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Eratosthenes Festivity 21 June

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21 June

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Once Upon a Time...

The Greek scholars elaborated on the observations made by the ancient Egyptians. This led them to the concepts of demonstration and mathematical modeling, which are the fundamentals of modern science.

Considerable progress was made in the field of anatomical medicine thanks to the use of human dissection; a practice previously banned for religious reasons and performed for the first time by Alexandrian doctors.

From the third century BCE to the fifth century CE, scientists broadened their scope of investigation. The conquests of Alexander the Great provided more detailed knowledge of the world; geography became more precise, and exploration became more scientific. Moreover, scholars received substantial backing, particularly financial, from Hellenistic kings, including the Ptolemies in Alexandria and the Attalids in Pergamon.

Who am I?

Profile

Name: Eratosthenes
Nationality: Lybian
Birth: 276 BCE, Cyrene (Libyan Arab Jamahiriya)
Education: Athens and Alexandria
Profession: Scientist and third Librarian of the Ancient Library of Alexandria.
Death: 194 BCE

My name is Eratosthenes. I was born in Cyrene; currently the Libyan Arab Jamahiriya. Along with Alexandria and Athens, Cyrene was one of the most prominent cultural cities of the Mediterranean. I studied in Athens and then moved to Alexandria where I lived for many years.

I lived at a time of eminent Greek scholars, including my good friend Archimedes(1), who was one of the greatest mathematicians in history. I earned the nickname Beta(2), the second letter in the Greek alphabet, from some of my contemporaries who claimed that I was second best among my peers in everything.

I was interested in and practiced numerous fields of science including astronomy, geography, literature, poetry, philosophy, and mathematics. In 236 BCE, I was appointed Librarian of the Library of Alexandria by Ptolemy III Euergetes I(3), becoming the third Librarian of this illustrious edifice.

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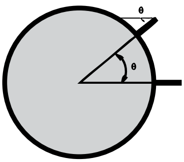

What Did I Do?

I wrote distinctive works of mathematics, and set some significant geometric and arithmetic definitions. I also wrote the first treaty on mathematical geography, including a map of the world. I was the one who invented the word “Geography”, which is derived form the Latin word “Geographikos” meaning “Science of Map Drawing”.

Moreover, I prepared a map of the Nile valley southward to the latitude of the modern Khartoum. I also reported that heavy rains hadbeen observed to fall in the upper reaches of the river and that these were sufficient to account for the flooding.

Furthermore, I described “Eudaimon Arabia”; now known as Yemen; as inhabited by four major peoples. Based on my categories, contemporary scholars began to speak of Minaeans, Sabaeans, Qatabanians, and Hadramites.

Other than that, I calculated the ecliptic orbit, as well as the distance between the Earth and the Moon, and the Earth and the Sun.

However, my most remembered feat is that I measured the circumference of the Earth with outstanding precision through the latitude difference between the ancient city of Syene (Aswan) and Alexandria, in Egypt.

Unfortunately, none of my works survived; however, many extracts have been quoted by my successors.

How Big is the Earth?

Can you imagine how easy it is to measure the circumference of the Earth?

I measured the circumference of the Earth applying a very simple method that every one of you can easily use.

I had heard that if you looked into one of the wells in the city of Syene (Aswan) at noon, on a midsummer's day, you could see the Sun reflected in the water at the bottom and that there was no shadow.

As the water surface in the well is horizontal, I realized this must mean the Sun was exactly overhead. I deduced that the sun rays were perpendicular; thus, reaching the bottom of wells and the shadow of objects was centered around them, making it seem as if they had no shadow at all (Fig.1).

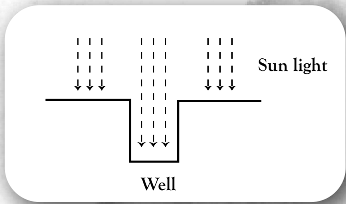

Fig.1

On the other hand, in Alexandria, the sun rays were not perpendicular and the same objects had short shadows (Fig.2).

