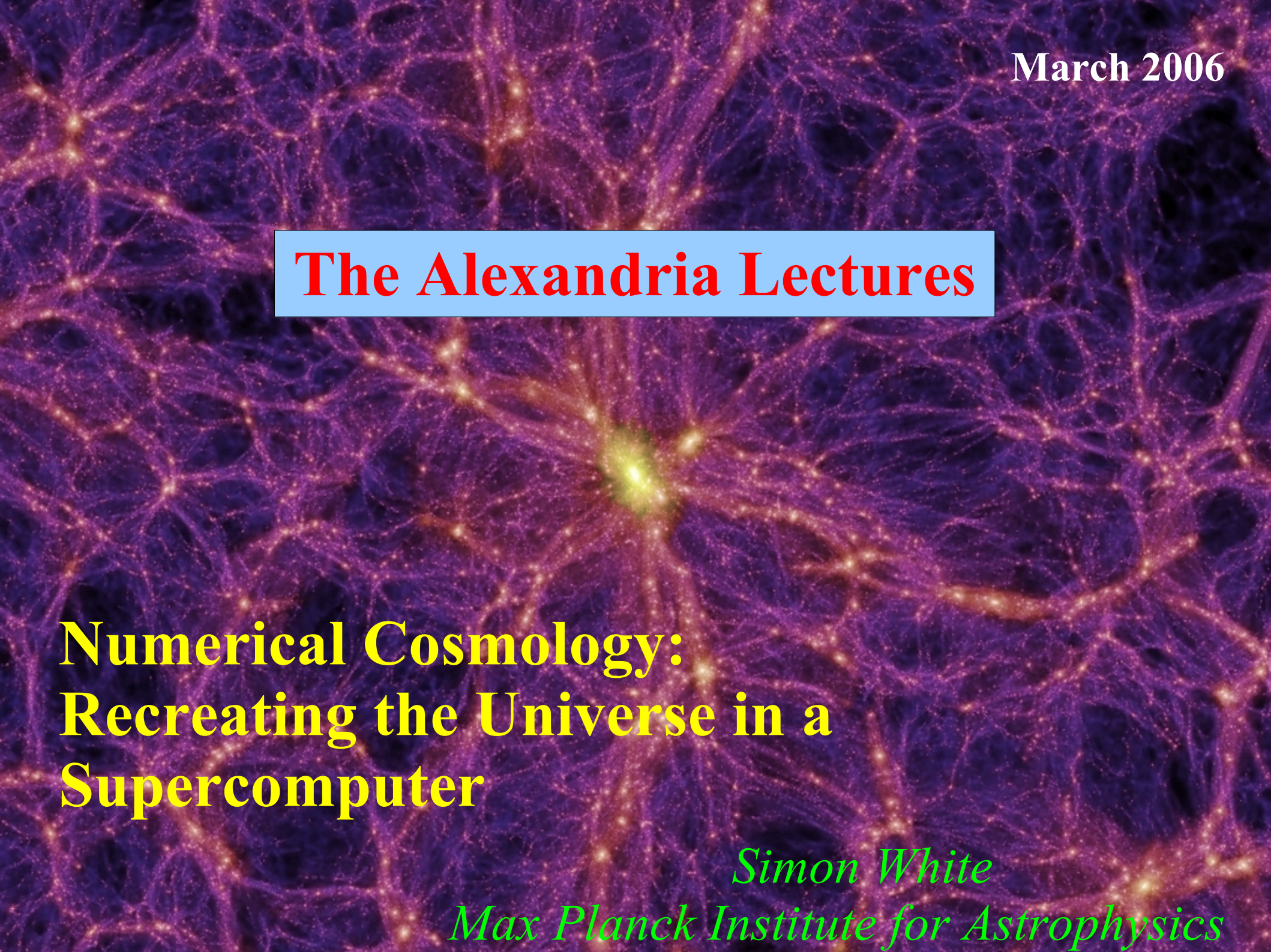


The background of the slide is a detailed visualization of the cosmic web, showing a complex network of dark matter filaments and galaxy clusters. The filaments are depicted as thin, branching structures in shades of purple and blue, while the clusters are represented by denser, glowing yellow and orange regions. The overall effect is a sense of vast, interconnected space.

March 2006

The Alexandria Lectures

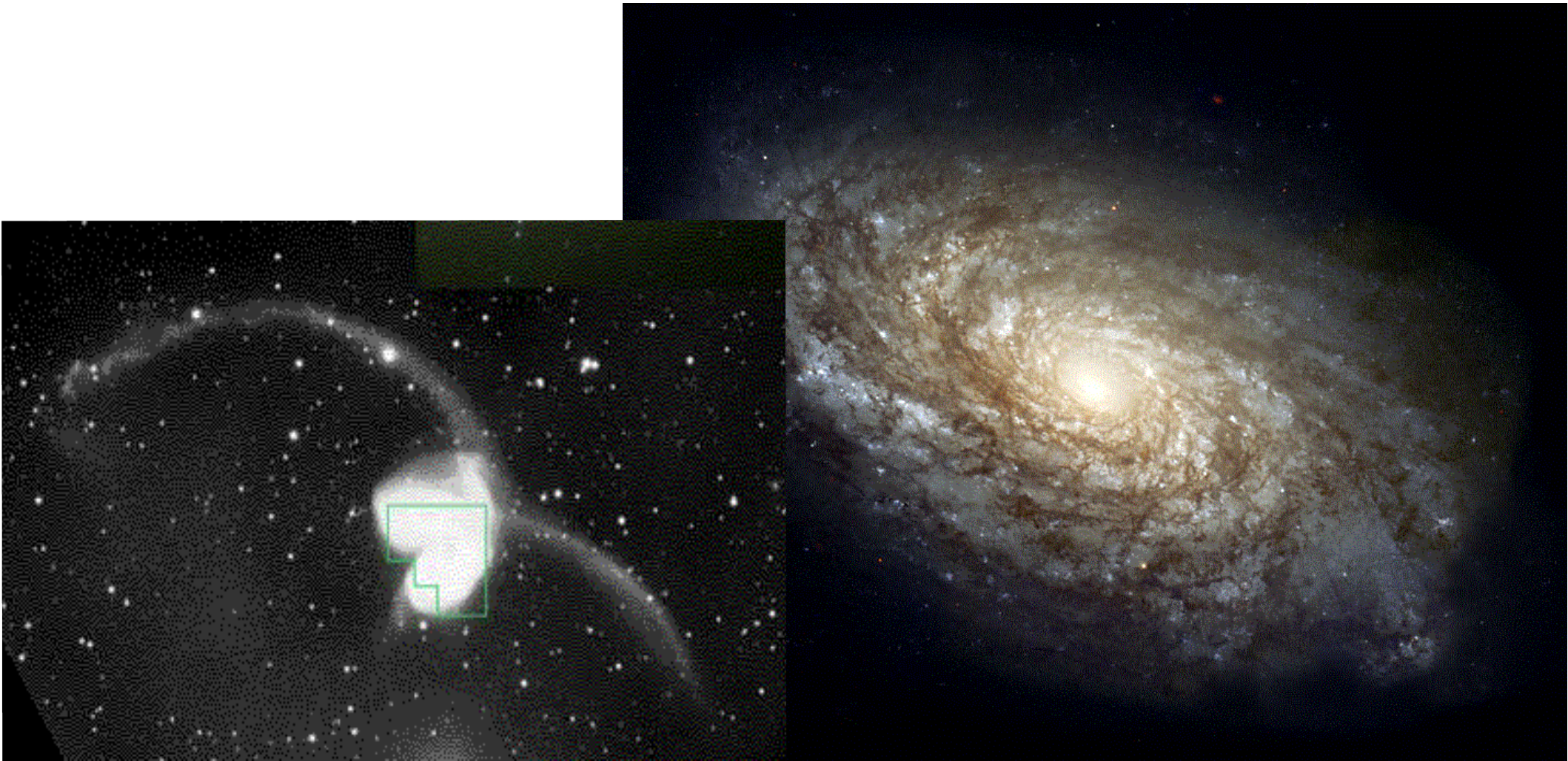
**Numerical Cosmology:
Recreating the Universe in a
Supercomputer**

Simon White

Max Planck Institute for Astrophysics

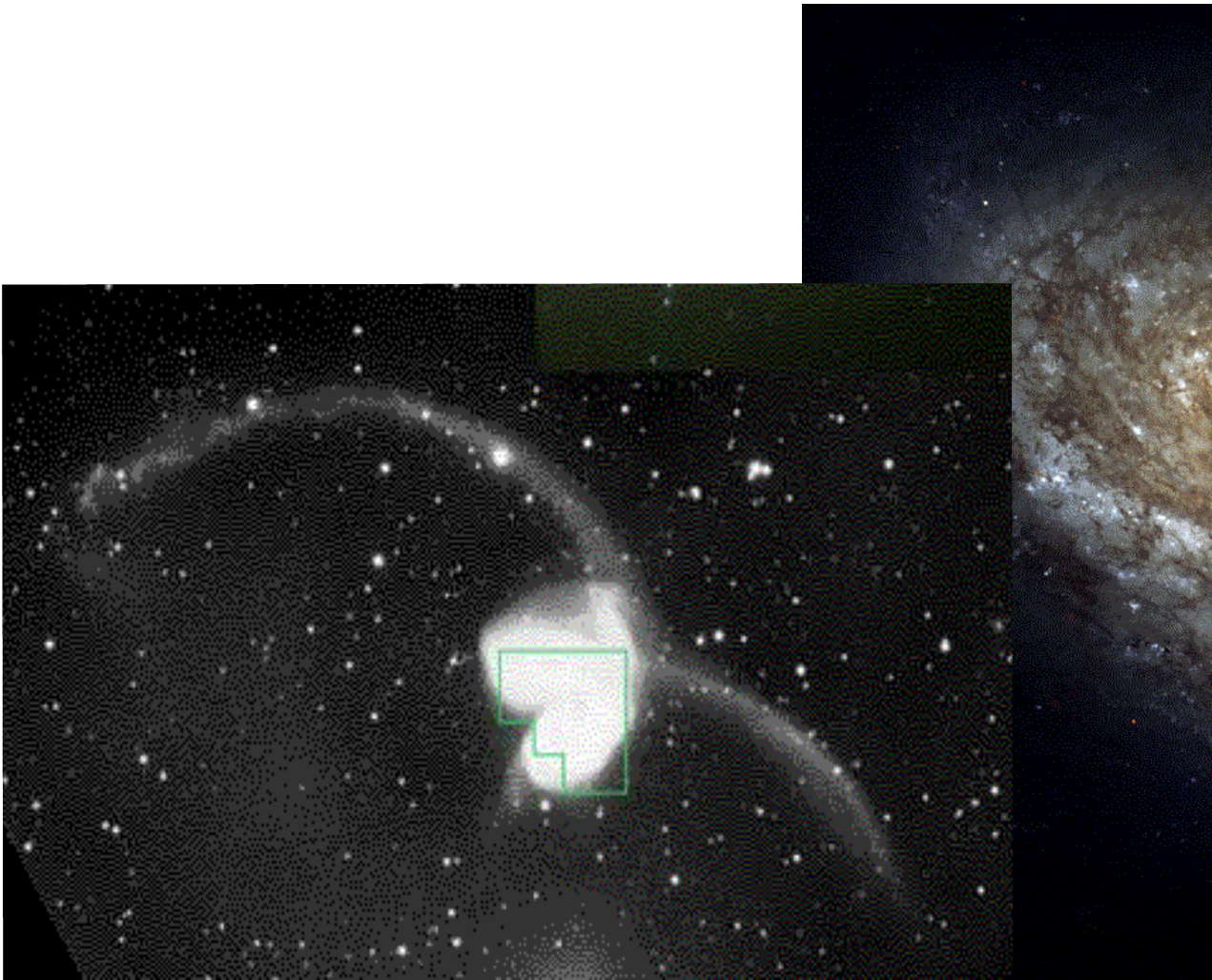
The Three-fold Way to Astrophysical Truth

OBSERVATION

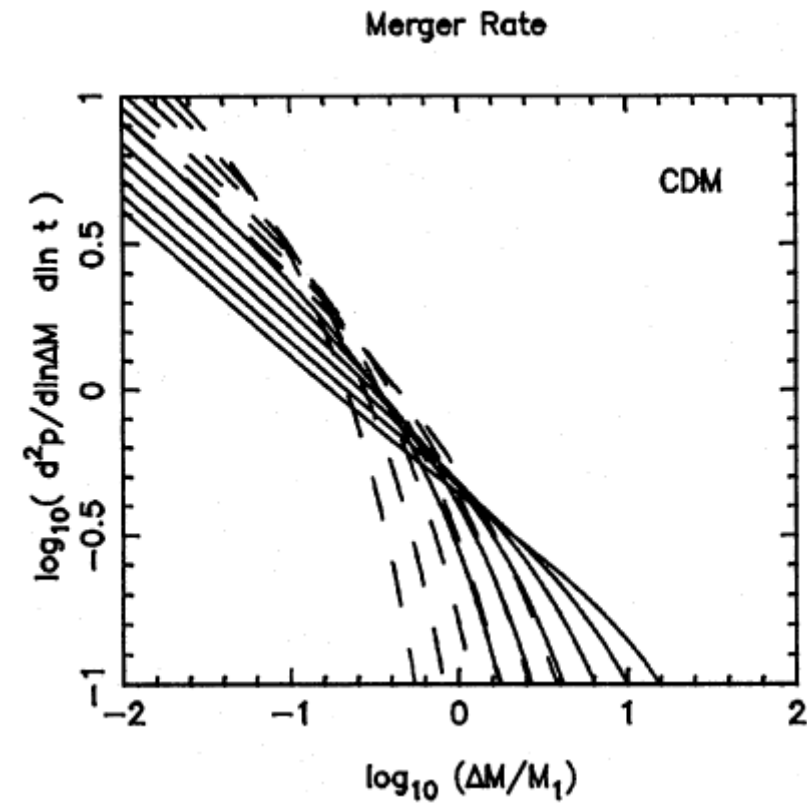


The Three-fold Way to Astrophysical Truth

OBSERVATION



THEORY

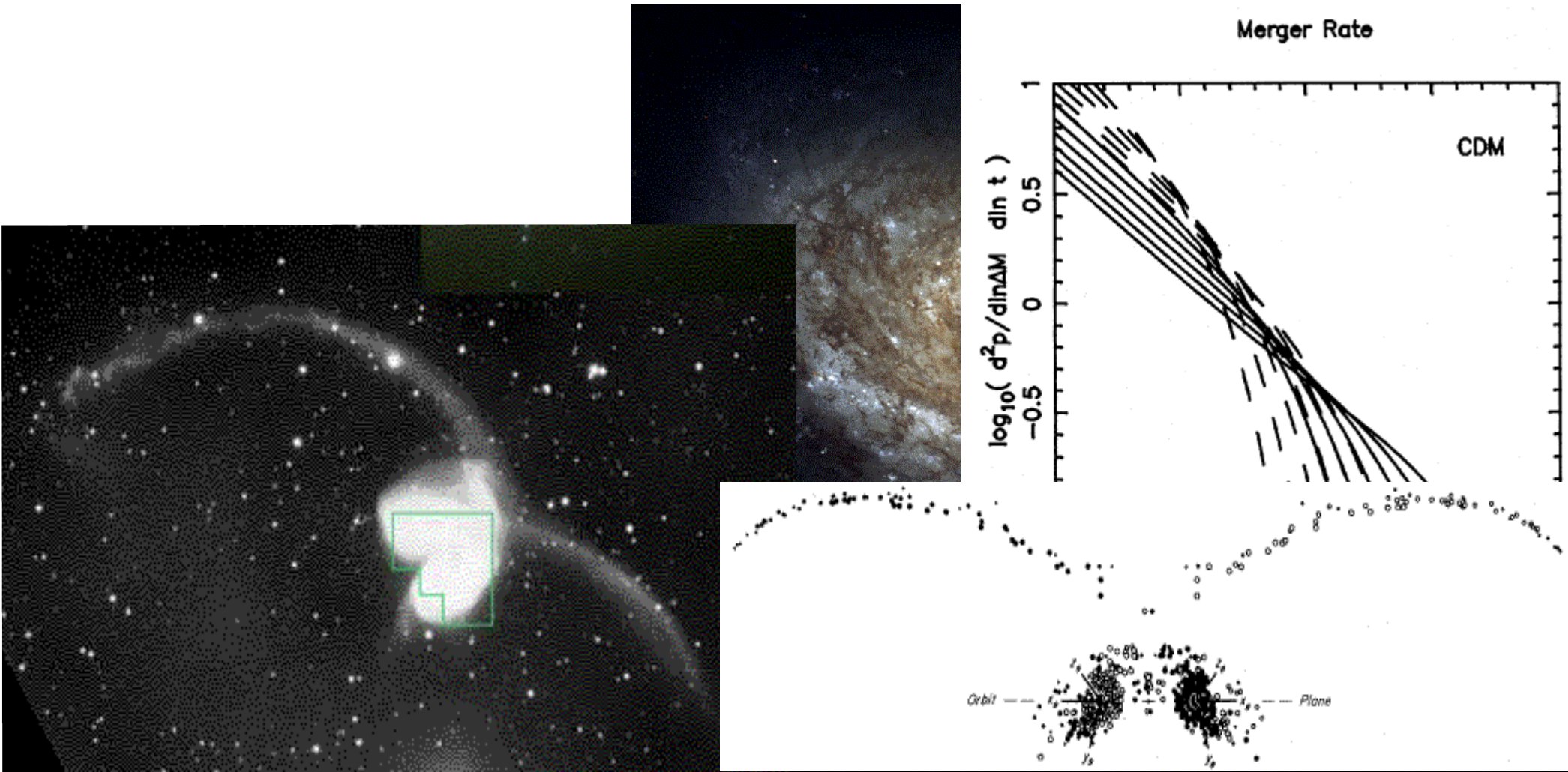


The Three-fold Way to Astrophysical Truth

OBSERVATION

SIMULATION

THEORY



INGREDIENTS FOR SIMULATIONS

- The physical contents of the Universe
 - Ordinary (baryonic) matter – protons, neutrons, electrons
 - Radiation (photons, neutrinos...)
 - Dark Matter
 - Dark Energy
- The Laws of Physics
 - General Relativity
 - Electromagnetism
 - Standard model of particle physics
 - Thermodynamics
- Initial and boundary conditions
 - Global cosmological context
 - Creation of “initial” structure
- Astrophysical phenomenology (“subgrid” physics)
 - Star formation and evolution

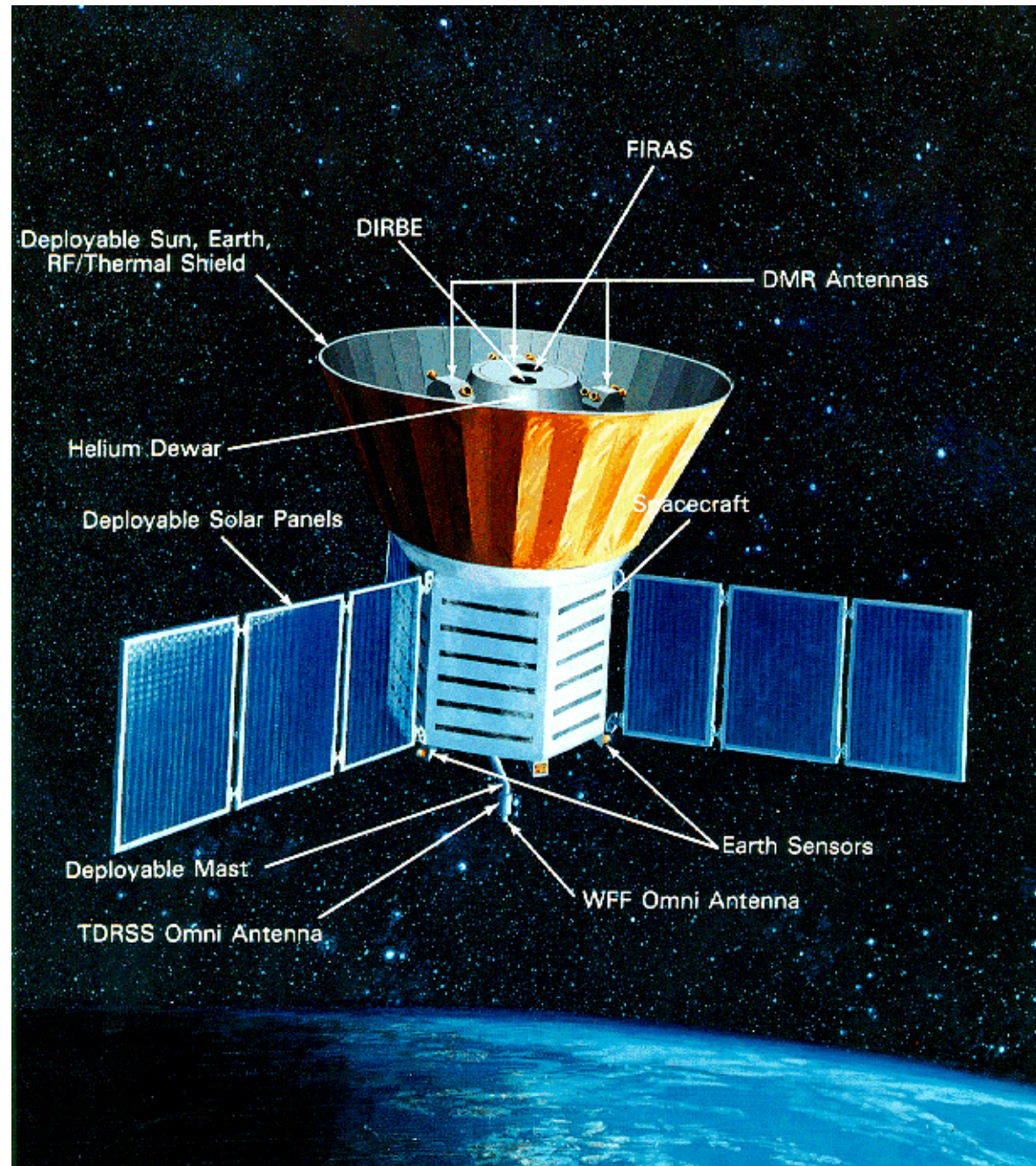
The COBE satellite (1989 - 1993)

- Three instruments

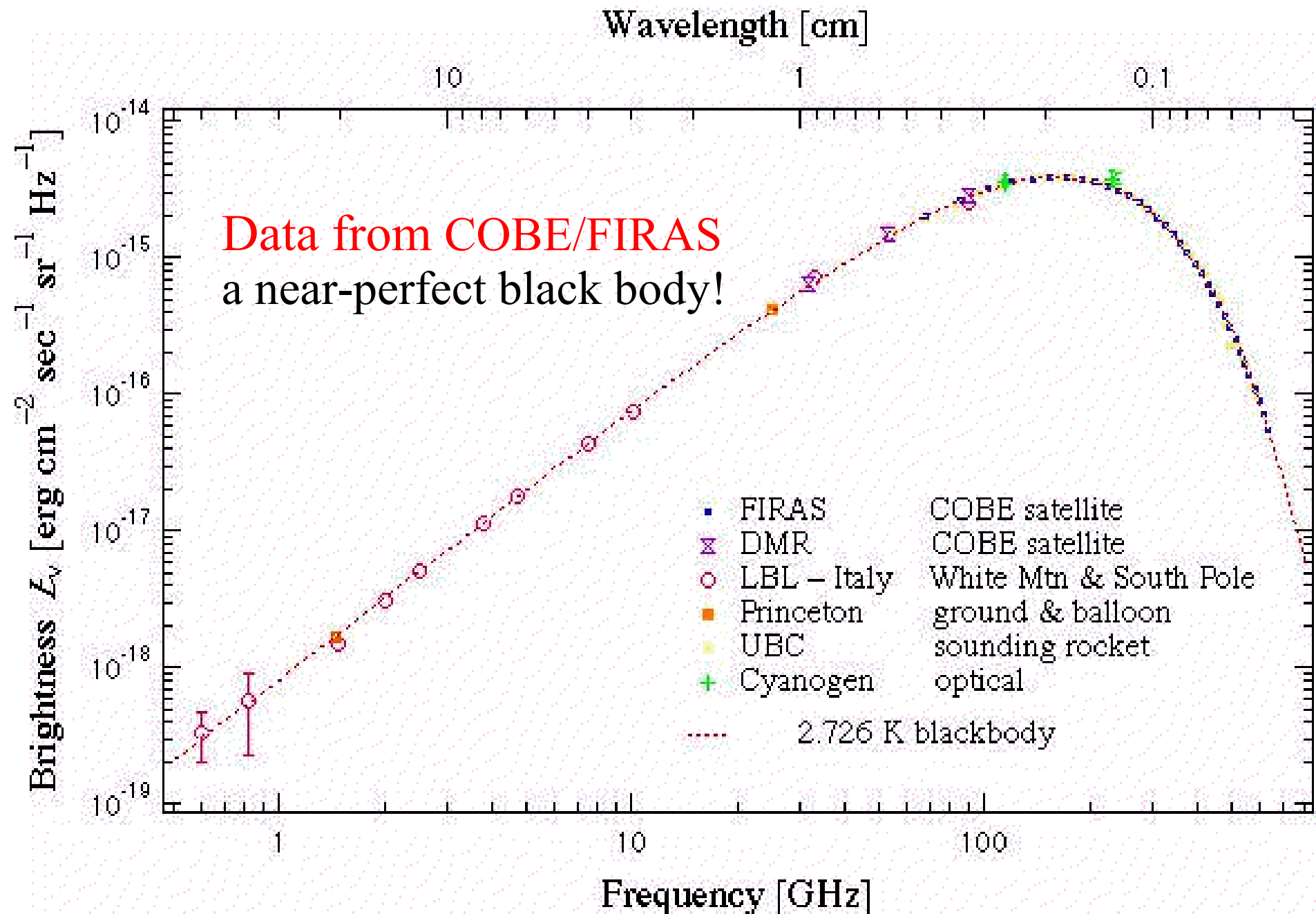
Far Infrared Absolute Spectroph.

Differential Microwave Radiom.

Diffuse InfraRed Background Exp



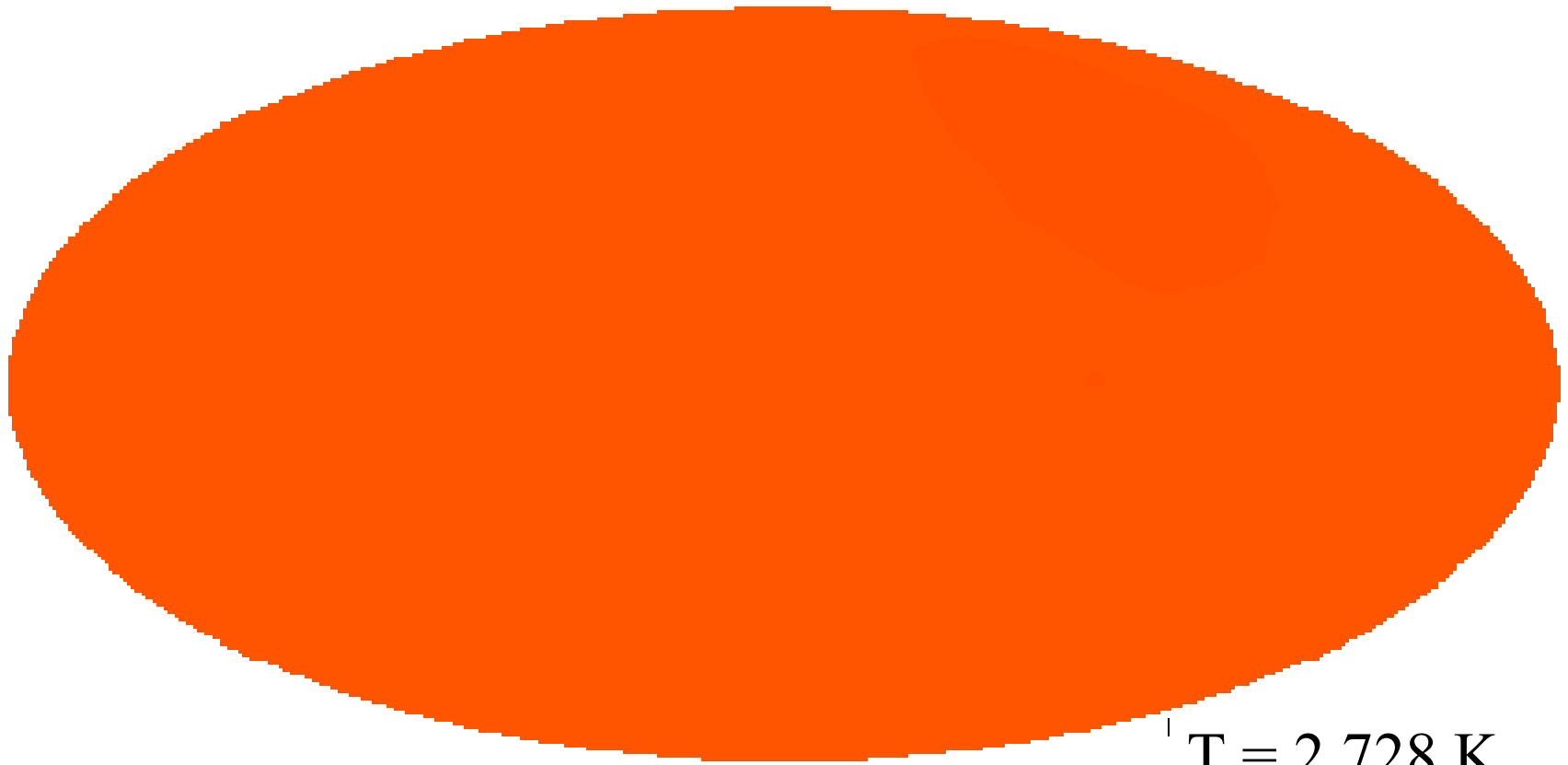
Spectrum of the Cosmic Microwave Background



What do we learn from the COBE spectrum?

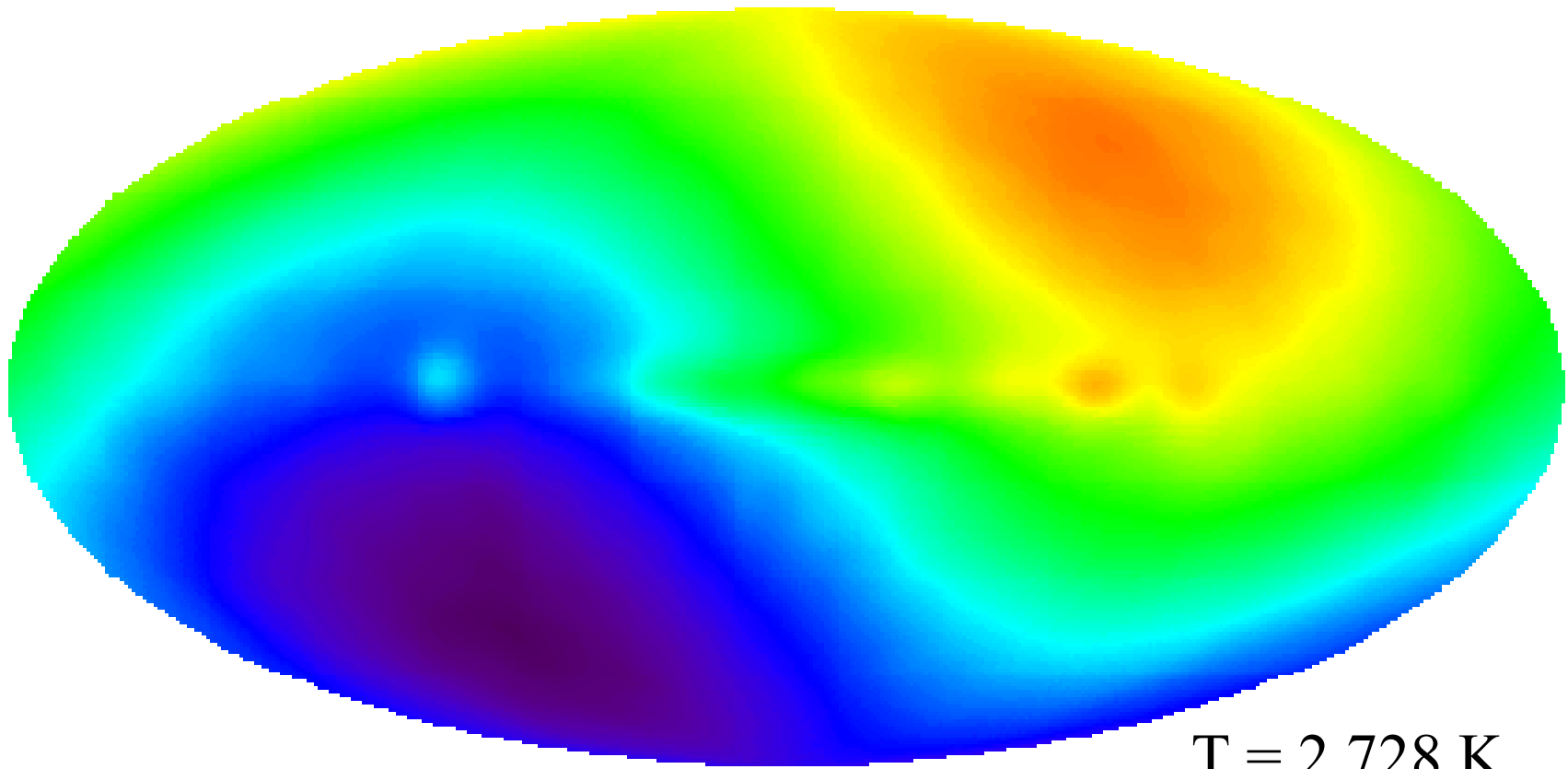
- The microwave background radiation looks like thermal radiation from a 'Planckian black-body'. This determines its temperature
 $T = 2.73\text{K}$
- In the past the Universe was hot and almost without structure
-- Void and without form --
At that time it was nearly in thermal equilibrium
- There has been no substantial heating of the Universe since a few months after the Big Bang itself.

COBE's temperature map of the entire sky



$T = 2.728 \text{ K}$
 $\Delta T = 0.1 \text{ K}$

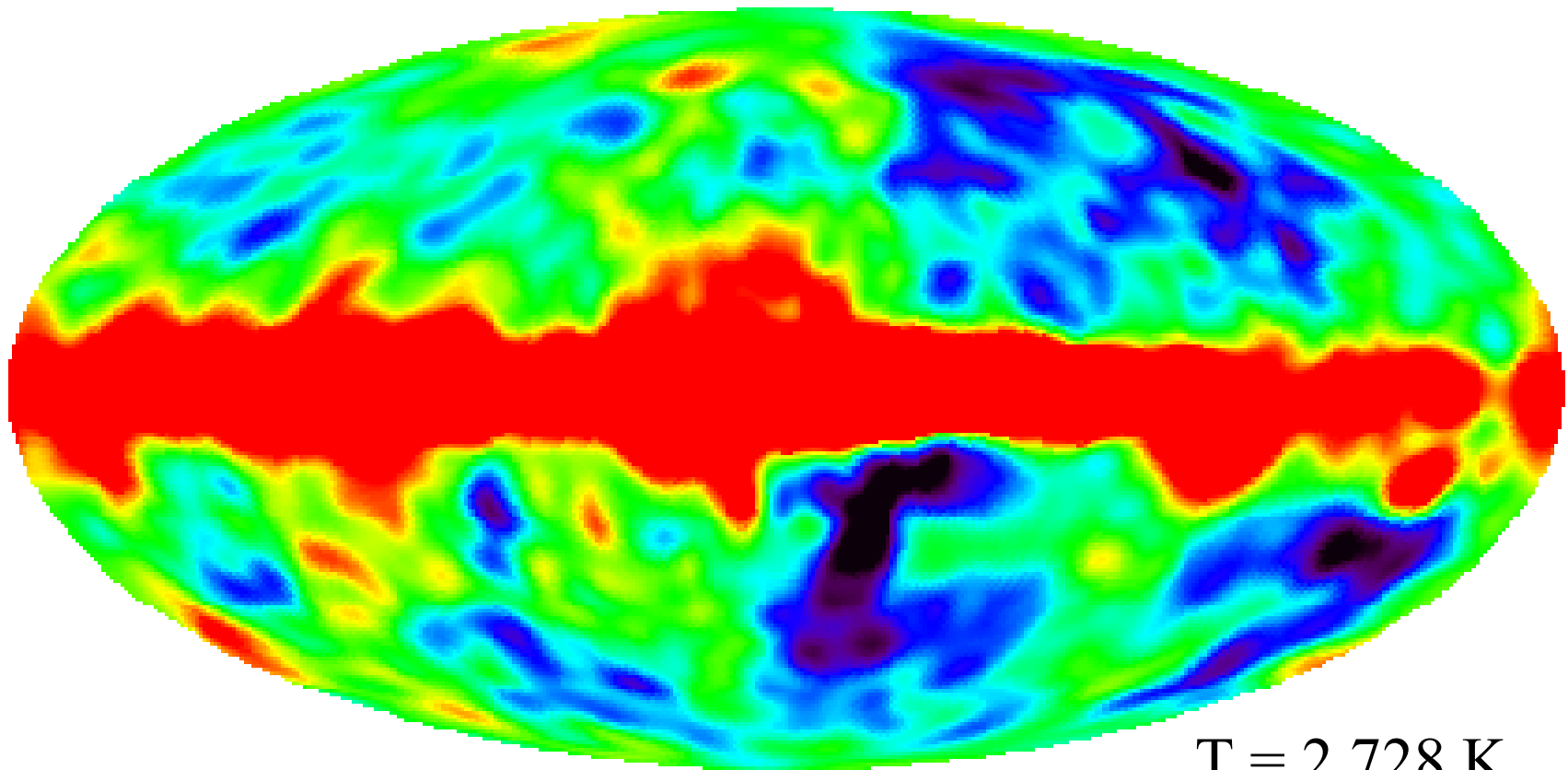
COBE's temperature map of the entire sky



$$T = 2.728 \text{ K}$$

$$\Delta T = 0.0034 \text{ K}$$

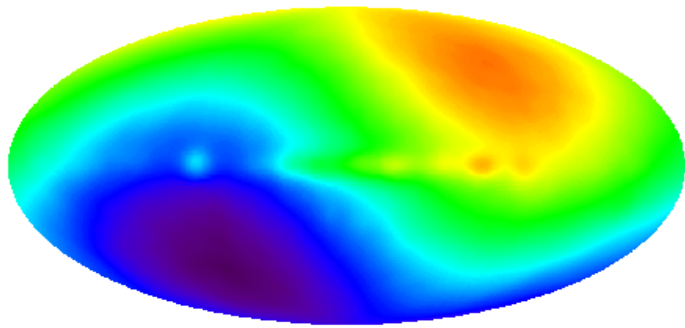
COBE's temperature map of the entire sky



$T = 2.728 \text{ K}$

$\Delta T = 0.00002 \text{ K}$

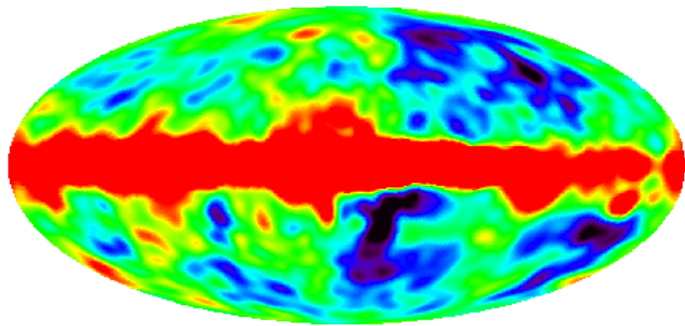
Structure in the COBE map



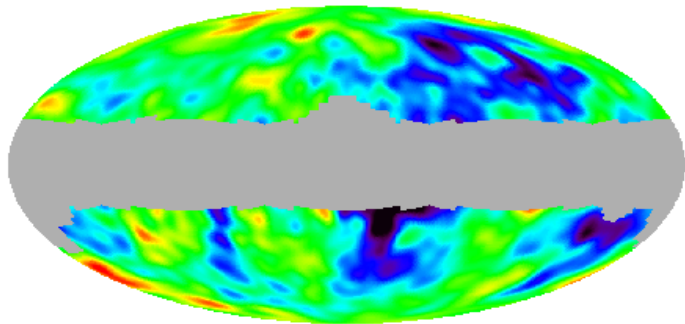
- One side of the sky is 'hot', the other is 'cold'

the Earth's motion through the Cosmos

→ $V_{\text{Milky Way}} = 600 \text{ km/s}$

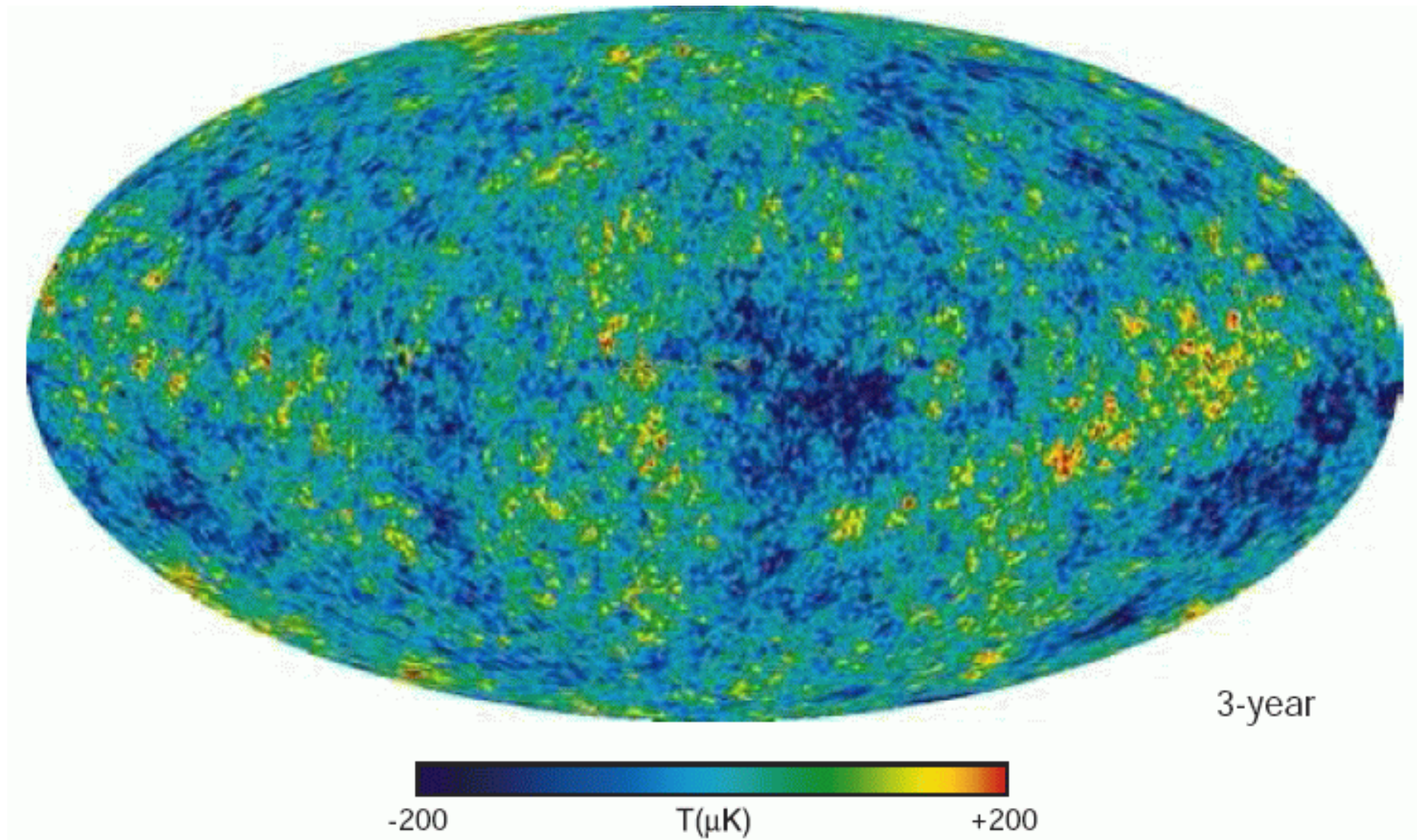


- Radiation from hot gas and dust in our own Milky Way



- Structure in the Microwave Background itself

WMAP's 2006 map of the entire CMB sky



Structure in the Microwave Background

Where is the structure?

In the cosmic 'clouds', 40 billion light years away

What are we seeing?

Weak sound waves in the clouds

When do we see these clouds?

When the Universe was 400,000 years old, and was 1,000 times smaller and 1,000 times hotter than today

How big are the structures?

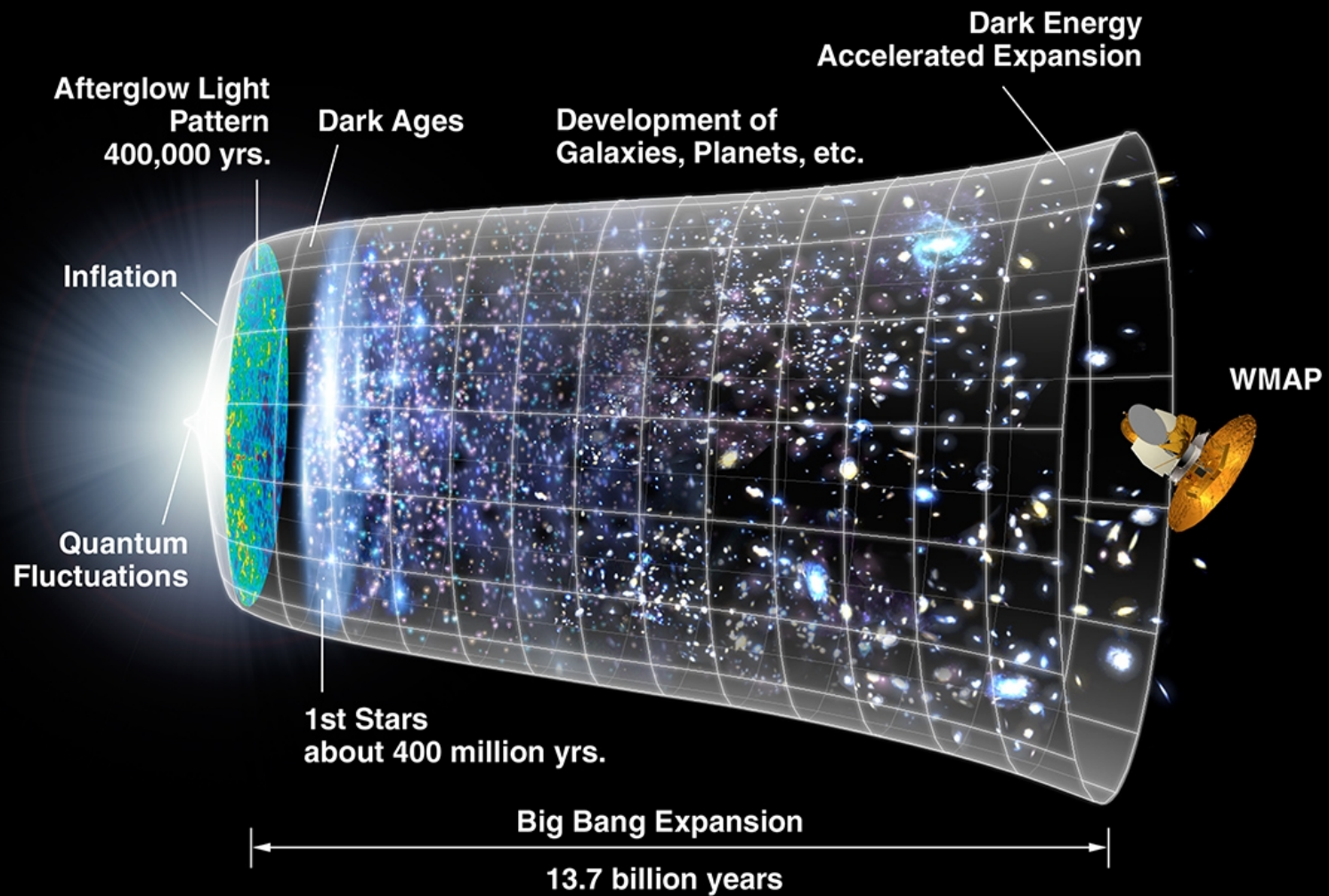
At least a billion light-years across (in COBE maps)

When were they made?

A tiny fraction of a second after the Big Bang

What did they turn into?