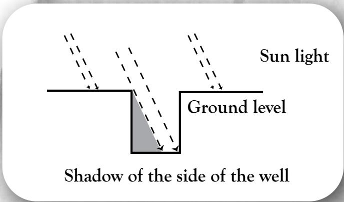

Fig.2

(Fig.2) Sunlight shining down a well in Alexandria at noon, on the same day as the observation shown in Fig.1; the Sun is not directly overhead. The ray bar at the bottom left shows the shadow cast by the side of the well.

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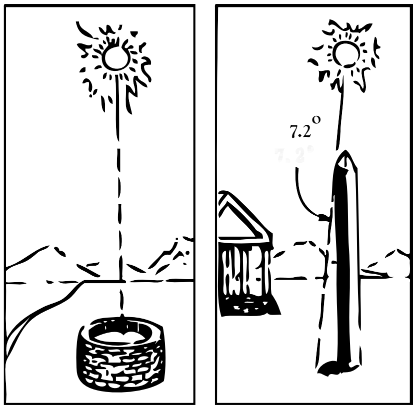
In 205 BCE, I proposed a rather simple and precise method of measuring the circumference of the Earth applying my observations of shadows.

Where?

In Alexandria and ancient Syene (Aswan).



When?

At the summer solstice[4], according to the local solar midday.

What did I do?

First, I measured the shadow of an obelisk in Alexandria and then estimated the angle between the sun rays and the obelisk (angle = 7.2°).
Meanwhile, in Syene there was no shadow for any vertical objects, such as the well, which means that the angle of the shadow equals 0°.

I concluded that:

The Earth is not flat and its surface is curved; maybe even completely spherical.

If the Earth is spherical, then if we extend the vertical line of the obelisk in Alexandria and that of the well in Syene, the two lines will meet exactly at the center of the Earth.

Knowing that Alexandria and Syene are located almost on the same meridian[5], I assumed that the rays of the sun are parallel.
Therefore, the angle formed by the two verticals at the center of the Earth is identical to the one of the obelisk shadow (7.2°).
I also knew that the distance between Alexandria and Syene was equal to 800 km (5,000 stadia).

Thus, the conclusion that the ratio of this angle 7.2° to the 360° of the circle is the same ratio of the distance between Alexandria and Syene (800 km) to the circumference of the circle (the Earth).

How?

$7.2°/360°=800/x$
$x=360X800/7.2°$
$x=40{,}000$

Then, the circumference of the Earth = 40,000 km (250,000 stadia)

Measuring in Stadia

The units that I used in my calculations of the Earth's circumference were called stadia because they were based on the distance that athletes ran in races held in Ancient Greek stadiums.

Some historians suggested that stadiums were built around racecourses that were initially measured out with paces, with 100 double paces (200 steps) from start to finish. Paces are not a very precise basis for defining a stadium, considering the fact that a pace varies from one individual to another.

The ancient stadium at Olympia is 1923 meters long, but there is evidence that stadia could have been defined to be anywhere from 157 meters to 211 meters long at different times and places in history.

Here are some of the distances I have calculated in stadia:

1- The distance from Alexandria to Syene=5,000 stadia.
2- The circumference of the Earth=250,000 stadia.
3- The distance from the Earth to the Moon=78,000 stadia.
4- The distance from the Earth to the Sun=804,000,000 stadia.

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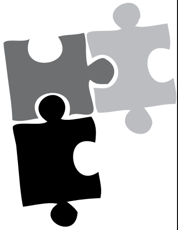
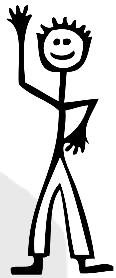


Play it Yourself

1. Shadow Dance



- A tube of glue or roll of tape
- A large piece of poster board
- A small piece of cardboard or Styrofoam
- A toothpick
- A flashlight

Objective

The purpose of this activity is to experiment with shadows and light sources, and to understand the relationship between the angle of illumination and the shadow's length.