Everything we see in the present Universe



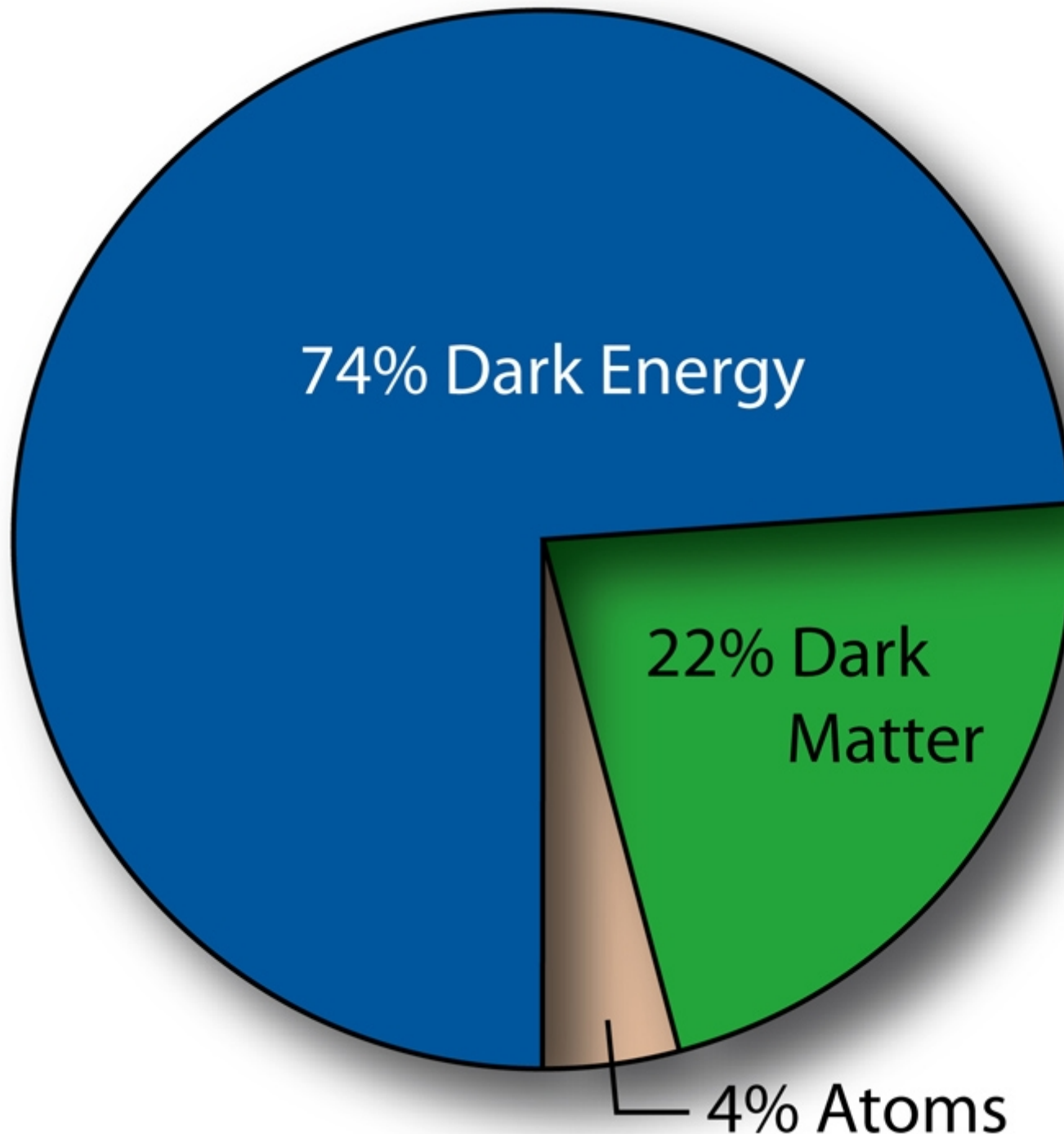
Cosmic History (according to NASA)

What can we learn from these structures?

The pattern of the structures is influenced by several things:

- the Geometry of the Universe
 - finite or infinite
 - eternal or doomed to end
- the Content of the Universe: its fractions in
 - normal (baryonic) matter
 - non-baryonic Dark Matter
 - unseen radiation (neutrinos?)
 - Dark Energy - a cosmological constant?
- the process which created the structure
 - Quantum effects during early inflation?
 - Topological knots from an early phase transition?

**The ingredients of
today's Universe
according to WMAP**



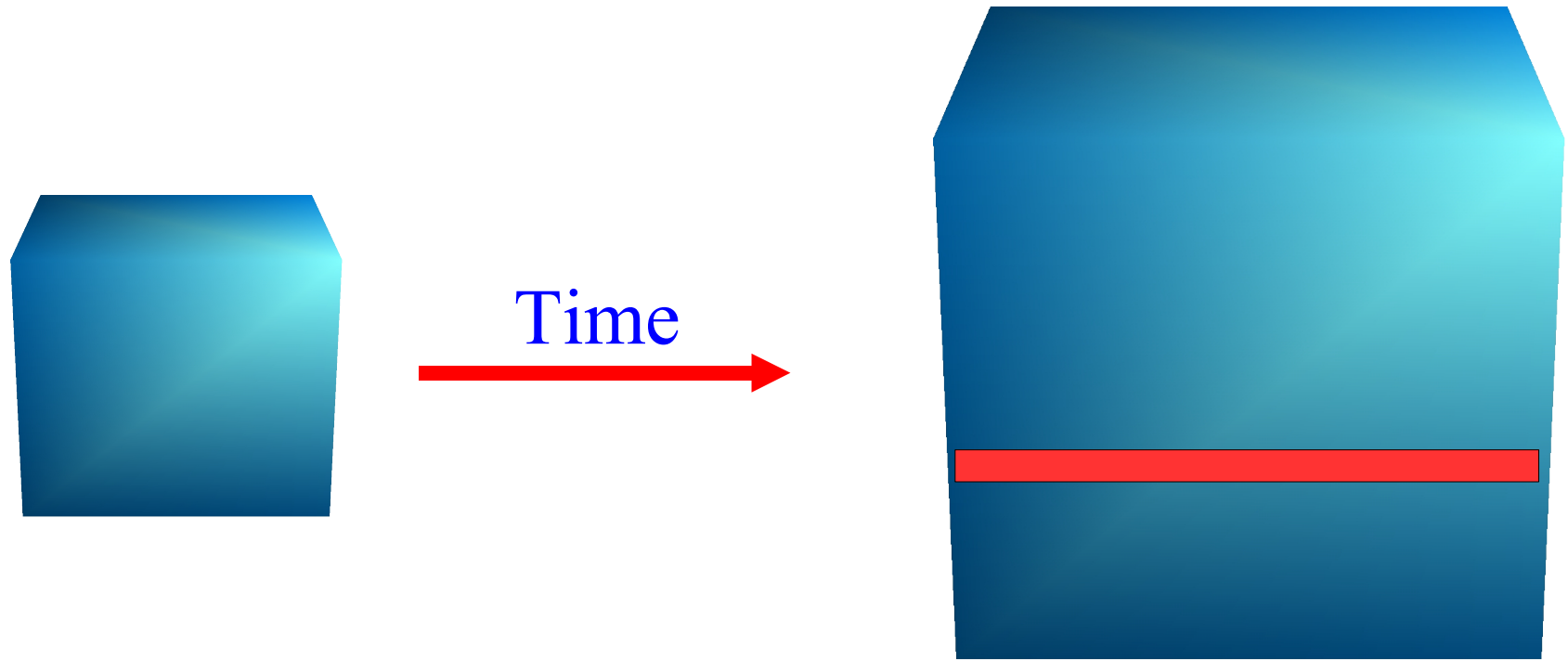
Radiation is about 10^{-4} of the total and is $\frac{3}{4}$ photons and $\frac{1}{4}$ neutrinos

How did the “boring” near-uniform 400,000 year-old Universe produce galaxies/planets/people?

Through the action of physical processes

- Gravity Rules! ..and drives the growth of all structure
- Microphysics of matter/radiation interactions drives the formation of heavy elements and the generation of light
- Macrophysics of (magneto)hydrodynamics and thermodynamics forms stars, galaxies and larger structures
- Phenomenology of complex systems controls the “weather” on galaxy scales and beyond

Evolving the Universe in a computer



- Follow the matter in an expanding cubic region
- Start 400,000 years after the Big Bang
- Match initial conditions to the observed Microwave Background
- Calculate evolution forward to the present day

Simulating the evolution of the DM distribution

- Represent the distribution of the N_{true} particles of DM in the simulated region by N_{sim} simulation particles.

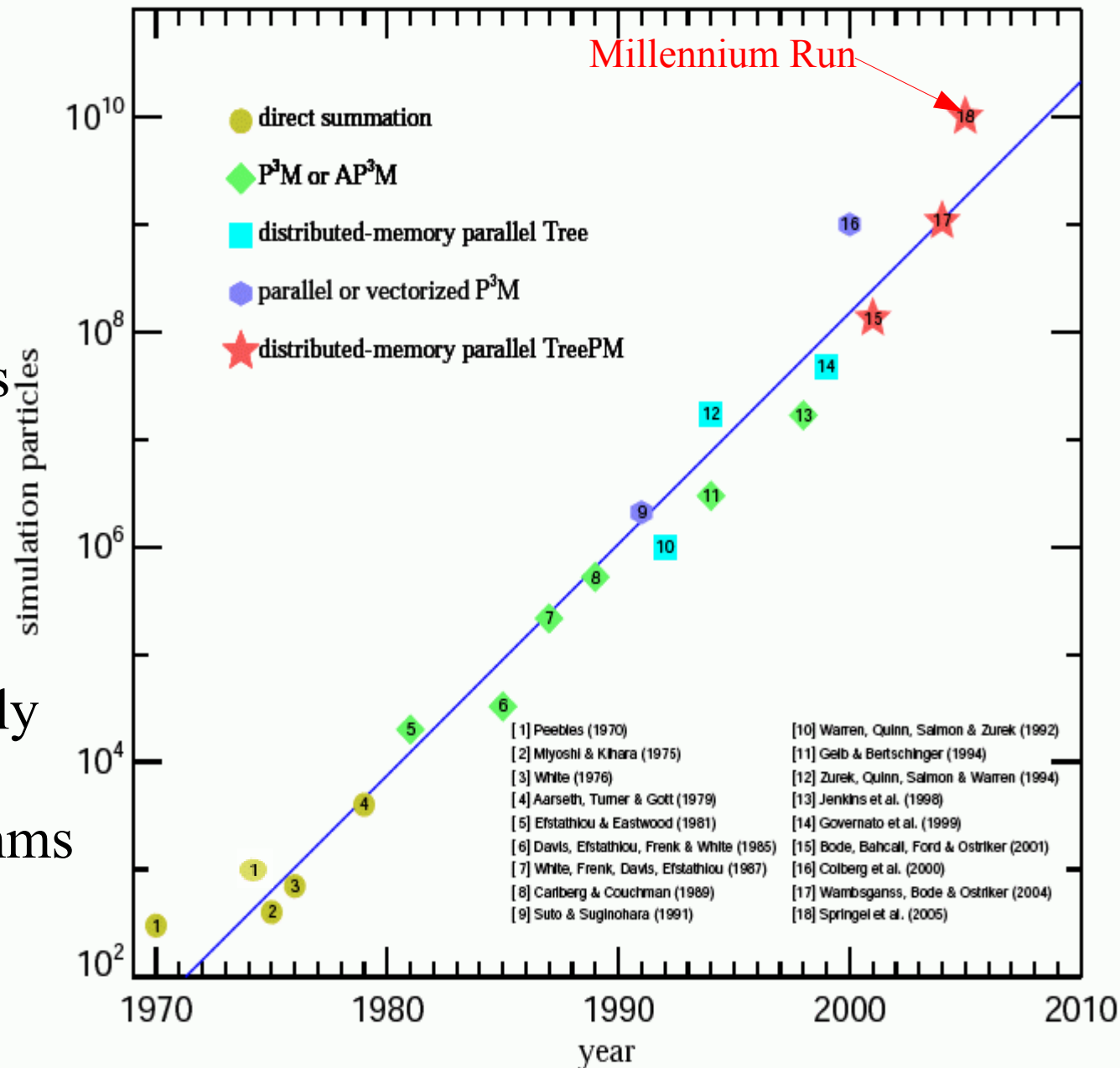
Typically $N_{\text{true}} / N_{\text{sim}} \sim 10^{60}$ or more!

- Place these uniformly within the computational volume
- Perturb their positions and velocities with a random realisation of the perturbation field predicted at early times by a theoretical model, e.g. Λ CDM – Cold Dark Matter + a cosmological constant + baryons + inflationary initial structure
- $$\ddot{\mathbf{x}}_i = \nabla_i \Phi = -\sum_j G m_i m_j \mathbf{r}_{ij} / r_{ij}^3 \quad i, j = 1, N$$
-  Solve for the gravitational forces
Step the particles forward a few million years  Repeat until the present

Moore's Law for Cosmological N-body Simulations

Springel et al 2005

- Computers double their speed every 18 months
- A naive N-body force calculation needs N^2 op's
- Simulations double their size every 16.5 months
- Progress has been roughly equally due to hardware and to improved algorithms

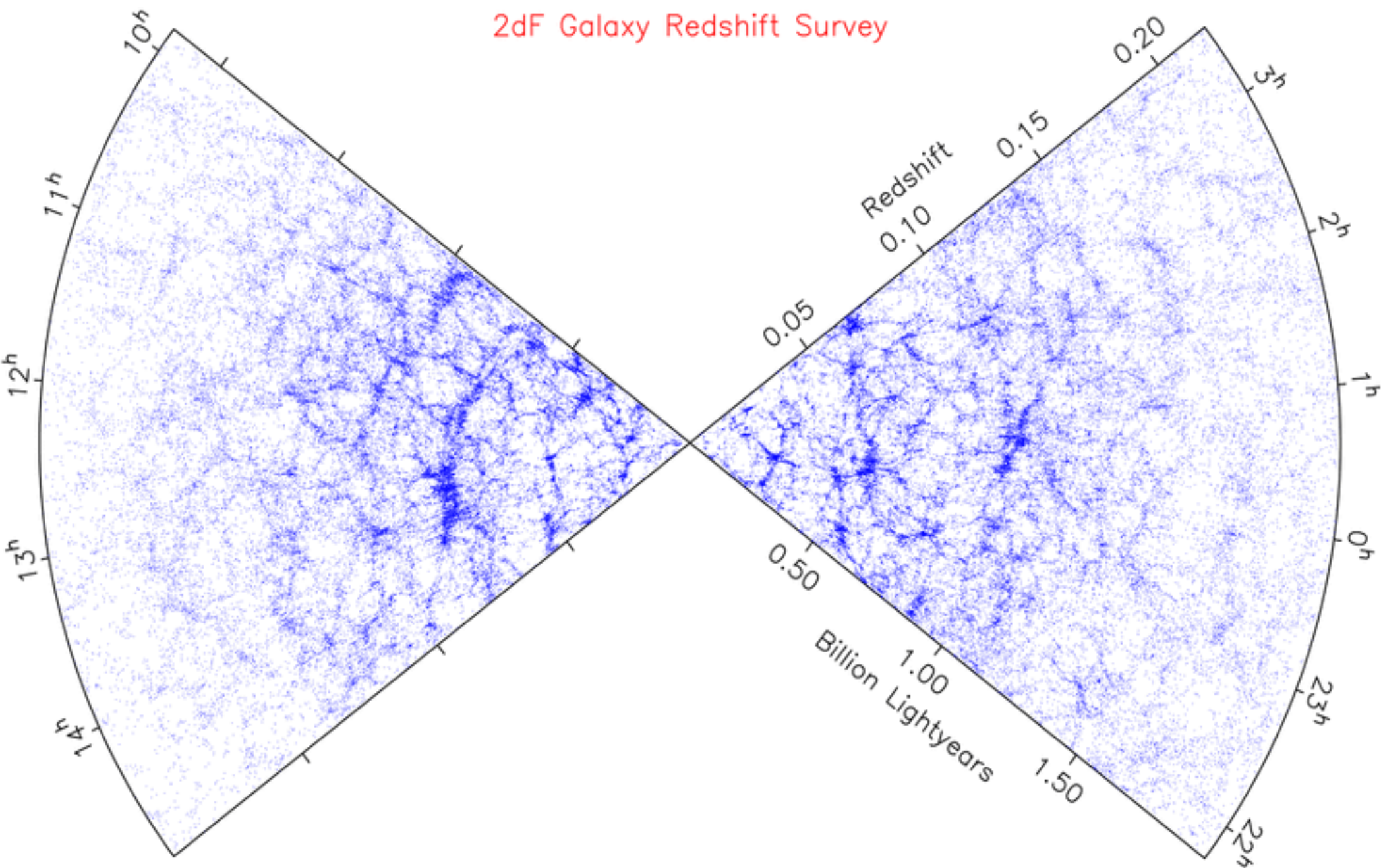


Large-scale structure in the Dark Matter

- Gravity sharpens and enhances the initial pattern
- The large-scale structure is a network of filaments
- On very large scales the Universe stays homogeneous
- Clumps form within filaments and at their intersections
- Flow along filaments channels small lumps onto big ones
- Big lumps have internal structure made of the remnants of smaller lumps which have fallen in to them
- All characteristic scales increase with time

Nearby large-scale structure

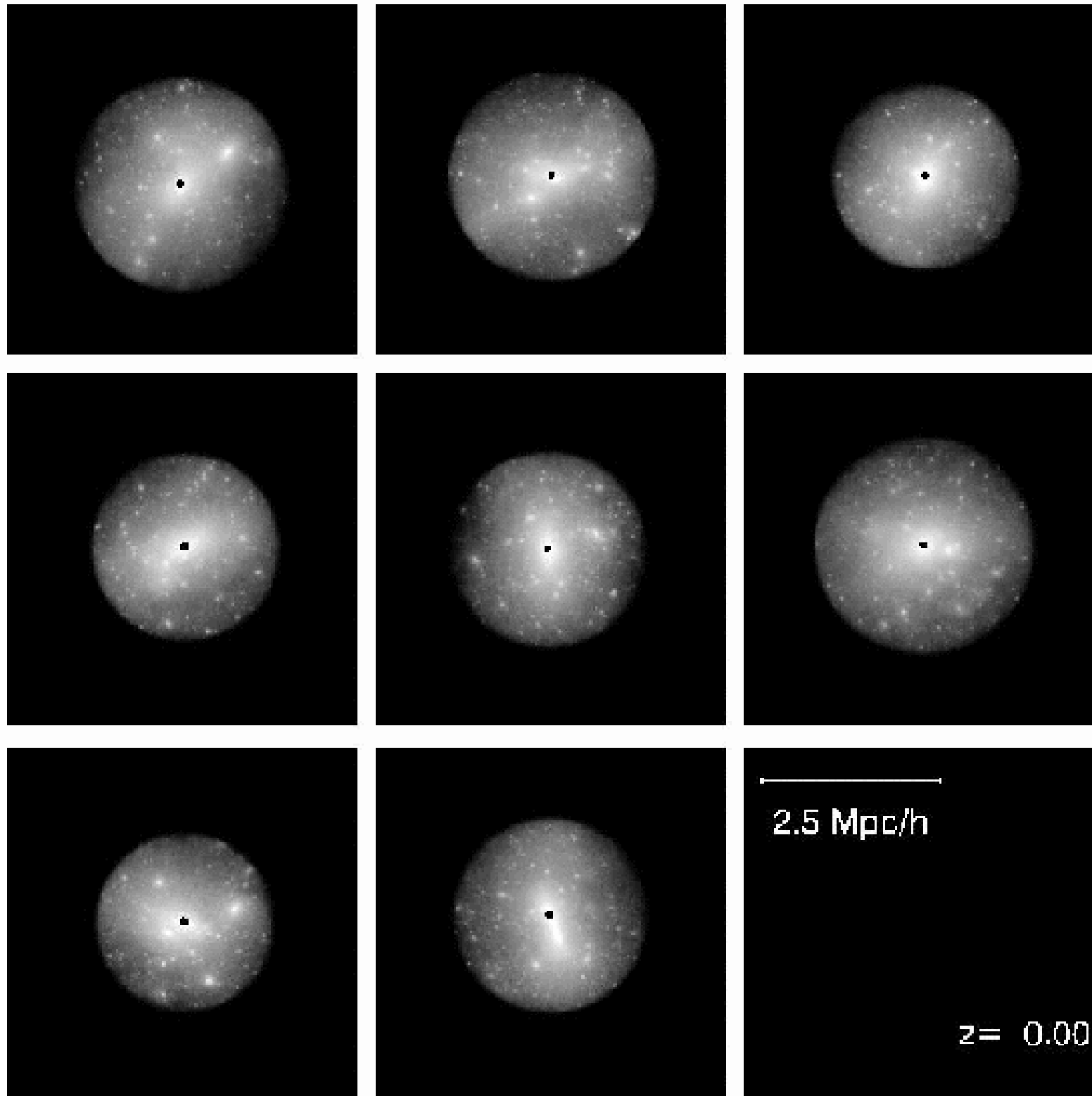
2dF Galaxy Redshift Survey



Cluster structure in Λ CDM

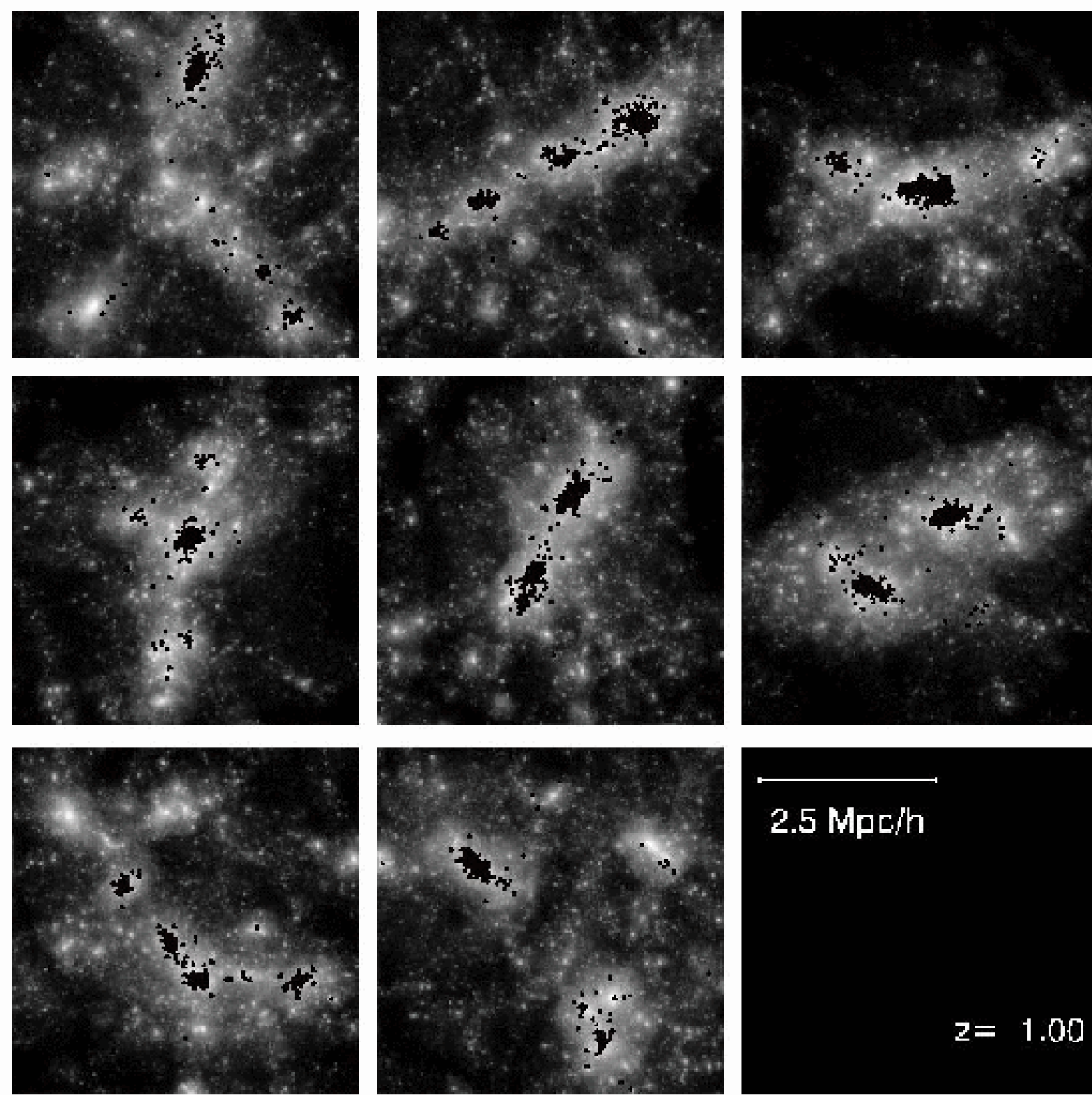
- 'Concordance' cosmology
- Final cluster mass $\sim 10^{15} M_{\odot}$
- DM within 20 kpc at $z = 0$ is shown black