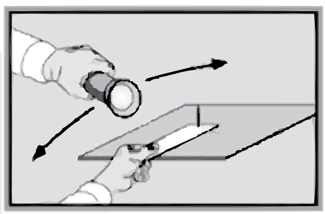
What to do?

1. Cut a small rectangular piece out of the cardboard or Styrofoam, approximately 3/4 of an inch wide (2 centimeters), 4 inches (10 centimeters) long.
2. Push a toothpick into the center of the small cardboard or Styrofoam.
3. Place this piece on top of a larger piece of cardboard or poster board.
4. Shine your flashlight on the toothpick to create a shadow line.

Try different ways of making the shadow line change its direction and its length.

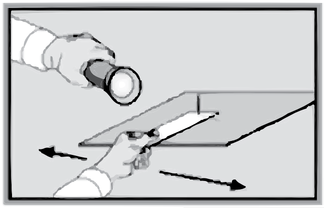
Moving the Flashlight

Without moving the toothpick, change the direction of the shadow line by moving the flashlight.



Moving the Toothpick

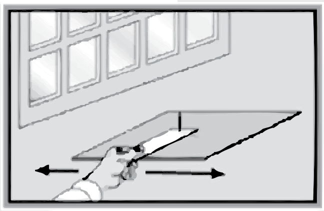
Without moving the flashlight, but keeping it pointed at the toothpick, move the toothpick around and observe what happens to the shadow.



Using the Sun

as a Light Source

You can also form a shadow line by placing a toothpick in direct sunlight.

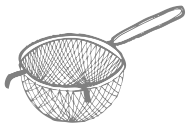


This can be done by going outside or to a window that faces the Sun. What happens to the shadow when you move the toothpick around in the sunlight? How is it similar, or different, to the shadow created by the flashlight?

What is going on?

You should notice some interesting differences between shadows created by the flashlight and those created by the Sun. As the Sun is so far away, you may have noticed that the angle of the shadow remained the same when you moved the toothpick. Since the light source of the flashlight was much closer, the shadow was different.

2. Sieve of Eratosthenes



The most efficient way to find all of the small primes (all those less than 10,000,000) is by using a sieve such as the Sieve of Eratosthenes (ca 240 BC):

Make a list of all the integers less than or equal to (n) (and greater than one).

Strike out the multiples of all primes less than or equal to the square root of (n), then the numbers that are left are the primes.

For example, to find all the primes less than or equal to 30; first list the numbers from 2 to 30:

(2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30)

The first number 2 is prime, so keep it (we will circle it) and cross out its multiples (we will cross them), so the red numbers are not prime.

(2)-3-~~4~~-5-~~6~~-7-~~8~~-9-~~10~~-11-~~12~~-13-~~14~~-15-~~16~~-17-~~18~~-19-~~20~~-21-~~22~~-23-~~24~~-25-~~26~~-27-~~28~~-29-~~30~~)

The first number left (still black) is 3, so it is the first odd prime. Keep it and cross out all of its multiples. We know that all multiples less than 9 (i.e. 6) will already have been crossed out, so we can start crossing out at $3^2=9$.

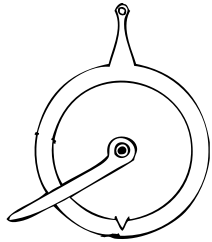
(2)(3)-~~4~~-5-~~6~~-7-~~8~~-~~9~~-~~10~~-11-~~12~~-13-~~14~~-~~15~~-~~16~~-17-~~18~~-19-~~20~~-~~21~~-~~22~~-23-~~24~~-25-~~26~~-~~27~~-~~28~~-29-~~30~~)

Now the first number left (still black) is 5, the second odd prime. So keep it also and cross out all of its multiples (all multiples less than 5^2=25 have already been crossed out, and in fact 25 is the only multiple not yet crossed out).