Gao et al 2004a



Cluster structure in Λ CDM

- 'Concordance' cosmology
- Final cluster mass $\sim 10^{15} M_{\odot}$
- DM within 20 kpc at $z = 0$ is shown black



2.5 Mpc/h

$z = 1.00$

Gao et al 2004a

Cluster structure in Λ CDM

- 'Concordance' cosmology
- Final cluster mass $\sim 10^{15} M_{\odot}$
- DM within 20 kpc at $z = 0$ is shown black

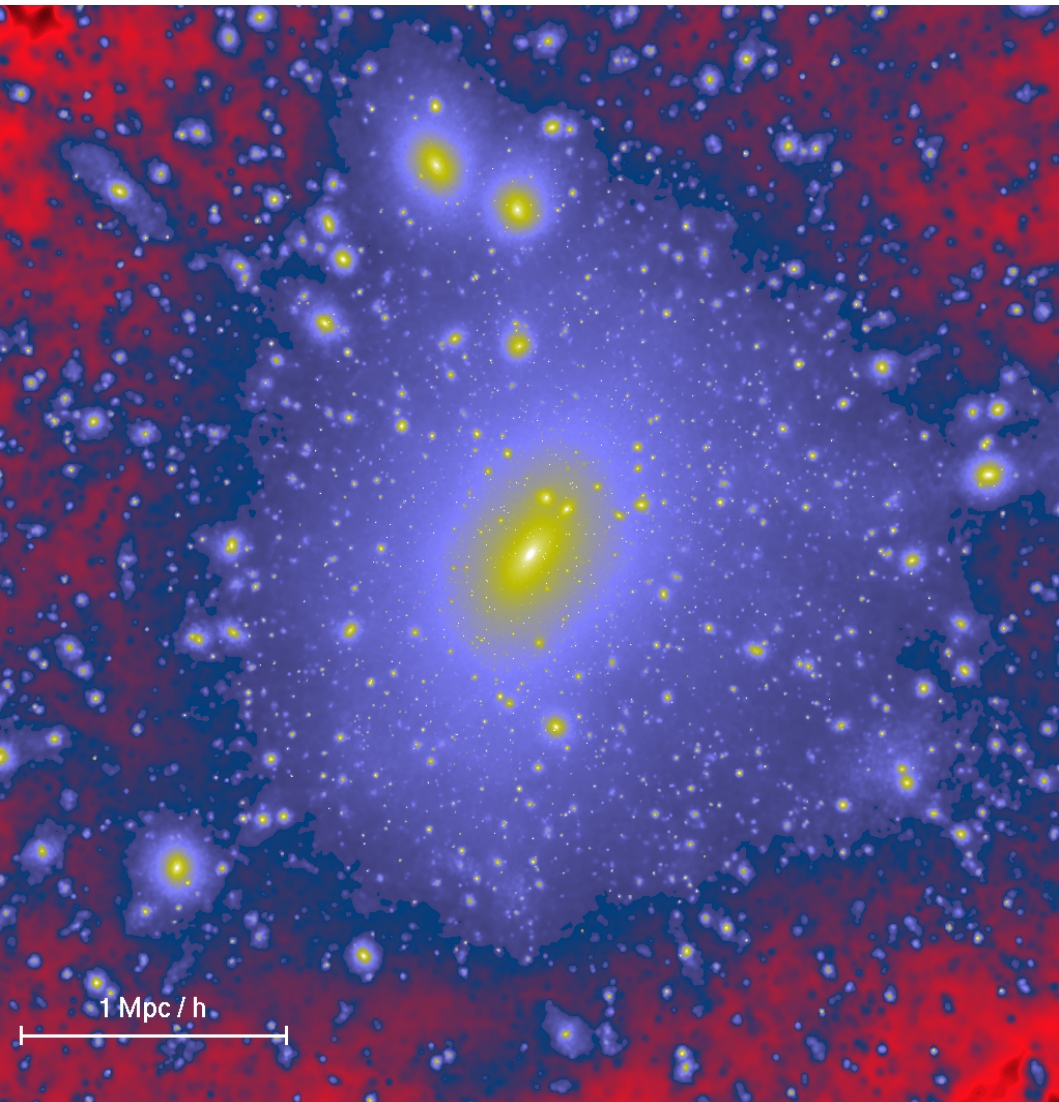
2.5 Mpc/h

$z = 3.00$

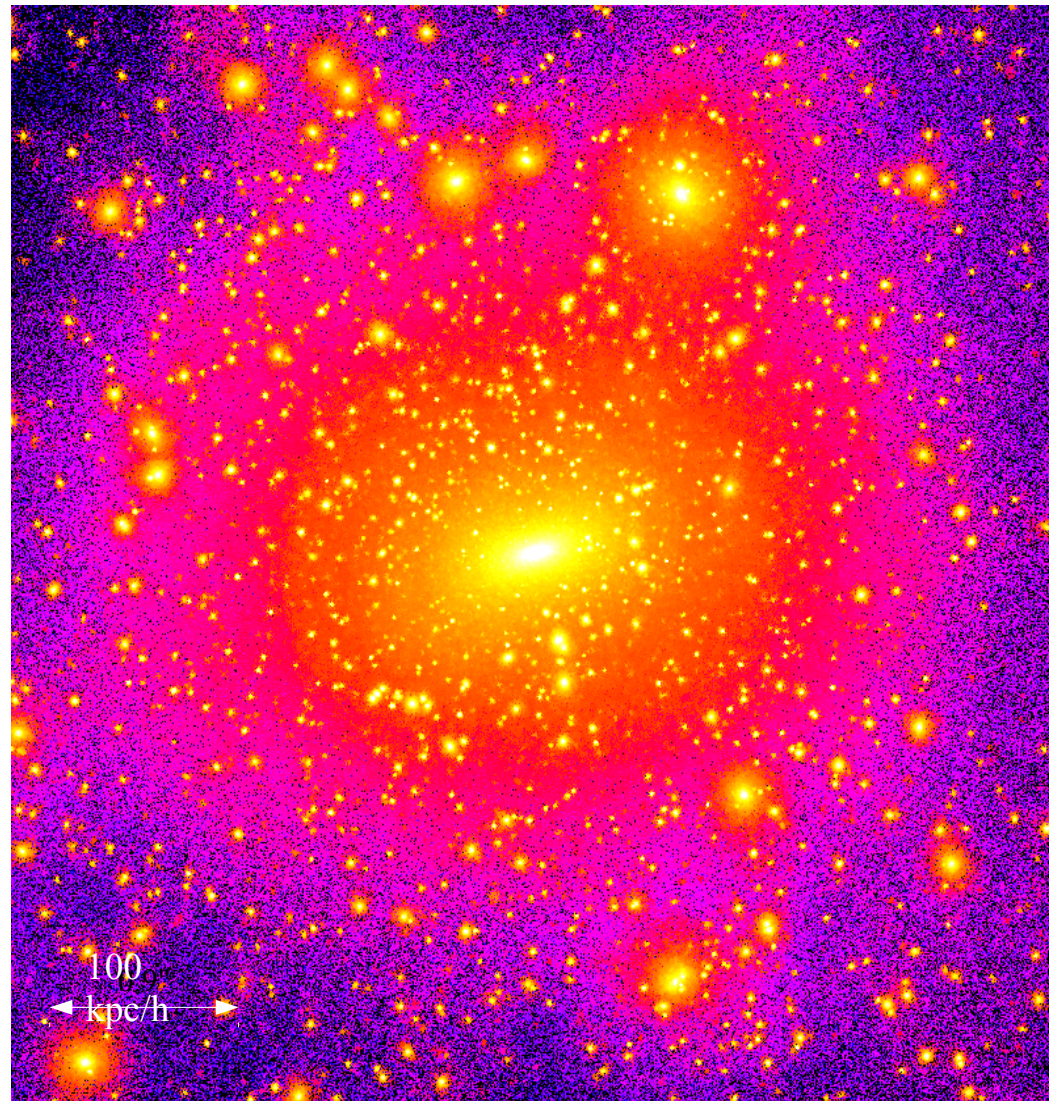
Gao et al 2004a

Small-scale structure in Λ CDM halos

A rich galaxy cluster halo
Springel et al 2001

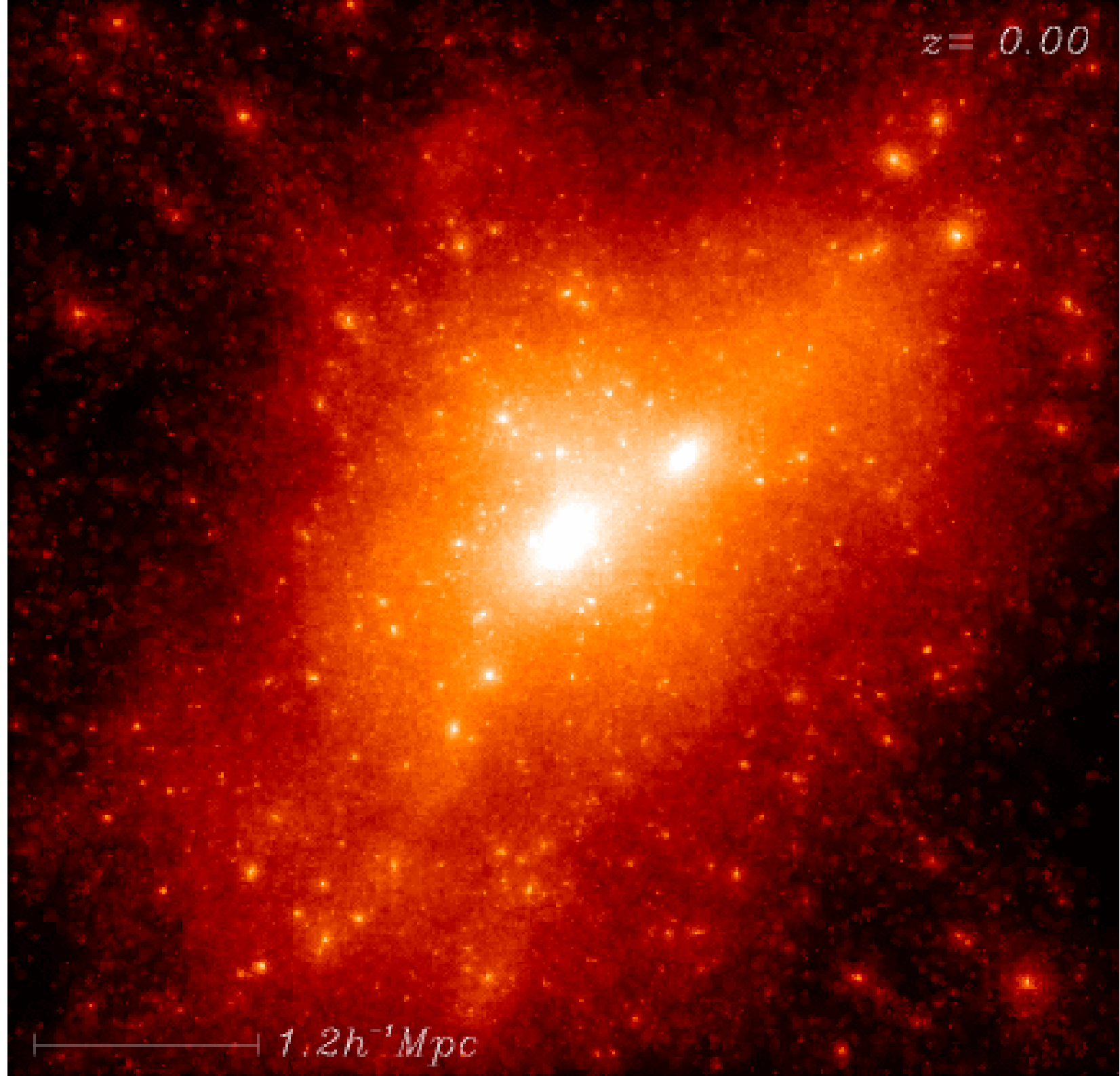


A 'Milky Way' halo
Power et al 2002



ROT

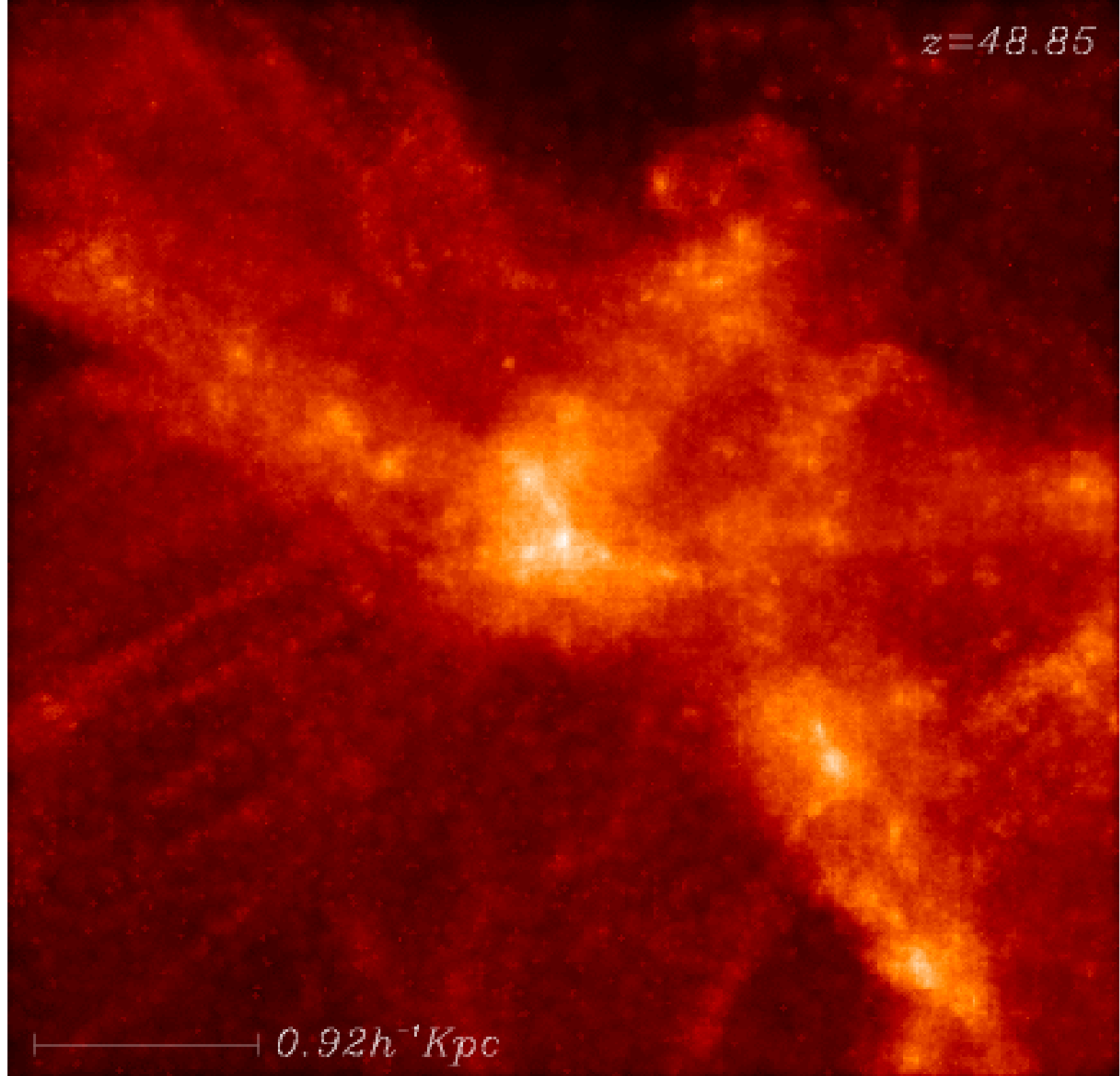
$z = 0.00$



EVOL

ROT

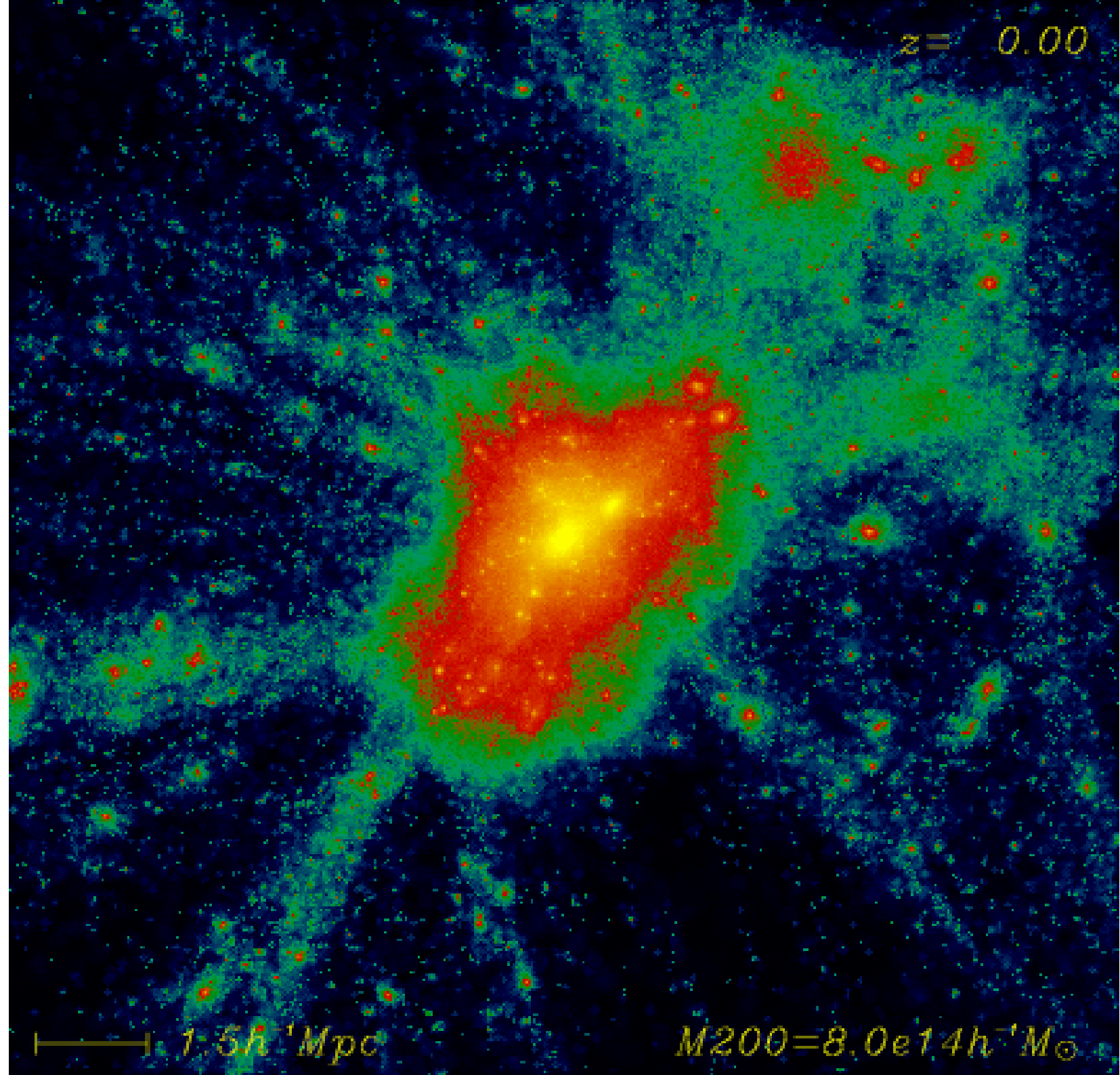
$z=48.85$



EVOL

0.92 h⁻¹ Kpc

$z = 0.00$



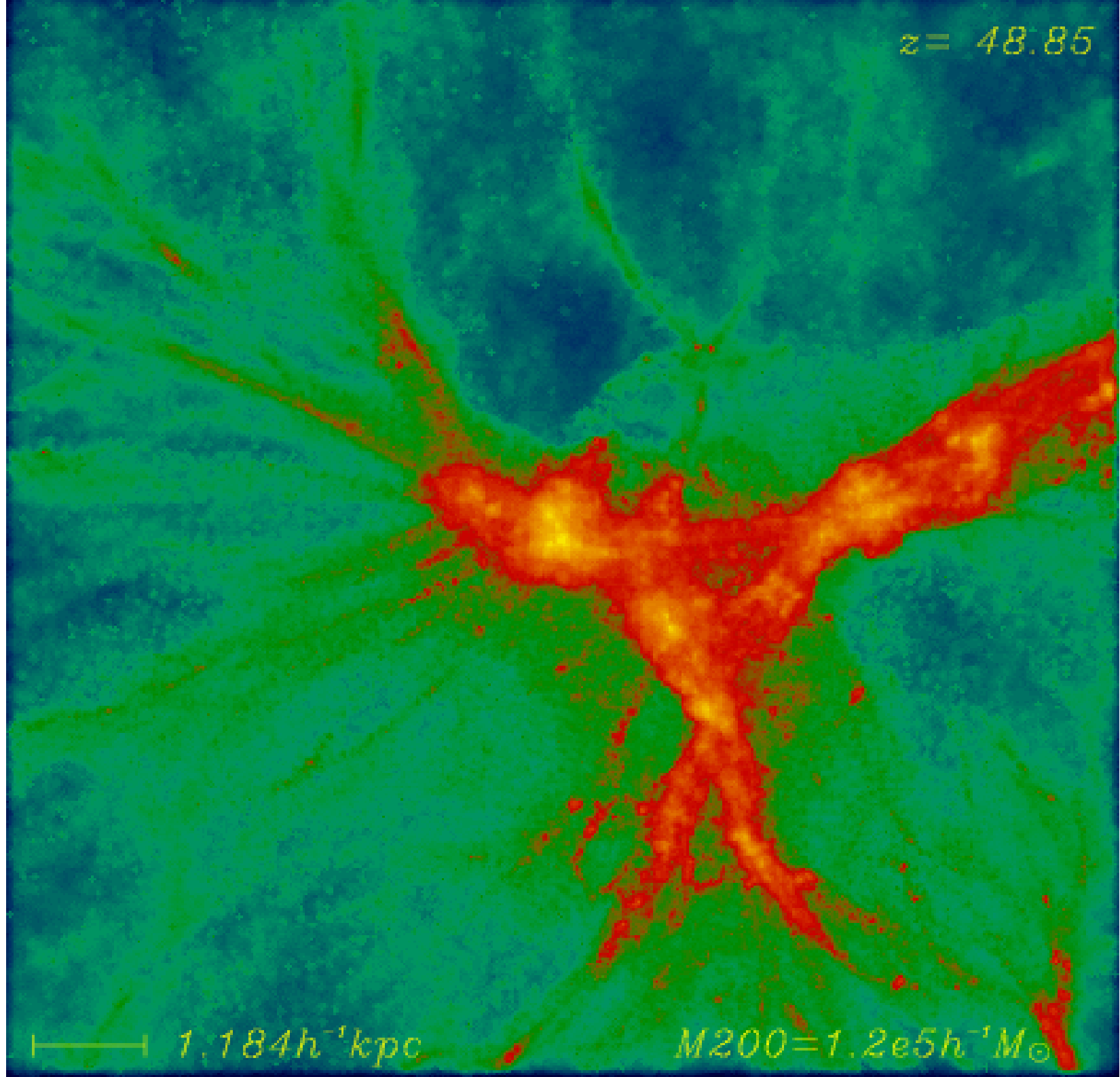
$1.5 h^{-1} \text{Mpc}$

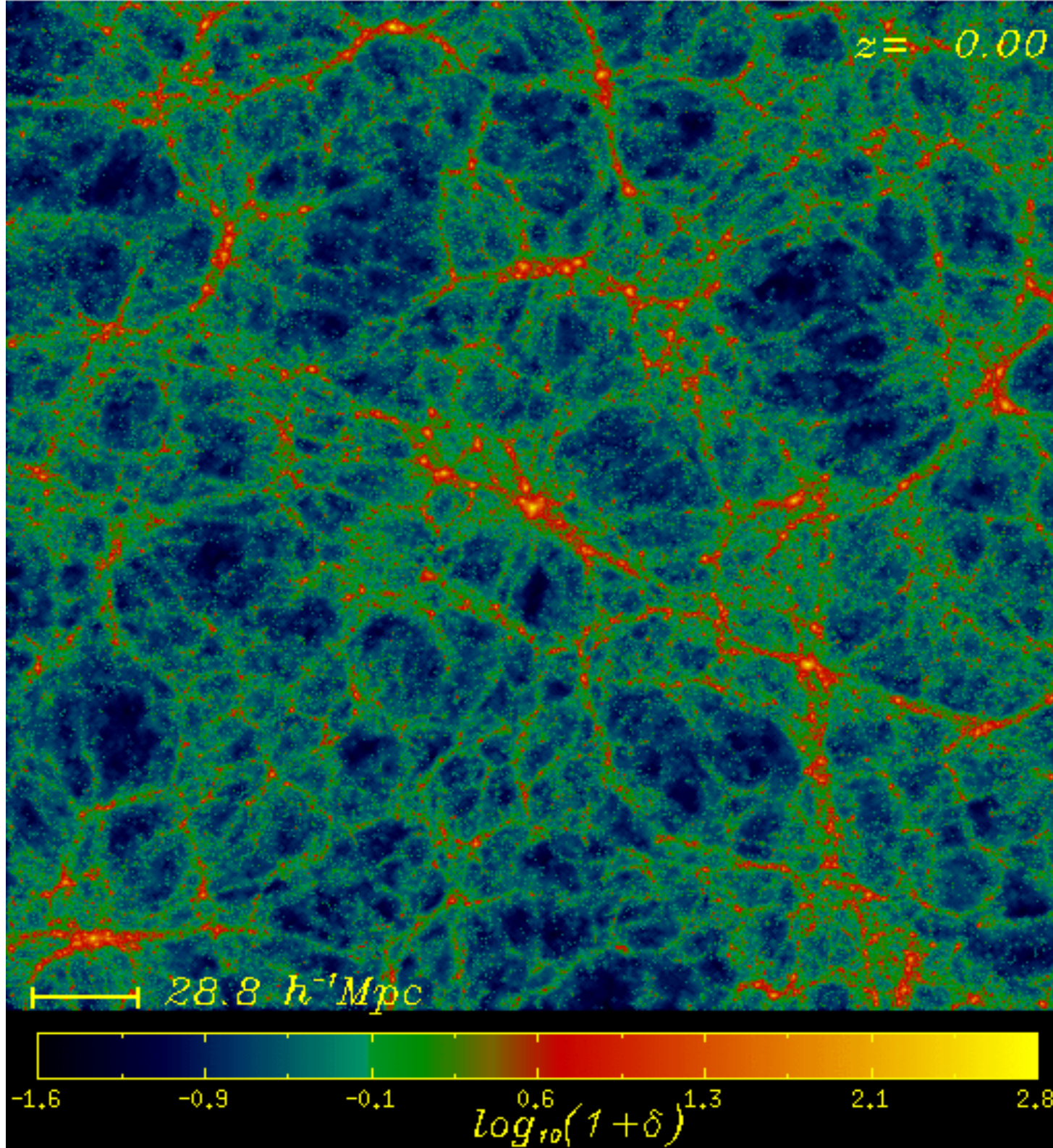
$M_{200} = 8.0e14 h^{-1} M_{\odot}$

$z = 48.85$

1.184 h^{-1} kpc

$M_{200} = 1.2 \times 10^5 h^{-1} M_{\odot}$





**Simulating
the growth
of a cluster
from $z=100$
to $z=0$**

Gao et al 2005

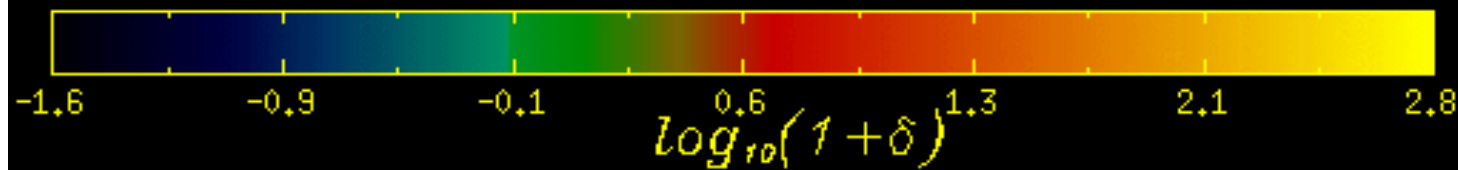
$z = 48.85$

**Simulating
the growth
of a cluster
from $z=100$
to $z=0$**

Gao et al 2005

Large-scale at
 $z=49$
does NOT look
like
that at $z=0$

$22.5 h^{-1} \text{kpc}$

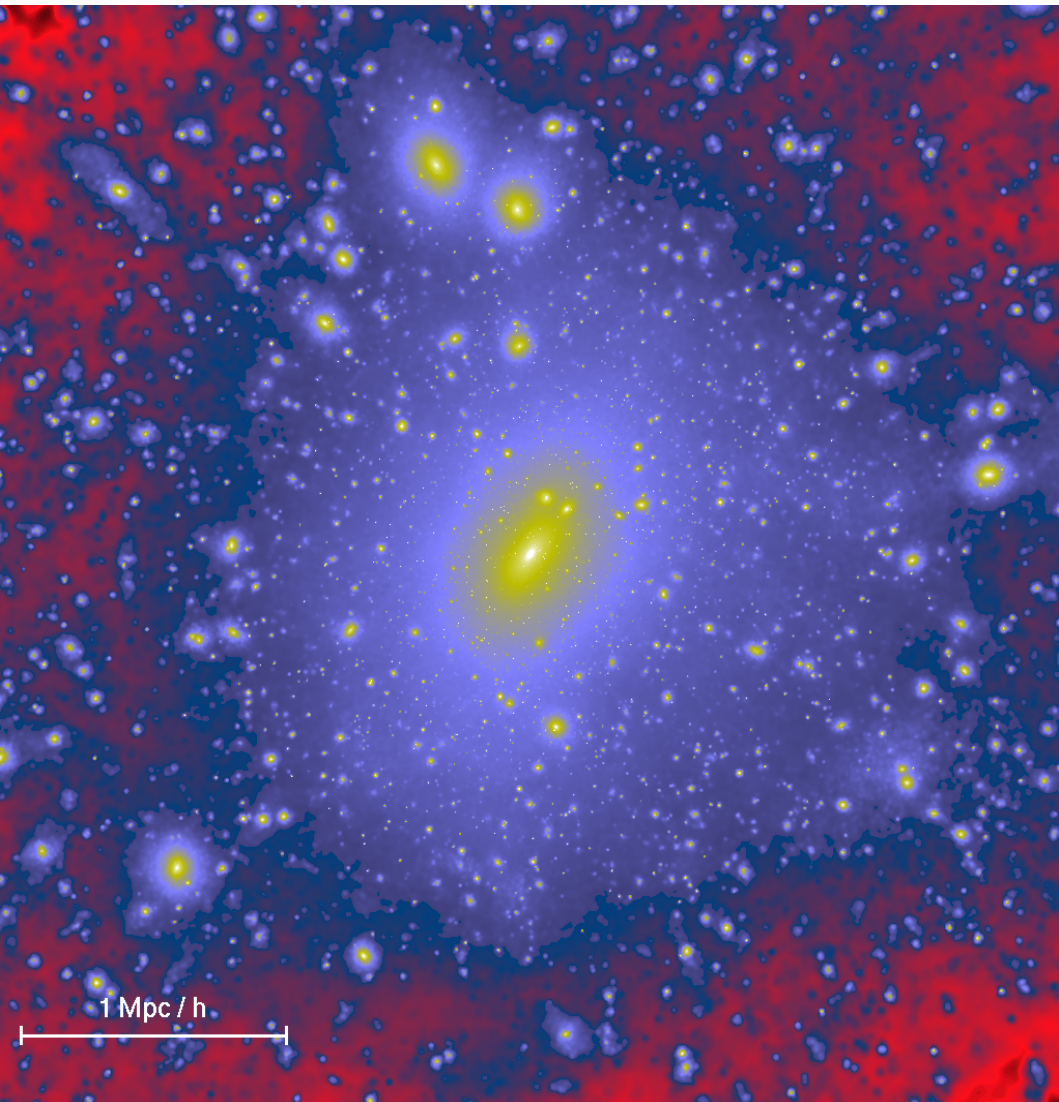


Λ CDM galaxy halos (without galaxies!)

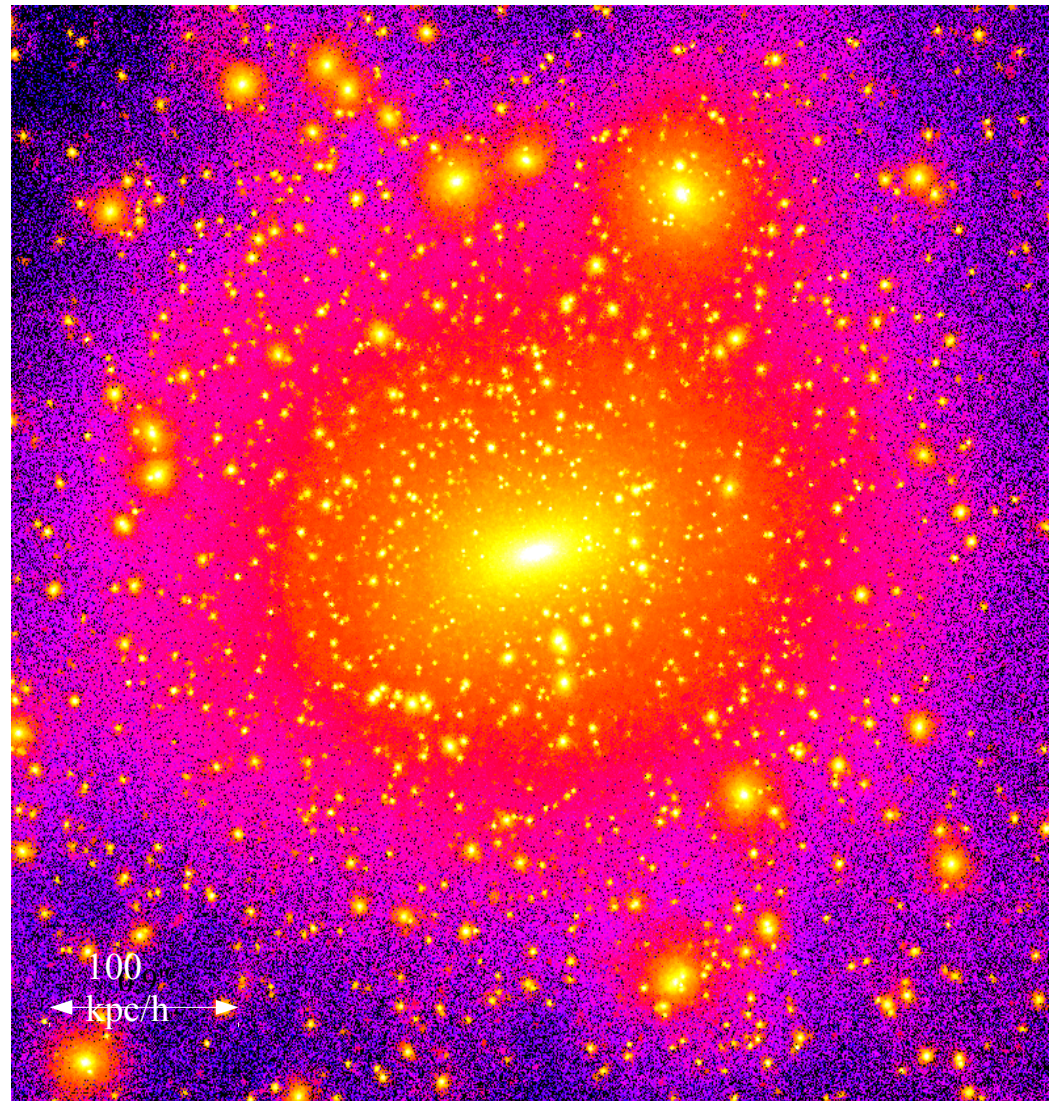
- Halos extend to ~ 10 times the 'visible' radius of galaxies and contain ~ 10 times the mass in the visible regions
- Halos are not spherical but approximate triaxial ellipsoids
 - more prolate than oblate
 - axial ratios greater than two are common
- "Cuspy" density profiles with outwardly increasing slopes
 - $d \ln \rho / d \ln r = \gamma$ with $\gamma < -2.5$ at large r
 $\gamma > -1.2$ at small r
- Substantial numbers of self-bound subhalos contain $\sim 10\%$ of the halo's mass and have $dN/dM \sim M^{-1.8}$
 - Most substructure mass is in most massive subhalos

Small-scale structure in Λ CDM halos

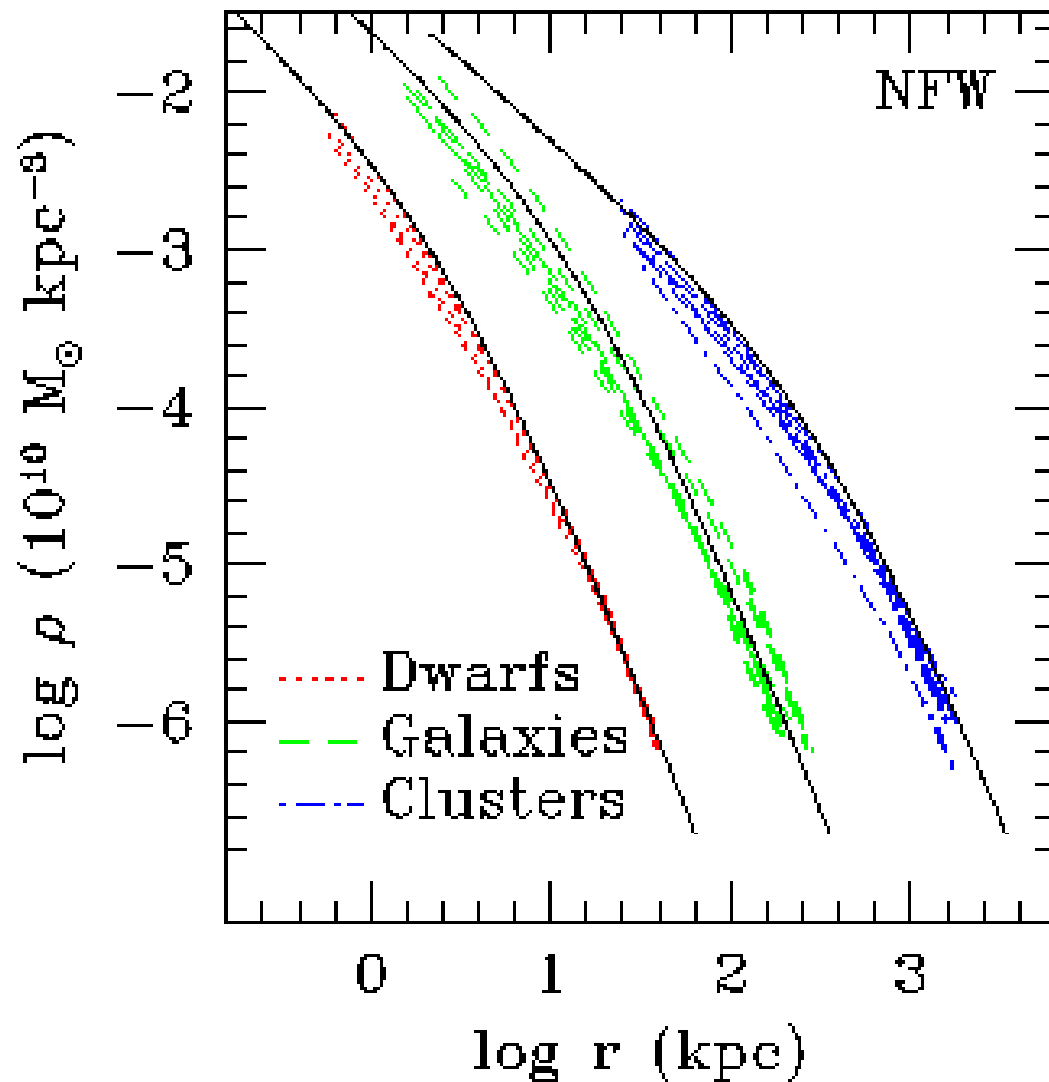
A rich galaxy cluster halo
Springel et al 2001



A 'Milky Way' halo
Power et al 2002



Density profiles of dark matter halos



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times

-- a universal profile shape --

$$\rho(r)/\langle\rho\rangle \approx \delta \, r_s / r (1 + r/r_s)^2$$

More massive halos and halos that form earlier have higher densities (bigger δ)

Dark halo tests of Λ CDM?

- Measures of the shape and density profile of halos from
 - gravitational lensing
 - dynamics of visible tracers (stars, satellite galaxies)
- Limits on central cusps from galaxy rotation curves
- Limits on amount of substructure from
 - numbers of observed satellite objects
 - image distortion of background lensed objects

Current results from these tests are in most cases controversial.

- The universal profile seems confirmed at large radii
- There are discrepancies at small radii
- Significance of substructure results is debated

NOTE: Predictions for all these properties require simulations

Dark Matter Annihilation

For certain kinds of Dark Matter particles

- Self-annihilation is possible
- Annihilation products will typically include γ -rays

The luminosity density of annihilation emission is

$$\mathcal{L}(\mathbf{x}) \propto n_{\text{DM}}(\mathbf{x})^2 \langle \sigma v \rangle$$

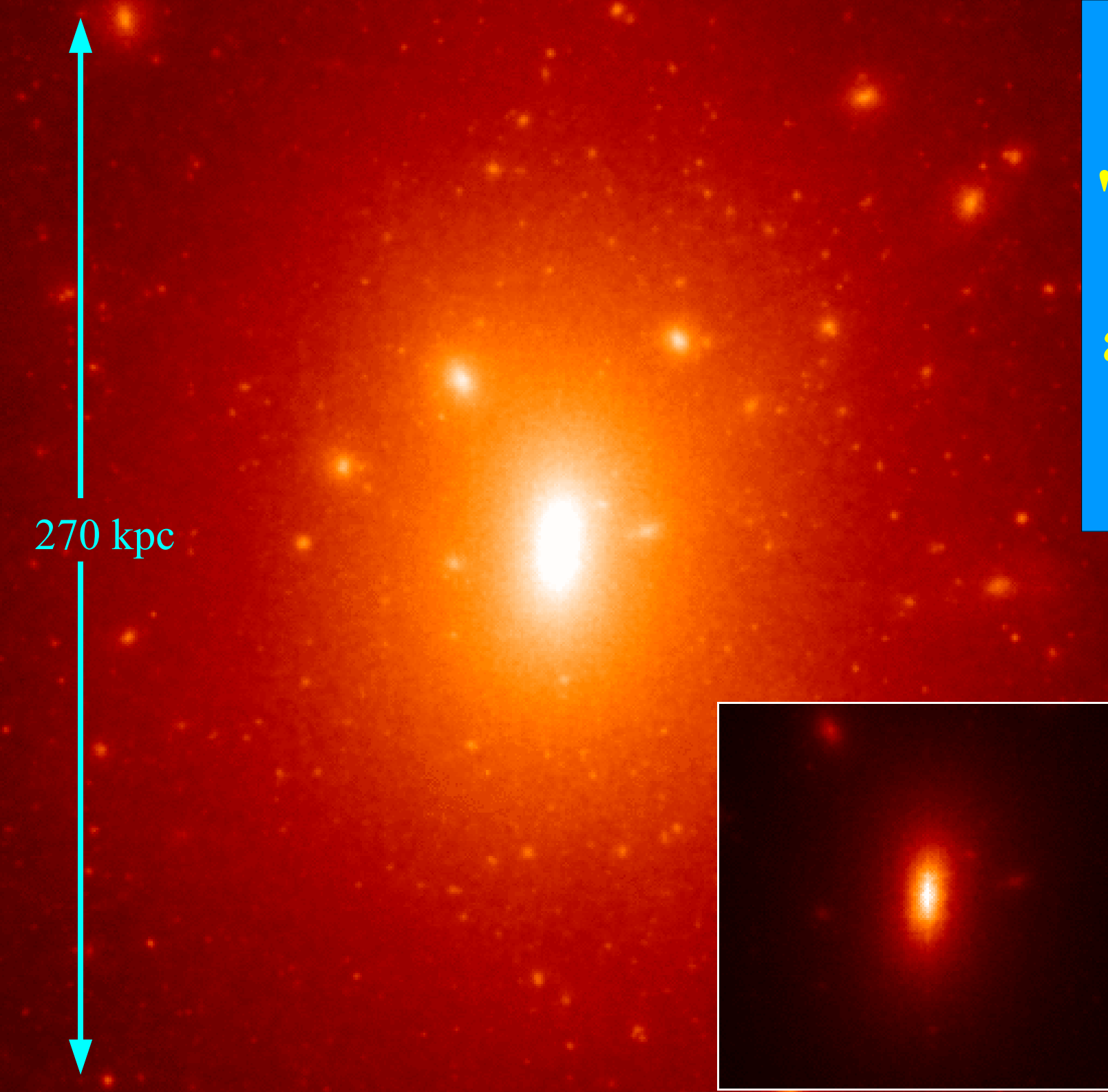
Thus the γ -ray luminosity of an object is

$$L \propto \langle \sigma v \rangle \int \rho^2 dV \propto \langle \sigma v \rangle \int \rho^2 r^2 dr$$

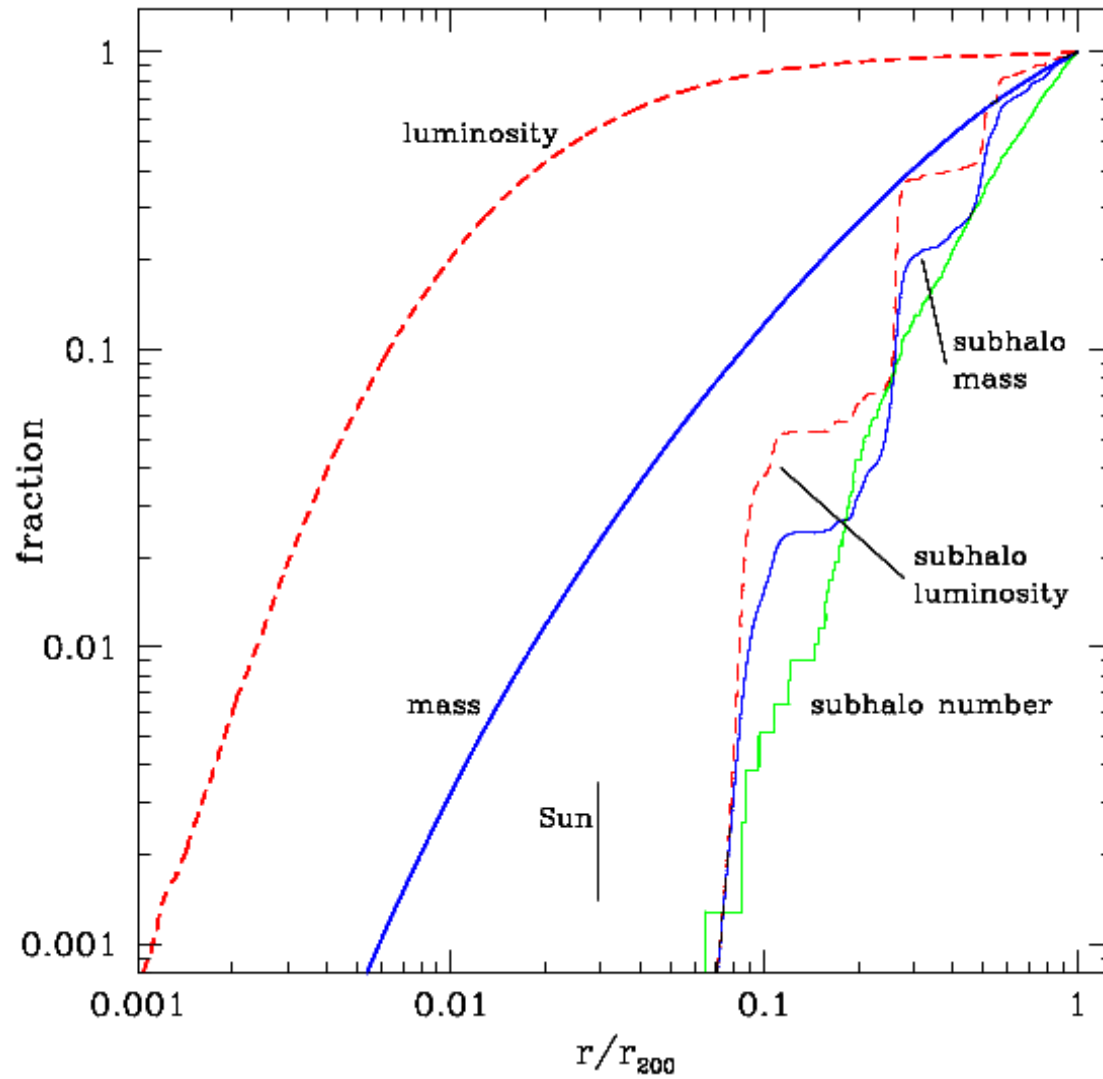
→ critical density exponent for convergence is $\rho \propto r^{-1.5}$