(2-3-~~4~~-5-~~6~~-7-~~8~~-~~9~~-~~10~~-11-~~12~~-13-~~14~~-~~15~~-~~16~~-17-~~18~~-19-~~20~~-~~21~~-~~22~~-23-~~24~~-~~25~~-~~26~~-~~27~~-~~28~~-29-~~30~~)

The next number left, 7, is larger than the square root of 30, so there are no multiples of 7 to cross out that have not already been crossed out; and therefore the sieve is complete. Therefore all of the numbers left are primes: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}. Notice that we have just found these primes without dividing.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



3. The "Pocket Sundial"

Materials:

- A pair of scissors
- Scotch tape
- A thin thread
- A compass

Steps:

1. Using the scissors, carefully cut out the "Pocket Sundial" and 15 cm of the thin thread, which will work as the pointer (or gnomon).

2. Use the scotch tape to fix the thread at the midpoint of the line connecting the two numbers 6, and the second end at the mark of your latitude.

3. Fold the "Pocket Sundial" in half along the horizontal line indicated in the figure, so that when you unfold the two parts of the sundial they are at right angle with each other.

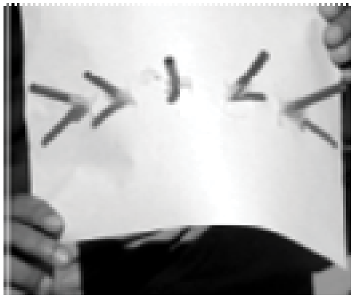
How to use your "Pocket Sundial"

Take your "Pocket Sundial" outside and aim it towards the North using the compass. The shadow cast by the thread (gnomon) indicates the time according to the hour lines illustrated.

4. Shadows on Earth

Materials:

- Five 4 cm straw pieces
- Tape or modeling clay
- Piece of paper (10 X 10 cm)



Model of shadows of sticks at noon, at different latitudes and the same longitude.

You are going to make a model of shadows at different points on Earth.

First, draw a straight line across the piece of paper and tape the five straw pieces, equally spaced, along this line, so the straws stand straight up.

The paper and straws could be a model of sticks placed at different locations on Earth by curving the paper, with the straws on the convex side.

To avoid damaging your eyes, never look directly at the sun.

How can the paper and straws model my experiment in sunlight?

Facing the sun, hold the paper at the ends of the long sides and curve it so the straws point out.

Turn the paper to make the shadow of one straw disappear. Make this straw point directly at the sun.

The straw without a shadow models the well at Aswan.

What happens to the shadows of the other straws? relate the shadow of each one to its position. Relate these shadows to the shadows I used to measure the Earth's circumference. See the figure.

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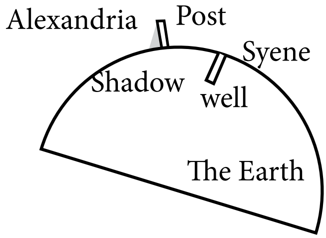
Test Yourself

1. In the picture, draw the Sun, and show where it must be in order to shine directly down the well at noon in Syene and create a shadow in Alexandria.



2. If you have a vertical post in Syene, and at the same time the Sun shines directly down the well, will the post cast a shadow?

Yes

No

Why do we see a shadow cast by the post at noon in Alexandria, and at the same time we see no shadow in Syene? What does this tell us about the shape of the Earth?

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Glossary

(1) **Archimedes:** An ancient Greek mathematician and philosopher who made fundamental discoveries in the fields of physics and engineering.

(2) **Beta:** The second letter of the Greek alphabet. It also refers to several symbols, according to the context it has been used in.

(3) **Ptolemy III Euergetes I:** The third ruler of the Ptolemaic Dynasty in Egypt. He was the eldest son of Ptolemy II Philadelphus and his first wife Arsinoe I.

(4) **Summer Solstice:** A solstice is either of the two events of the year when the sun is at its greatest distance from the equatorial plane. The summer solstice is the longest day of the year, in the sense that the length of time elapsed between sunrise and sunset on this day is a maximum for the year.

(5) **Meridian:** An imaginary great circle on the celestial sphere that passes through the zenith and the north point on the horizon.

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