**Image of a
'Milky Way'
halo in
annihilation
radiation**

270 kpc


$$S(\boldsymbol{\theta}) \propto \int \rho^2 dl$$

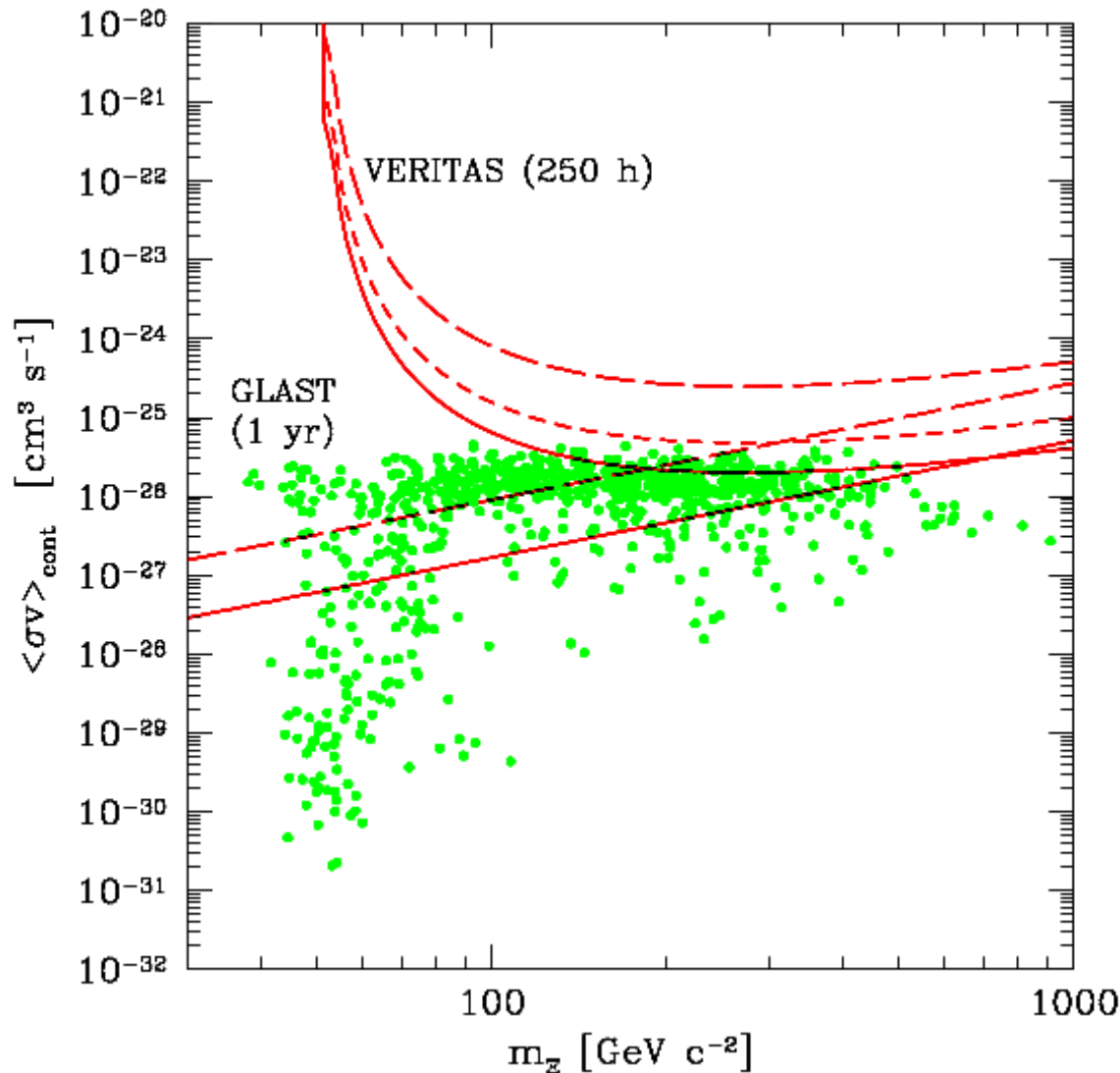
Cumulative radial distributions of mass and light



- Half mass/light radii of the diffuse halo component are **90 kpc** and **7 kpc**
- Half mass/light radii of the subhalo component are both **130 kpc**
- Total light from subhalo component is 25% that from the diffuse component
- The Sun is *much* closer to the peak of the diffuse emissivity than to a subhalo

➡ Observed flux dominated by diffuse emission from inner Galaxy

Could GLAST or VERITAS see the Signal?



Possible parameters for the DM in a simple supersymmetric theory

- For VERITAS (a Čerenkov detector with 1.75° FOV) the detectability of the G.C. depends on poorly resolved regions of the simulation and is marginal
- For GLAST (a satellite with 3 sterad. FOV) detection should be possible 20° to 30° from the G.C. in a very long integration and for most MSSM parameters. This does *not* depend on badly resolved regions of the simulation

Adding the baryons: hydrodynamics

- After recombination the baryons are in the form of a diffuse, near-uniform mixture of neutral H and He – no stars, no heavier elements, no magnetic fields (?)
- Need to solve hydrodynamics equations for the gas in addition to N-body equations for the DM
- $\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{u}) = 0$ Mass conservation
 $\partial(\rho\mathbf{u})/\partial t + \mathbf{u}\cdot\nabla(\rho\mathbf{u}) + \nabla p + \rho\nabla\Phi = 0$ Momentum cons.
+ Energy conservation
- Main solution techniques
 - discretise on a regular fixed mesh (Eulerian hydro)
 - discretise on a variable, adaptive mesh (AMR)
 - discretise using a finite set of fluid elements
(Smoothed Particle Hydrodynamics: SPH)

Hydrodynamics and cluster formation

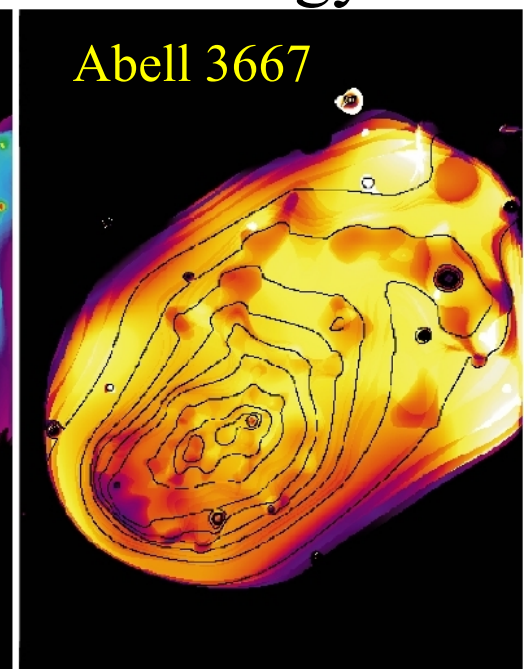
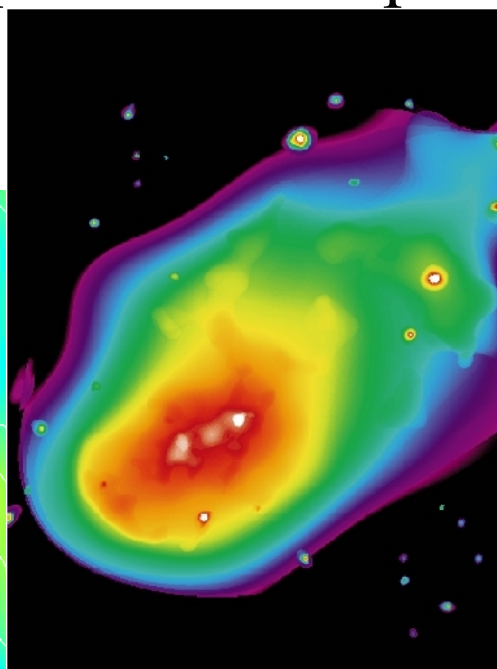
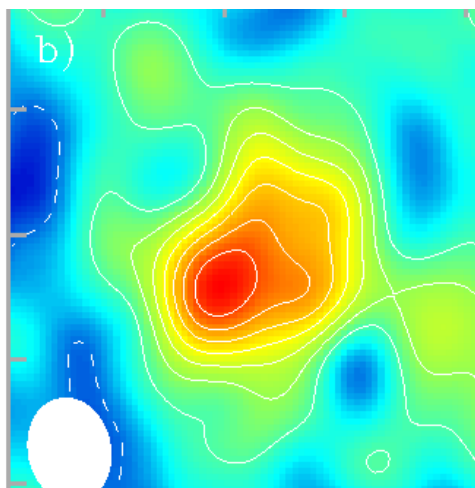
- Structure formation produces shocks which transform the K.E. of fluid motions into heat
- Bremsstrahlung and line emission from cluster gas is observable in X-rays

$$S = \int dl \rho^2 \Lambda(T)$$

- Cluster “shadows” are observable against the CMB through the Sunyaev-Zeldovich effect $\Delta T \propto \int dl p$

- Hierarchical growth of structure produces the phenomenology observed in images of clusters

- asymmetries and sublumps
- shocks
- cold fronts
- cold cores



Gas cooling and galaxy formation

- Bremsstrahlung and line emission cause shocked gas to radiate away its internal energy
- Gas settles into the centre of DM potential wells and starts to form the stellar populations of galaxies
- Gas cooling times are $t_{cool} \propto T / \rho \Lambda(T)$ while objects at time t have typical density $\rho \propto t^{-2}$ so $t_{cool} / t \propto t T / \Lambda(T)$
 - rapid cooling in lower mass objects and at early times
 - efficient galaxy formation

(See G. Kauffman's lectures)



Including the formation and evolution of stars

- Stars form where gas is dense, cold and self-gravitating
- Star formation is *not* resolved in galaxy formation simulations
→ a simple *sub-grid* prescription is needed, for example
$$\dot{\rho}_* = \alpha \rho_{\text{gas}} t_{\text{dyn}}$$
- Aging of stars affects their brightness and colour
- Supernovae, explosions of massive stars put energy and heavy elements into surrounding gas – *feedback*, *chemical enrichment*
- Feedback processes are also not resolved and must also be implemented with phenomenological recipes
- Implementation details *strongly* affect galaxy formation models

Simulating the formation of individual galaxies

- Systems which have a *major merger* after most of their stars have formed end up looking like **Elliptical galaxies**
- Systems which have no major merger at late times end up having a substantial disk and looking like **Spiral galaxies**
- It seems very hard to make spiral galaxies with small bulges

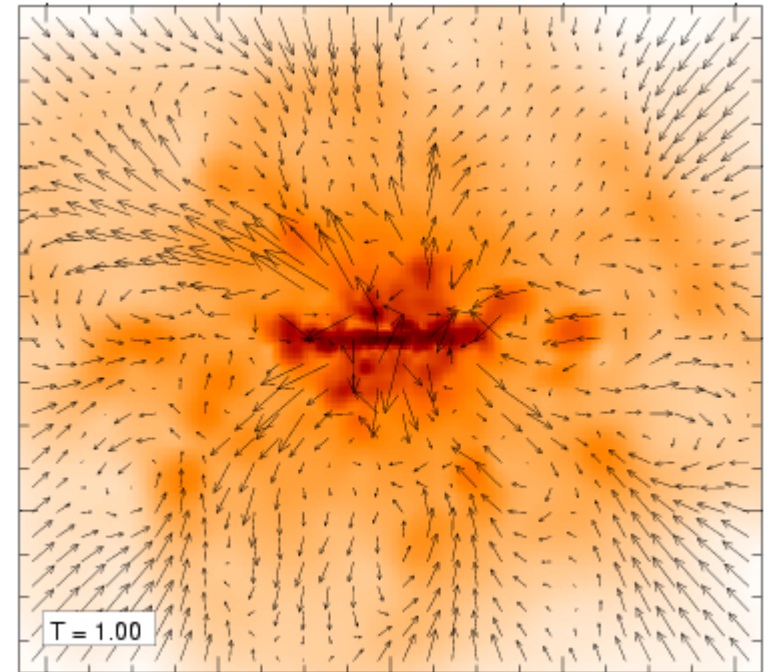


- It's also hard to make big disks
- ..or to get enough heavy elements out of galaxies

Problems with feedback recipes?

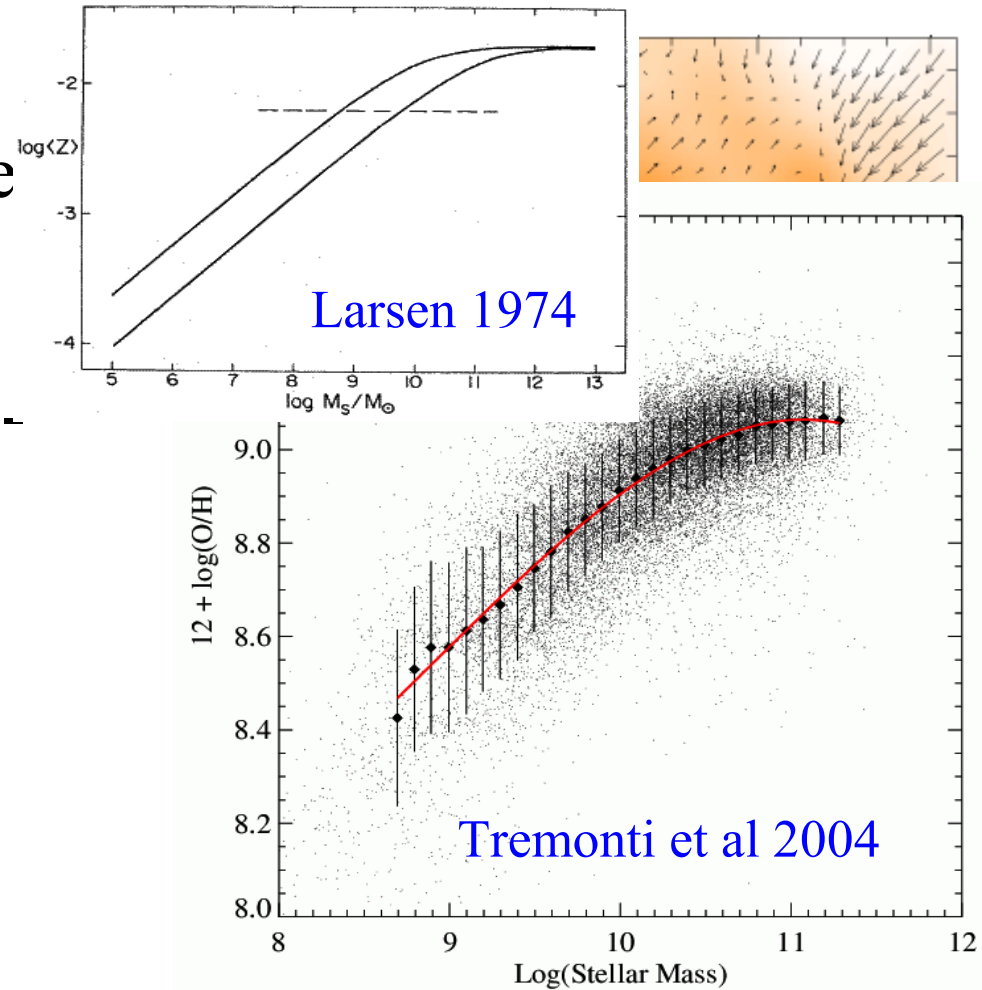
Feedback/galactic wind issues

- Can supernova feedback drive galactic winds?
- Can these reproduce the mass-element abundance relation?
- Can they enrich intergalactic gas with heavy elements?
- Can these enhance formation of disks over bulges?
- What about feedback from Active Galactic Nuclei?



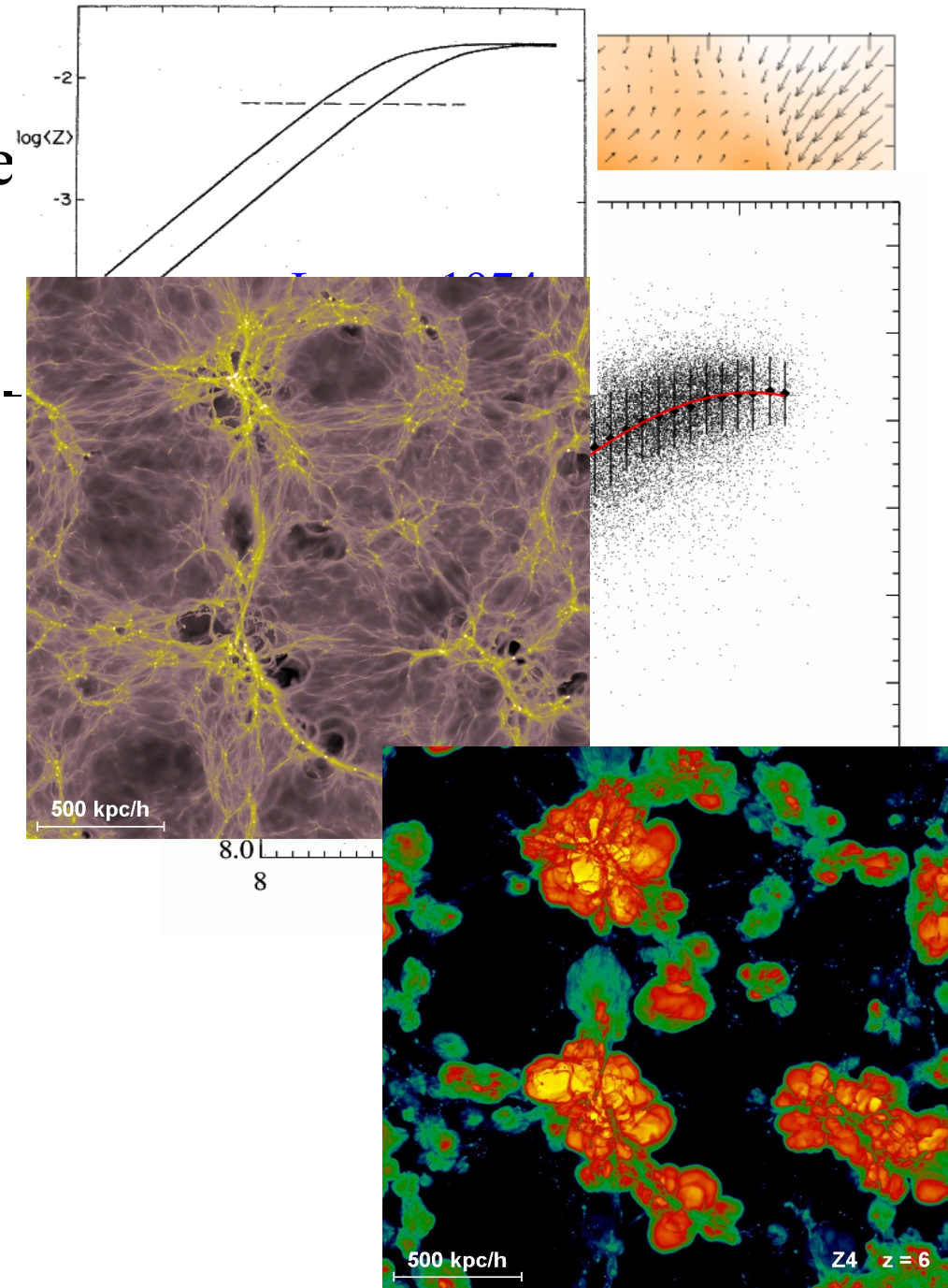
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Collisions and Mergers

- Galaxy collisions are “sticky”
 - the stars miss each other but feel each other's gravity
 - the same is true for dark matter particles
 - the gas clouds in the two galaxies do collide and shock
- Gravitational exchange of energy between different parts of the two systems
 - tidal effects distort the structure of the two galaxies
 - system orbital energy converts into internal energy
orbit decays
- Accelerating orbital decay leads to merging of the galaxies
 - a strong starburst
 - material dumped onto the black hole(s) → quasars?
- Remnant of the merger is a gas-less star pile -- an elliptical

There is a merger in our own Milky Way's future

M32

M31

NGC 205



Did mergers make real ellipticals?

- Are there enough of them?
- Do they make objects with the right masses and sizes?
- Do they make objects with the right internal structure?
- Do they make objects with the right stellar populations?
- Do they make objects in the right places?
- Can we see them happening?
- Do they predict the right history for ellipticals?
- Do they predict the relation between BH's and galaxies?

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- Do they predict the relation between BH's and galaxies?

They do a reasonable job, but many aspects remain controversial
LOTS LEFT TO DO!

Goals for simulating galaxy/AGN populations

- Explore the physics of galaxy formation
- Understand the links between galaxy and SMBH formation
- Clarify why galaxy properties are related to clustering
- Determine how environment stimulates galaxy activity
- Interpret new multi-wavelength surveys of galaxies
- Check if such surveys can provide precision tests of and parameter estimates for the standard Λ CDM paradigm

Physics for Galaxy Formation Modelling

Gas Cooling and Condensation

Sensitive to metal content, phase structure, UV background...

Star Formation

No *a priori* understanding -- efficiency? IMF?

Stellar Feedback

SF regulation, metal enrichment, galactic winds

Stellar Aging

Population synthesis  luminosities, colours, spectra, (dust?)

AGN physics

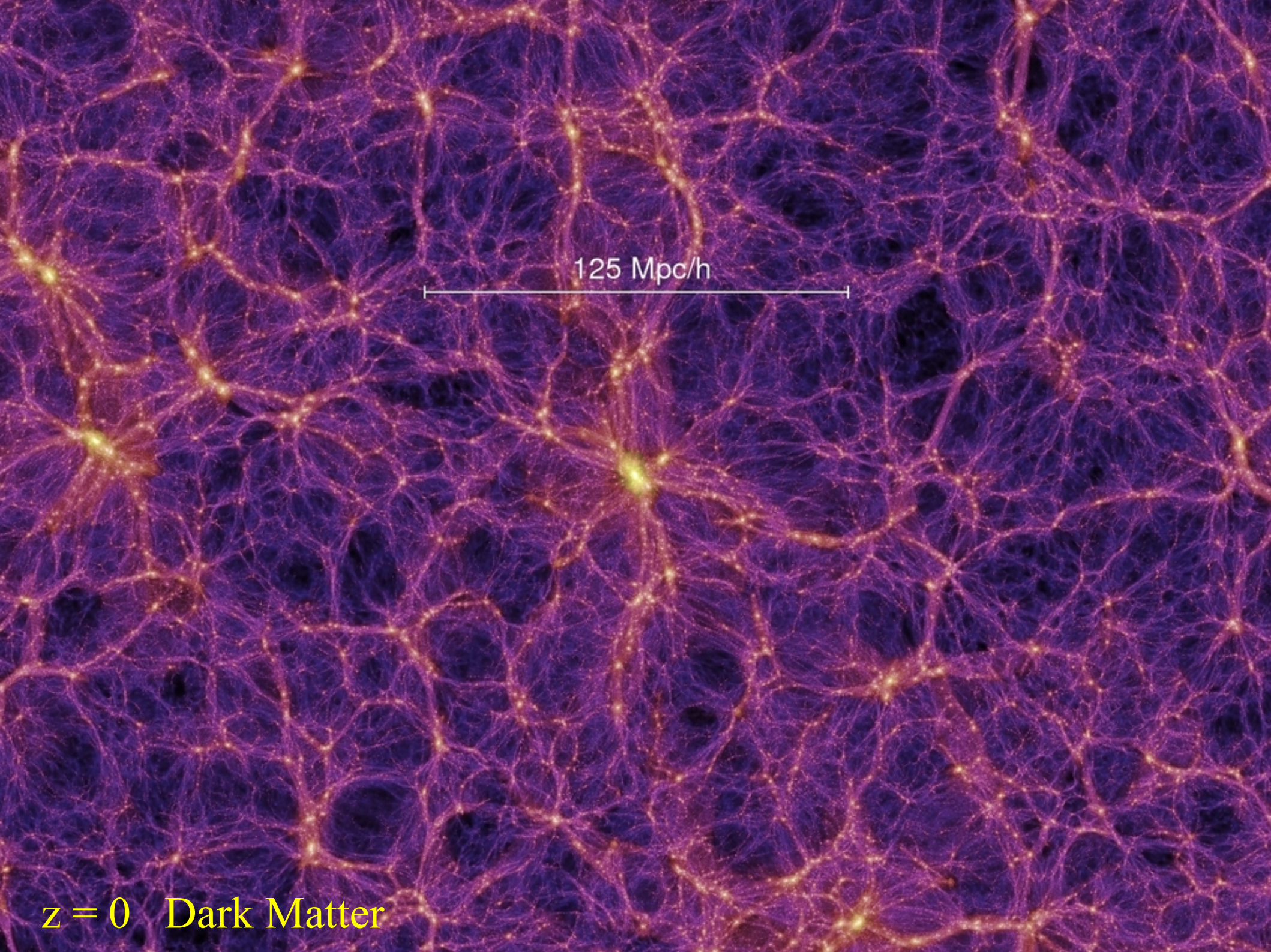
Black hole formation, feeding, AGN phenomenology, feedback

Environment interactions

Galaxy mergers, tidal effects, ram pressure effects

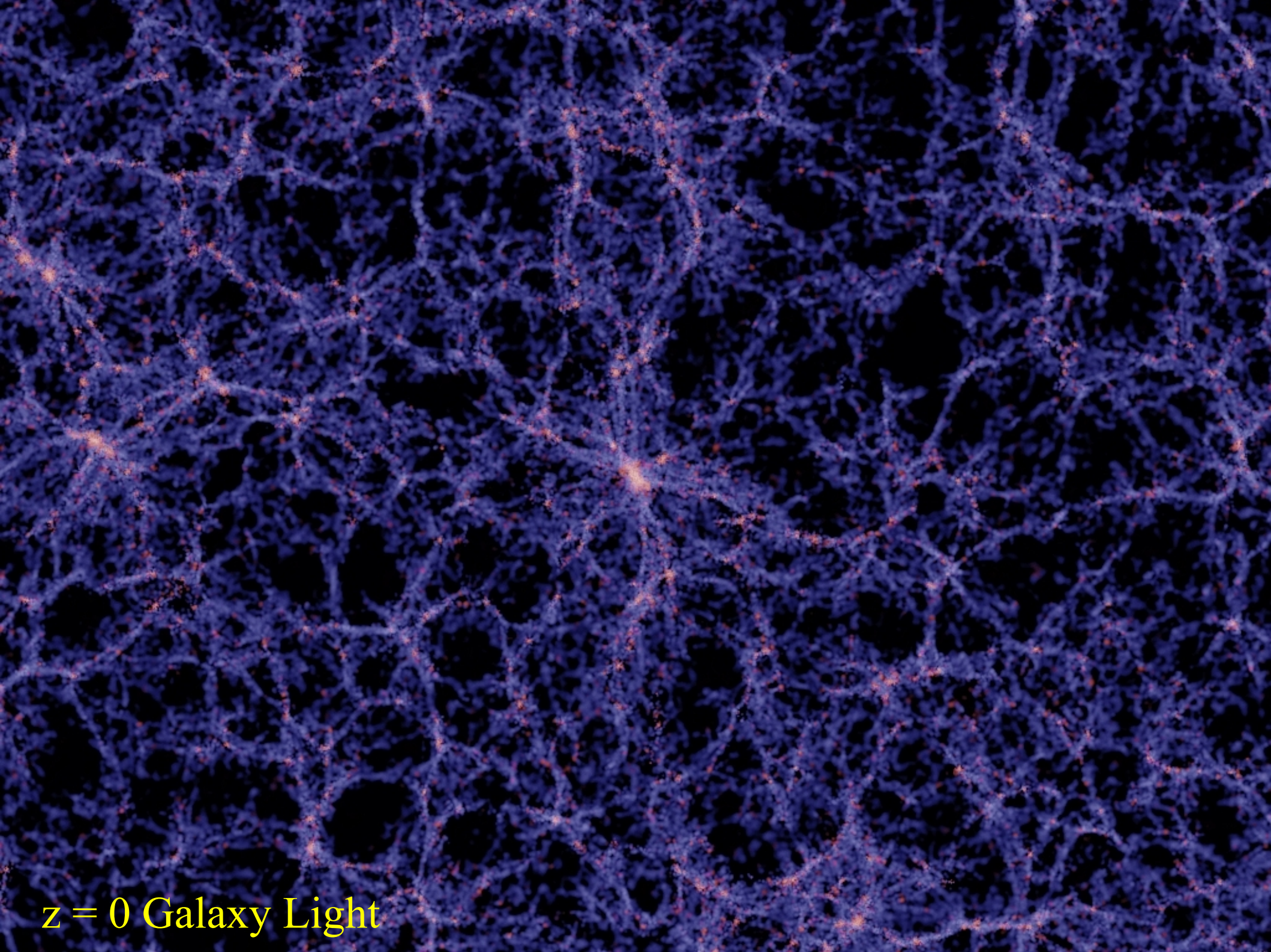
Millennium Run as a testbed for simulating evolution of the galaxy/AGN population

- Particle number: $N = 2160^3 = 10,077,696,000 \approx 10^{10}$
- Box size: $L = 500 \text{ Mpc/h}$, Softening: $\epsilon = 5 \text{ kpc/h}$ $\longrightarrow L/\epsilon = 10^5$
- Initial redshift: $z_{\text{init}} = 127$
- Cosmology: $\Omega_{\text{tot}}=1$, $\Omega_{\text{m}}=0.25$, $\Omega_{\text{b}}=0.045$, $h=0.73$, $n=1$, $\sigma_8=0.9$
- 343,000 processor-hours on 512 nodes of an IBM Regatta
(28 machine days @ 0.2 Tflops using 1 Tbyte RAM)
- Full raw and reduced data stored at 64 redshifts
 $\longrightarrow$ **27 Tbytes of stored data**
A testbed for studying galaxy formation models

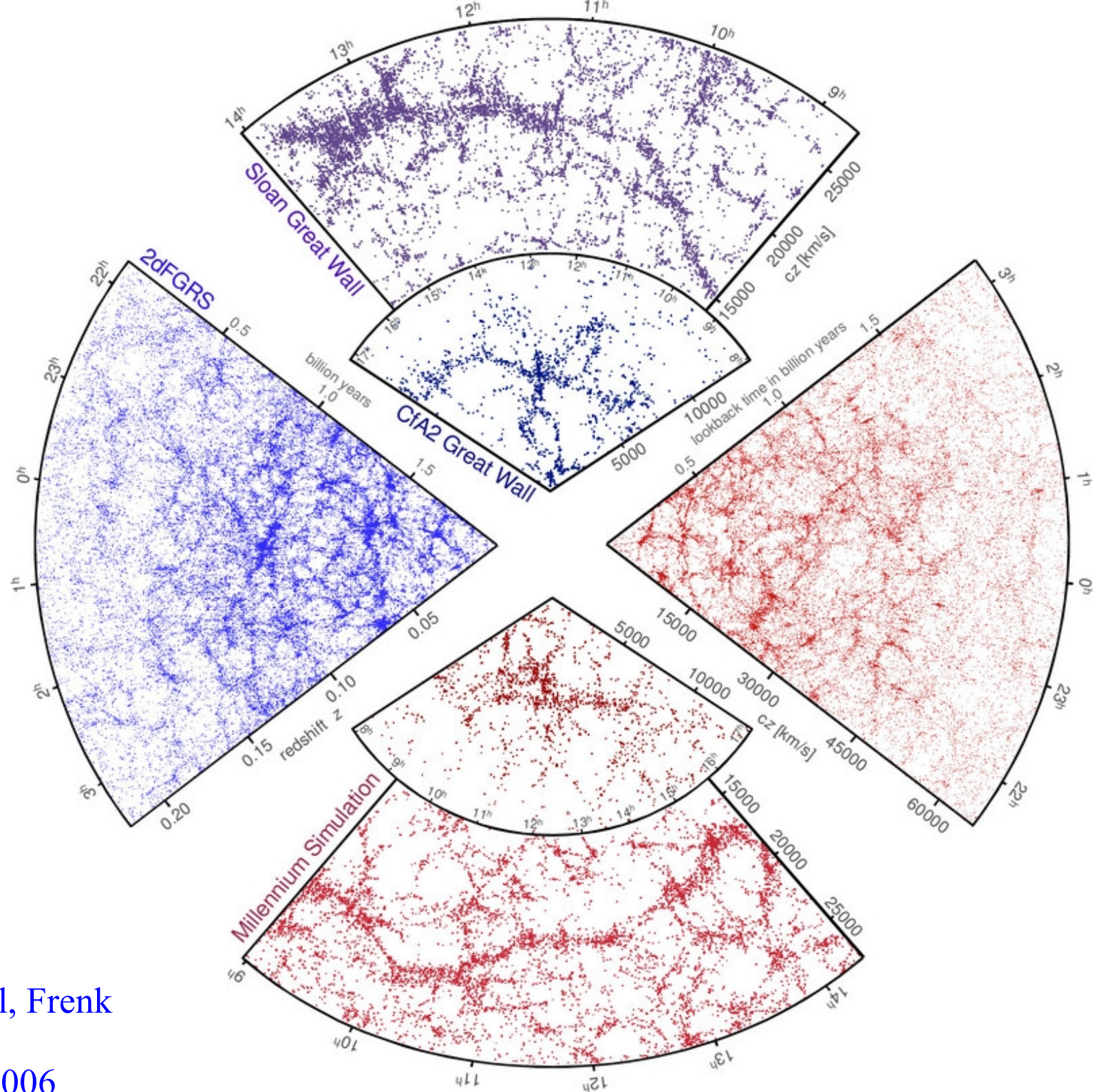


125 Mpc/h

$z = 0$ Dark Matter

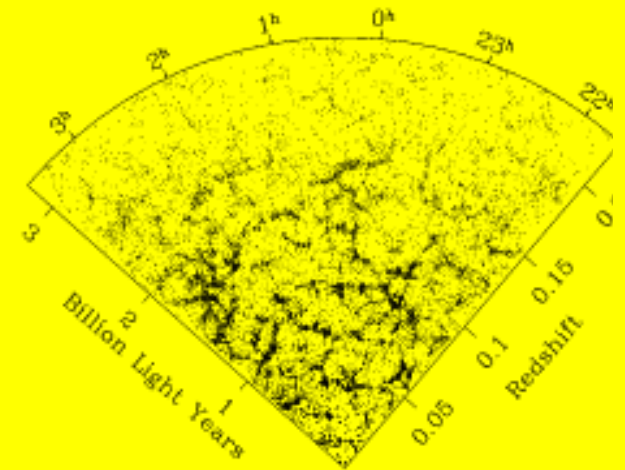
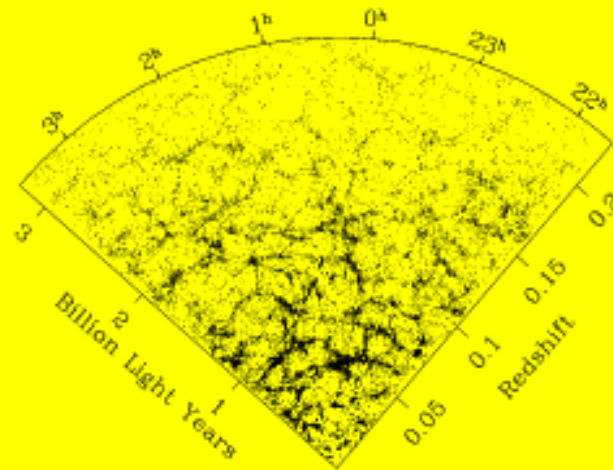
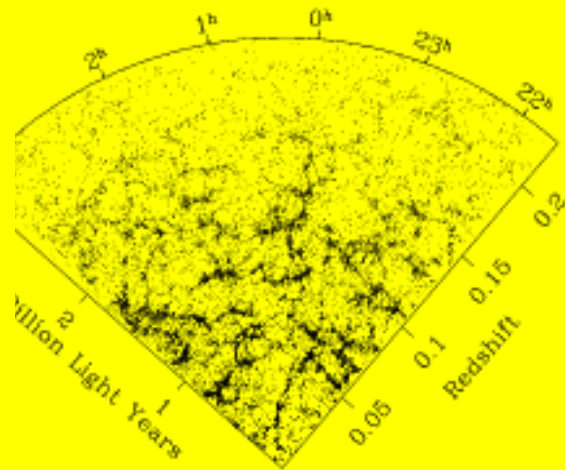
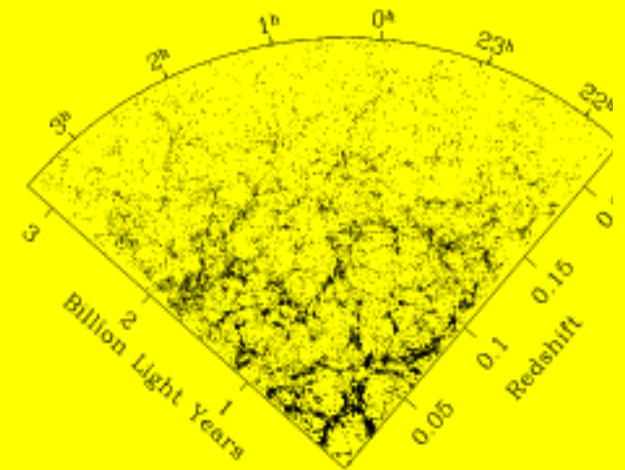
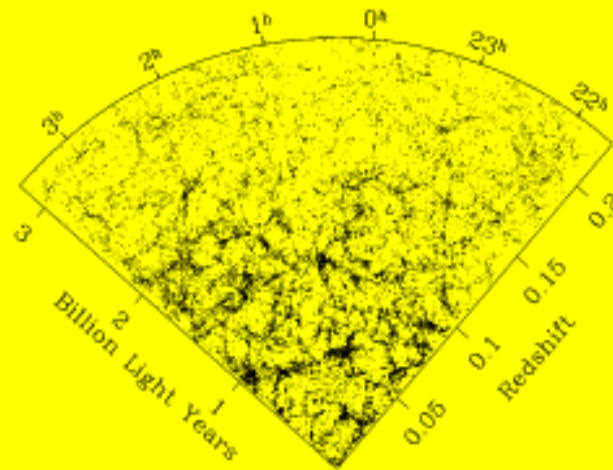
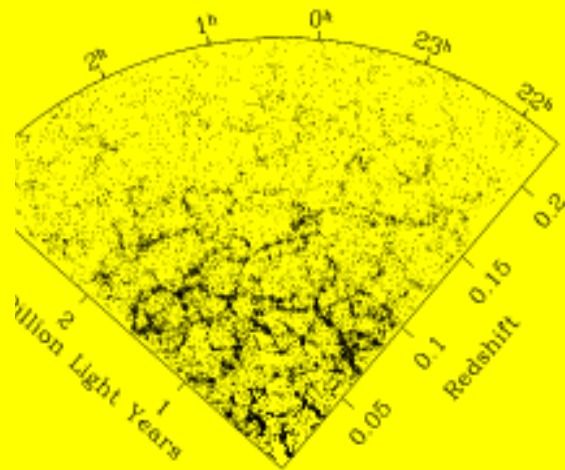


$z = 0$ Galaxy Light



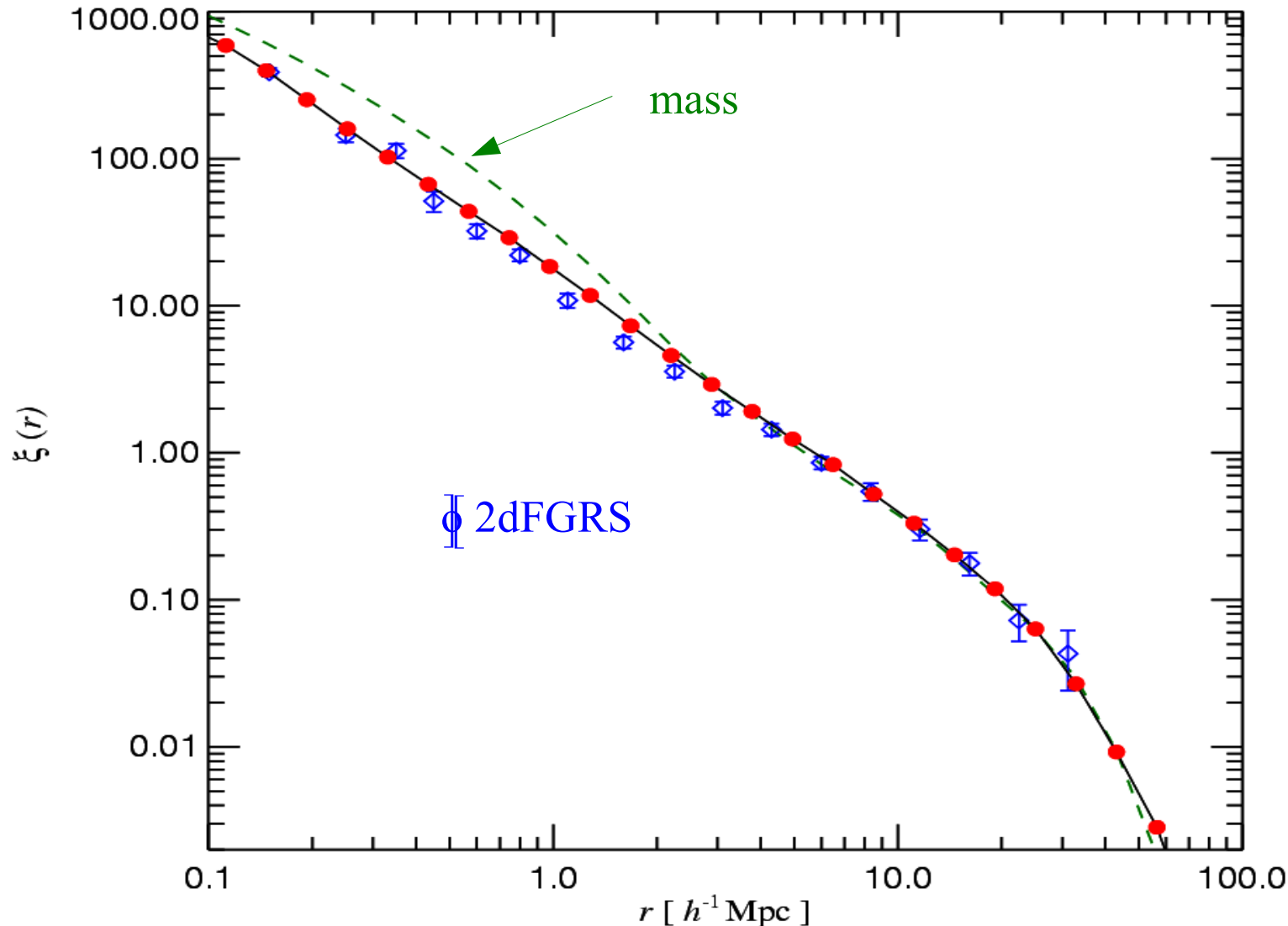
Springel, Frenk
&
White 2006

VIRTUAL vs REAL UNIVERSES I



Galaxy autocorrelation function

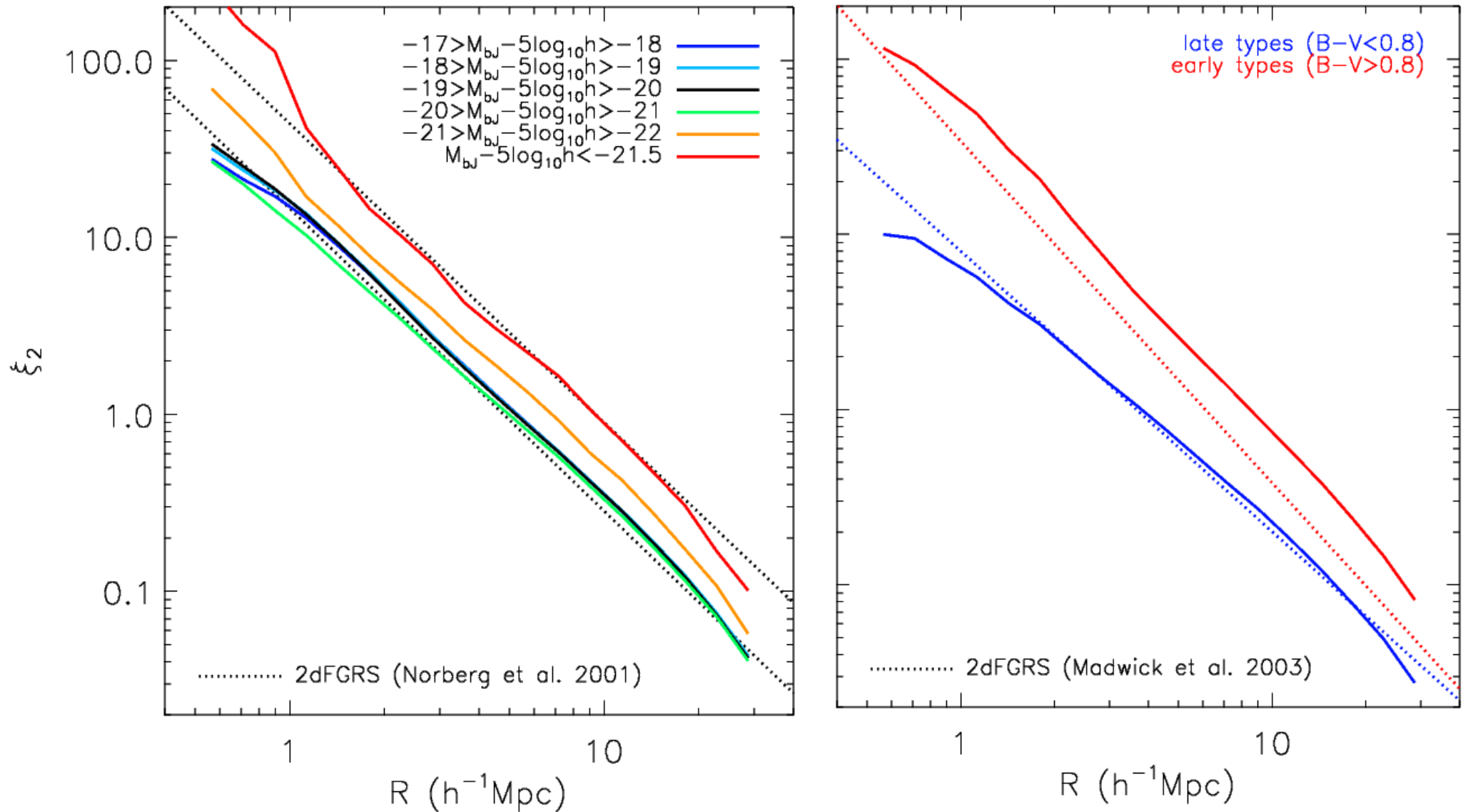
Springel et al 2005



For such a large simulation the purely statistical error bars are negligible on ξ even for the **galaxies**

Correlation functions depend on L and colour

Croton et al 2005

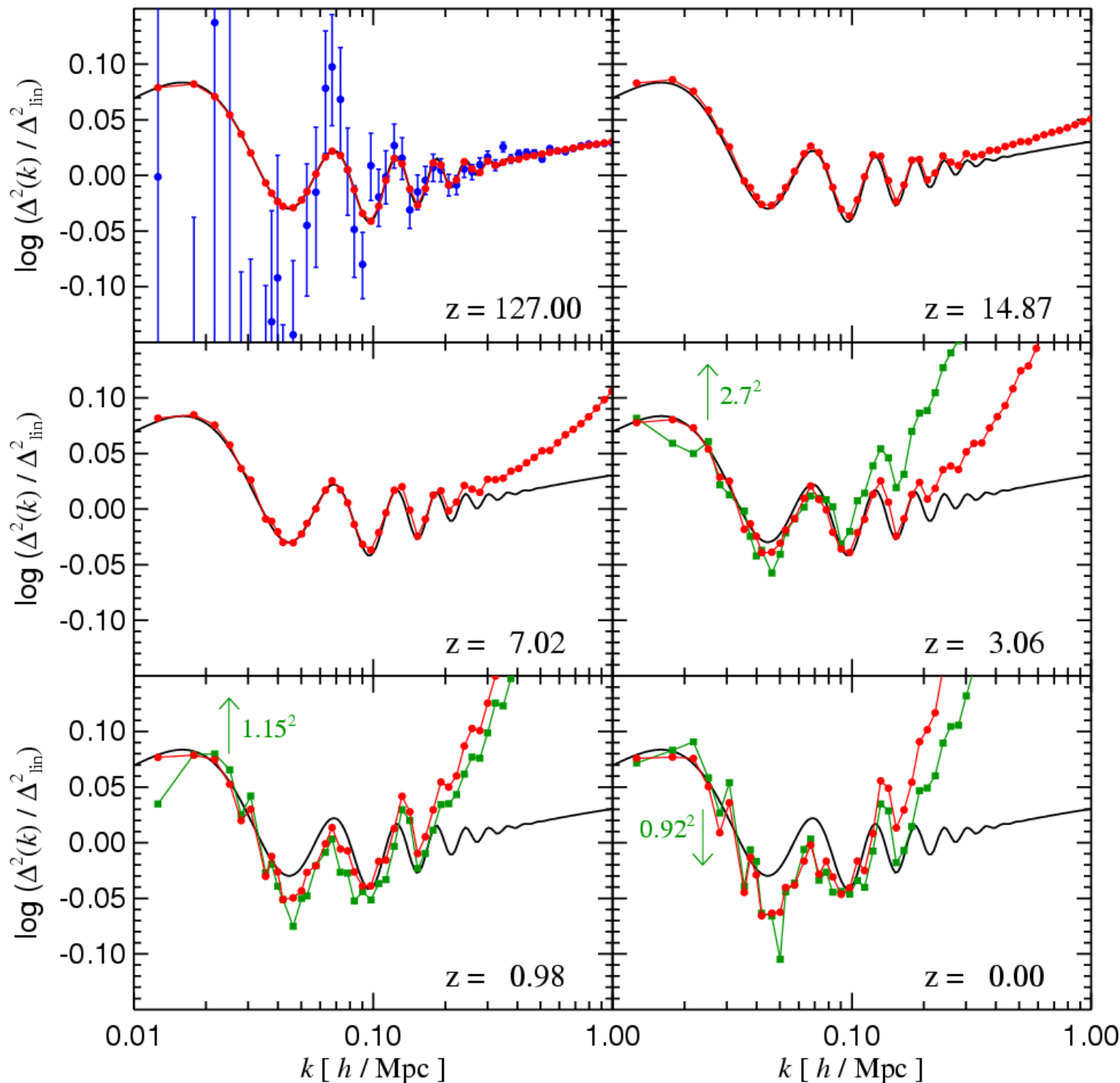


Baryon wiggles in the galaxy distribution

Springel et al 2005

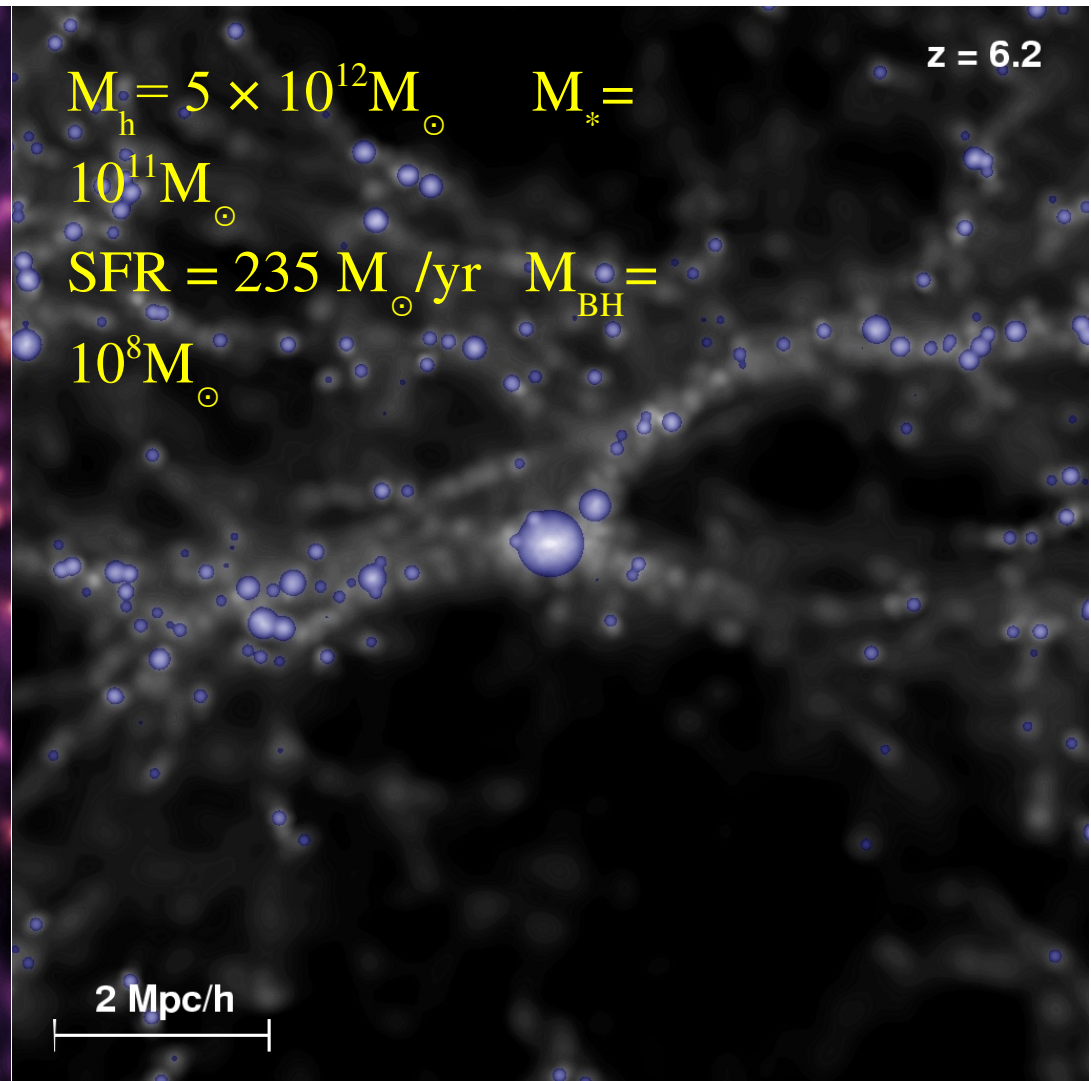
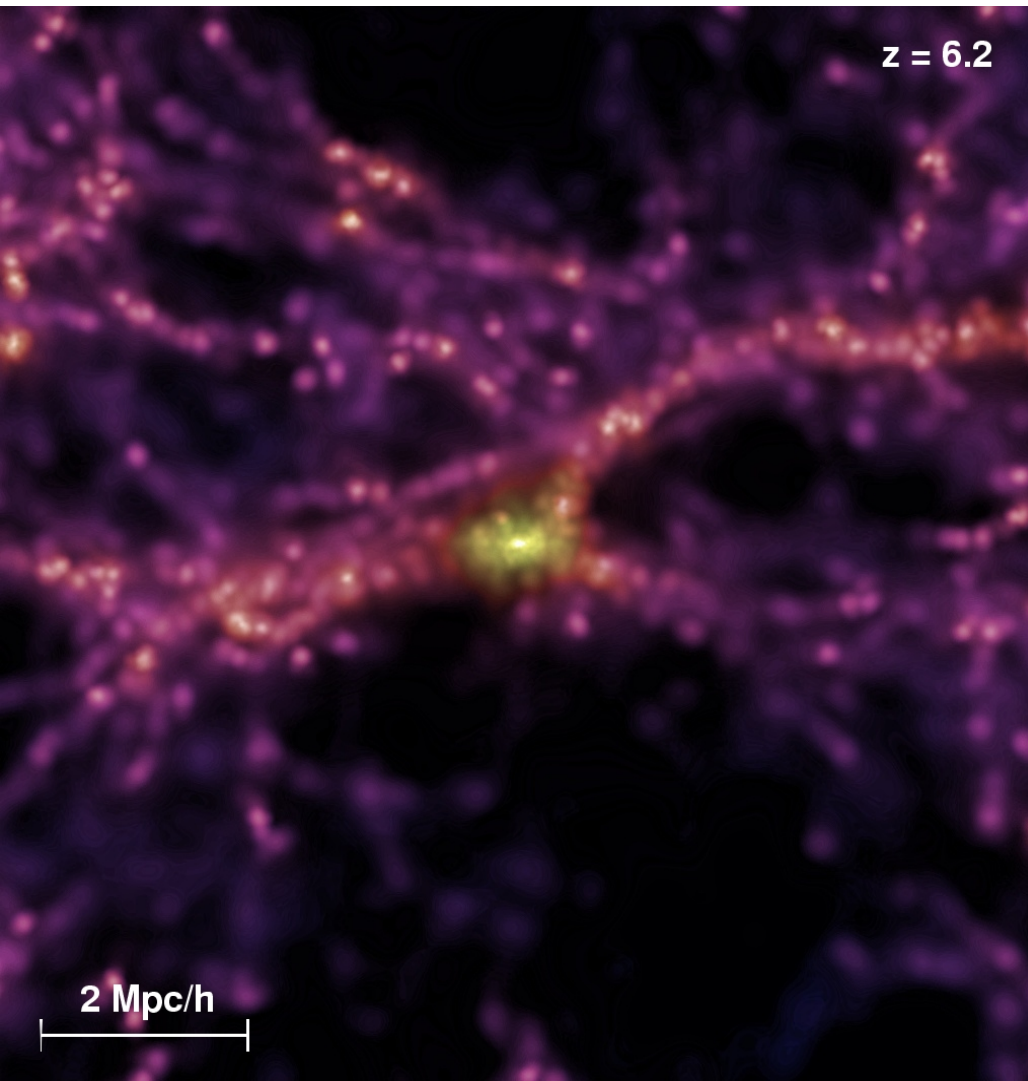
Power spectra from the Millennium run divided by a baryon-free Λ CDM spectrum

Galaxy samples are matched to plausible large observational surveys at given z



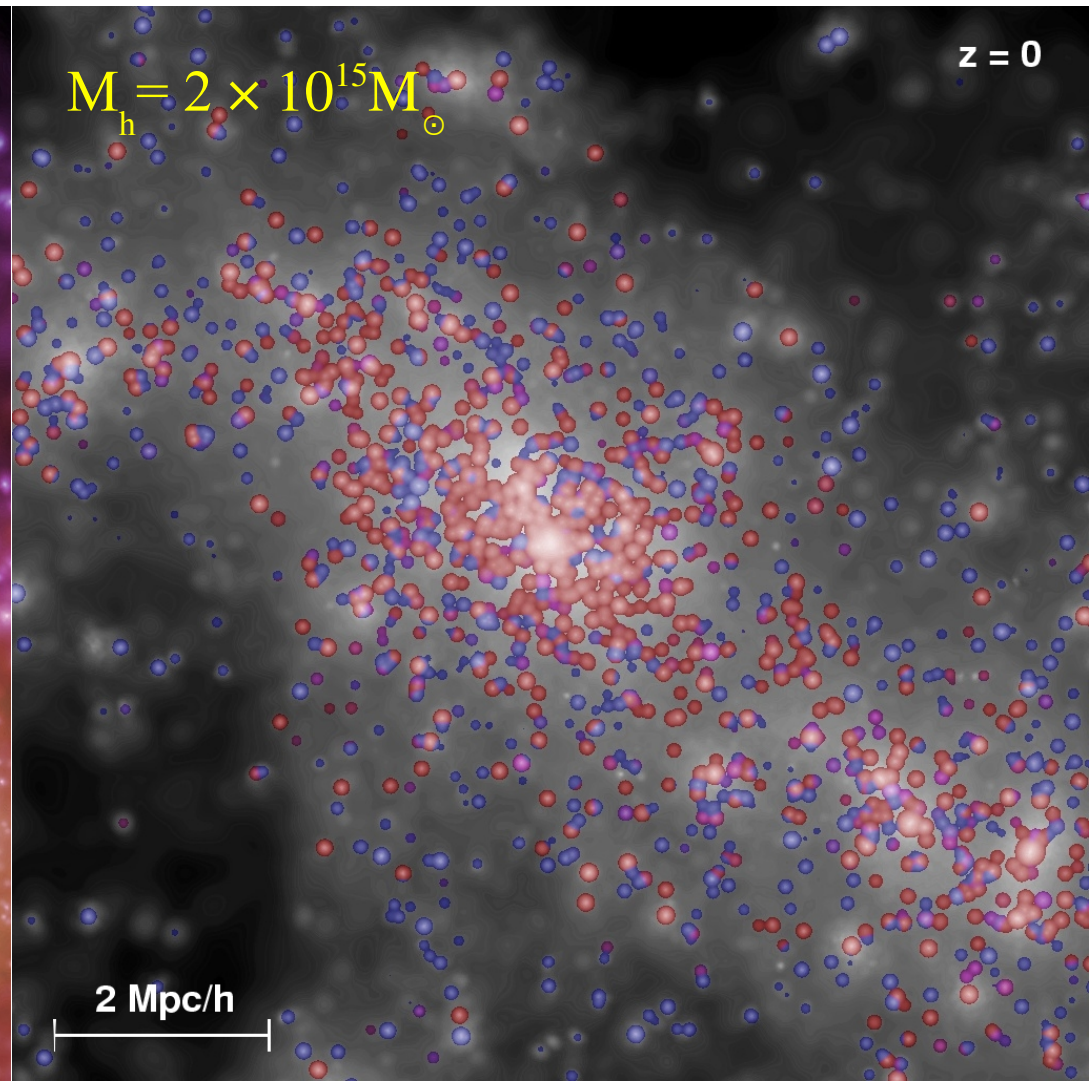
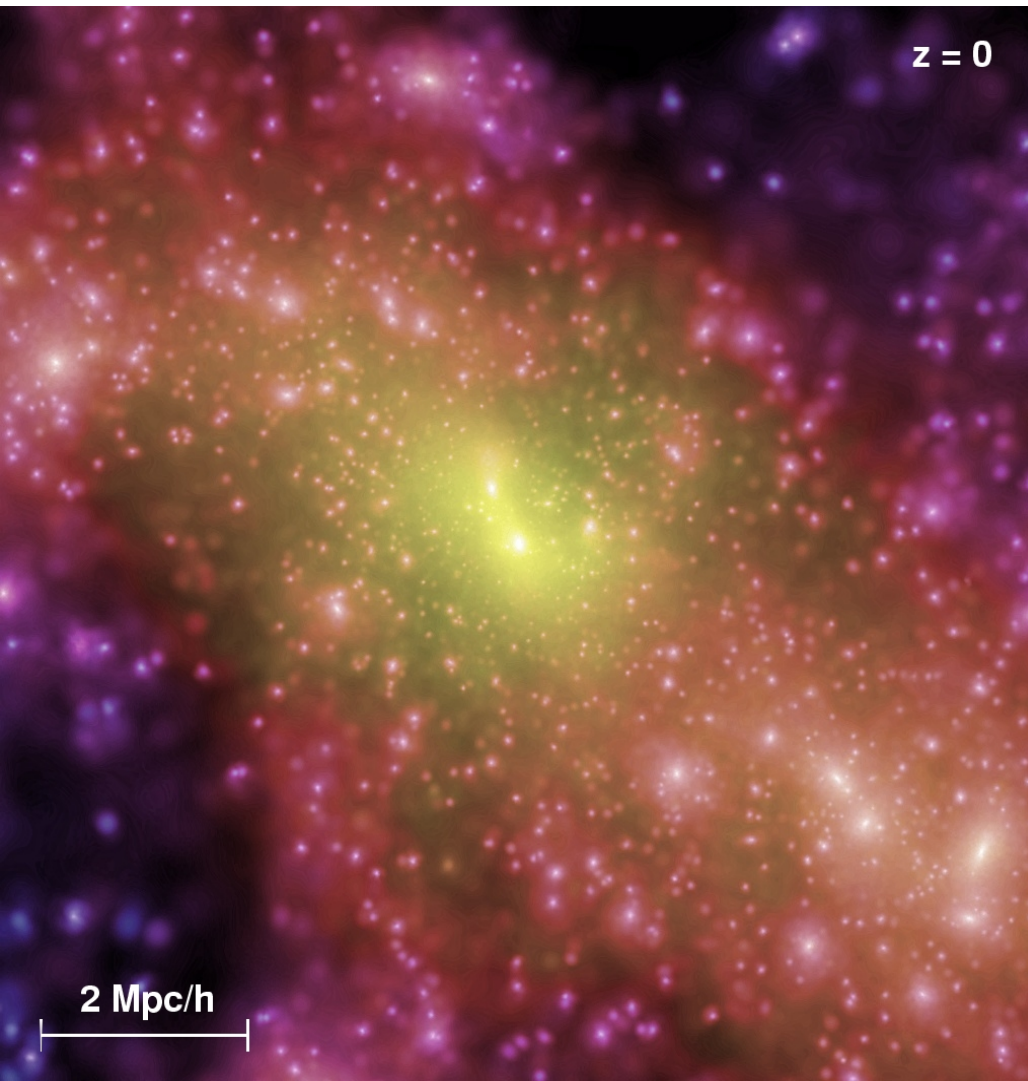
A bright quasar and its surroundings at 1 billion years

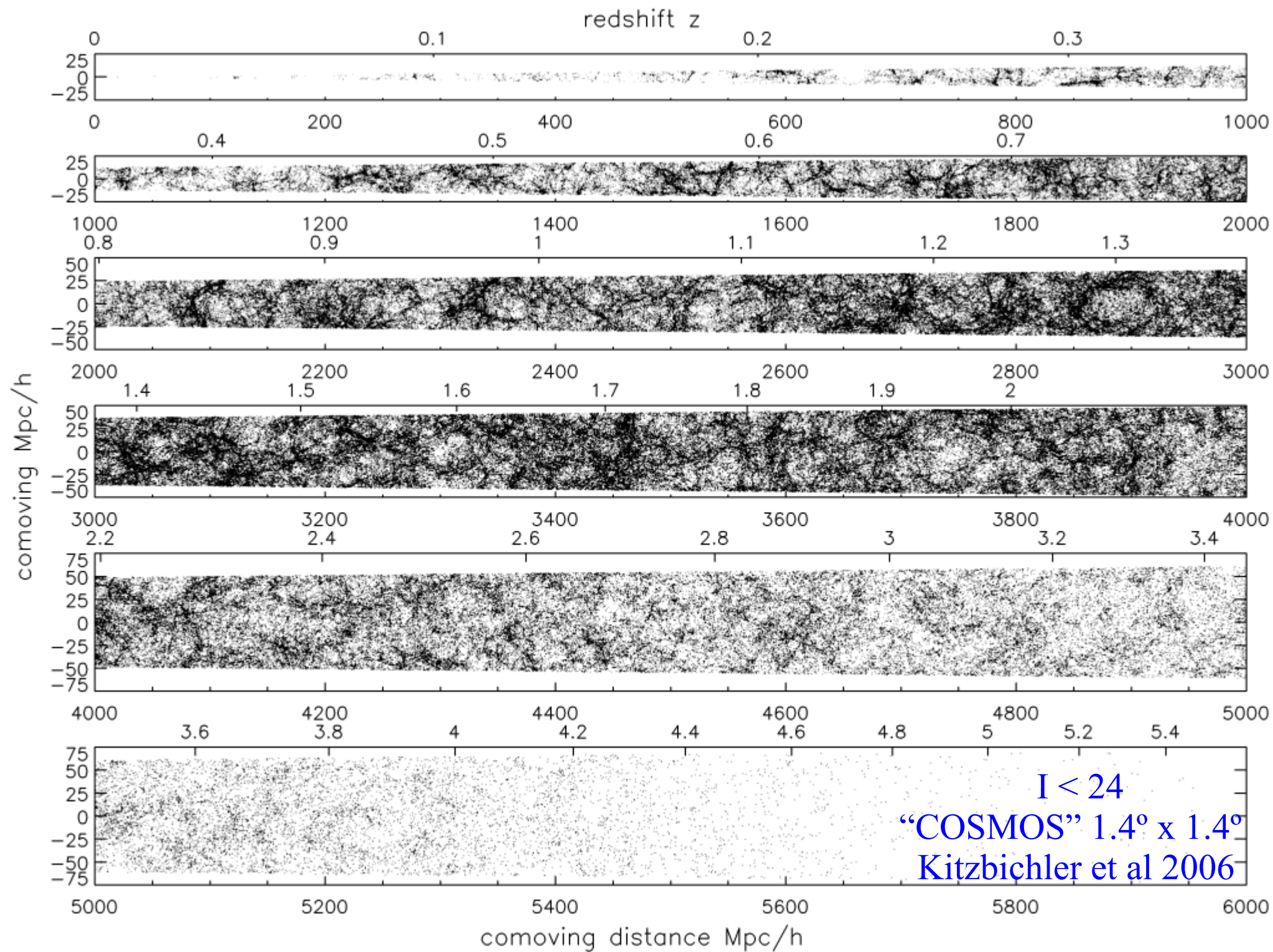
One of the most massive dark matter clumps, containing one of the most massive galaxies and most massive black holes.



The quasar's descendant and its surroundings today, at $t = 13.7$ billion years

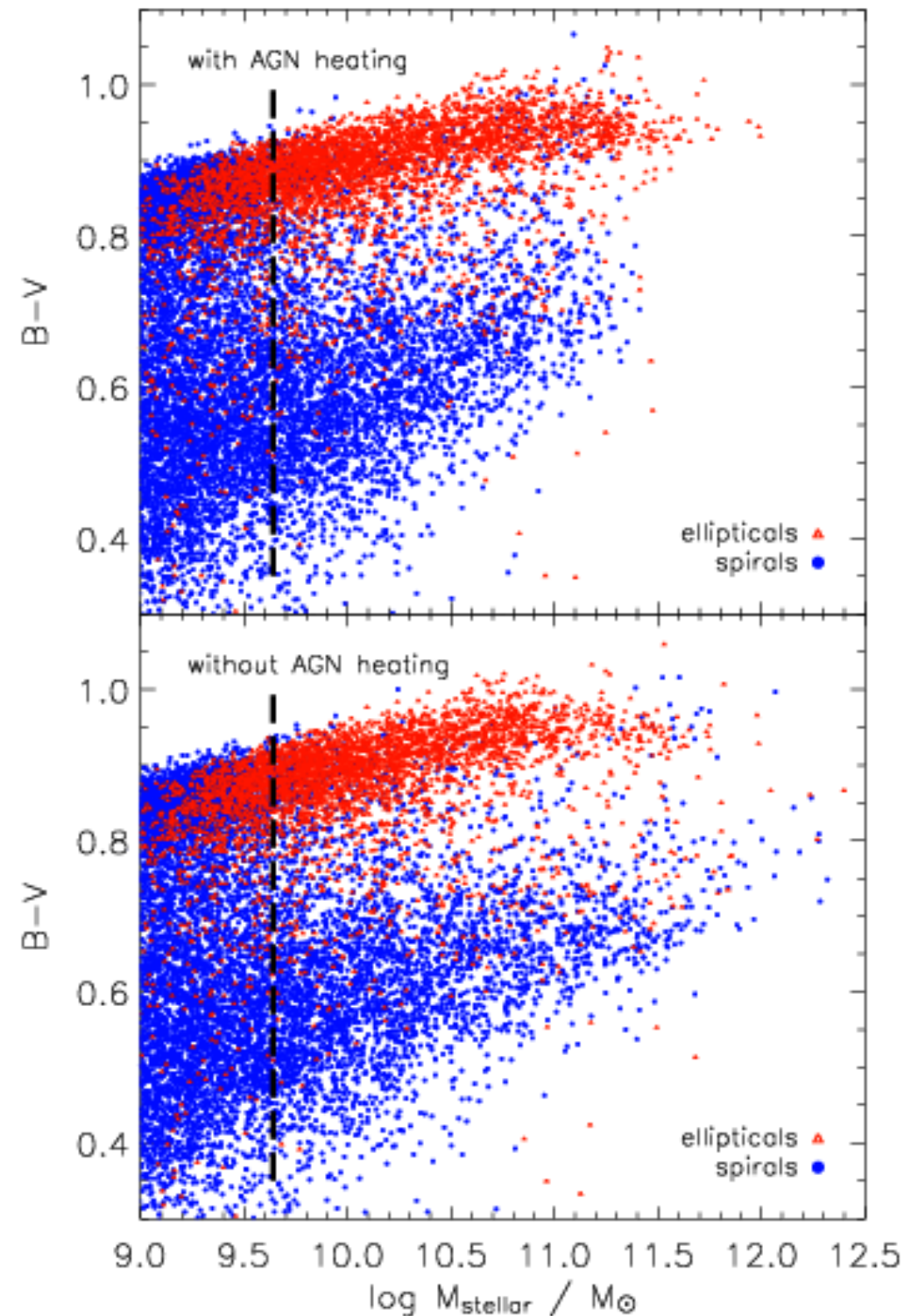
One of the most massive galaxy clusters. The quasar's descendant is part of the central massive galaxy of the cluster.





The effects of “radio mode” feedback on $z=0$ galaxies

Croton et al 2005



- In the absence of a “cure” for the cooling flow problem, the most massive galaxies are:
 - too bright
 - too blue
 - disk-dominated
- With cooling flows suppressed by “radio AGN” these galaxies are
 - less massive
 - red
 - elliptical

What are simulations good for?

- They enable “experiments” on astrophysical systems
- They compress cosmic evolution into human timescales
- They can follow complex structures and complex physics
- They can be “observed” in exactly similar ways to the real sky, enabling direct comparison of theory and observation
- They allow computer geeks to do astronomy and make beautiful movies