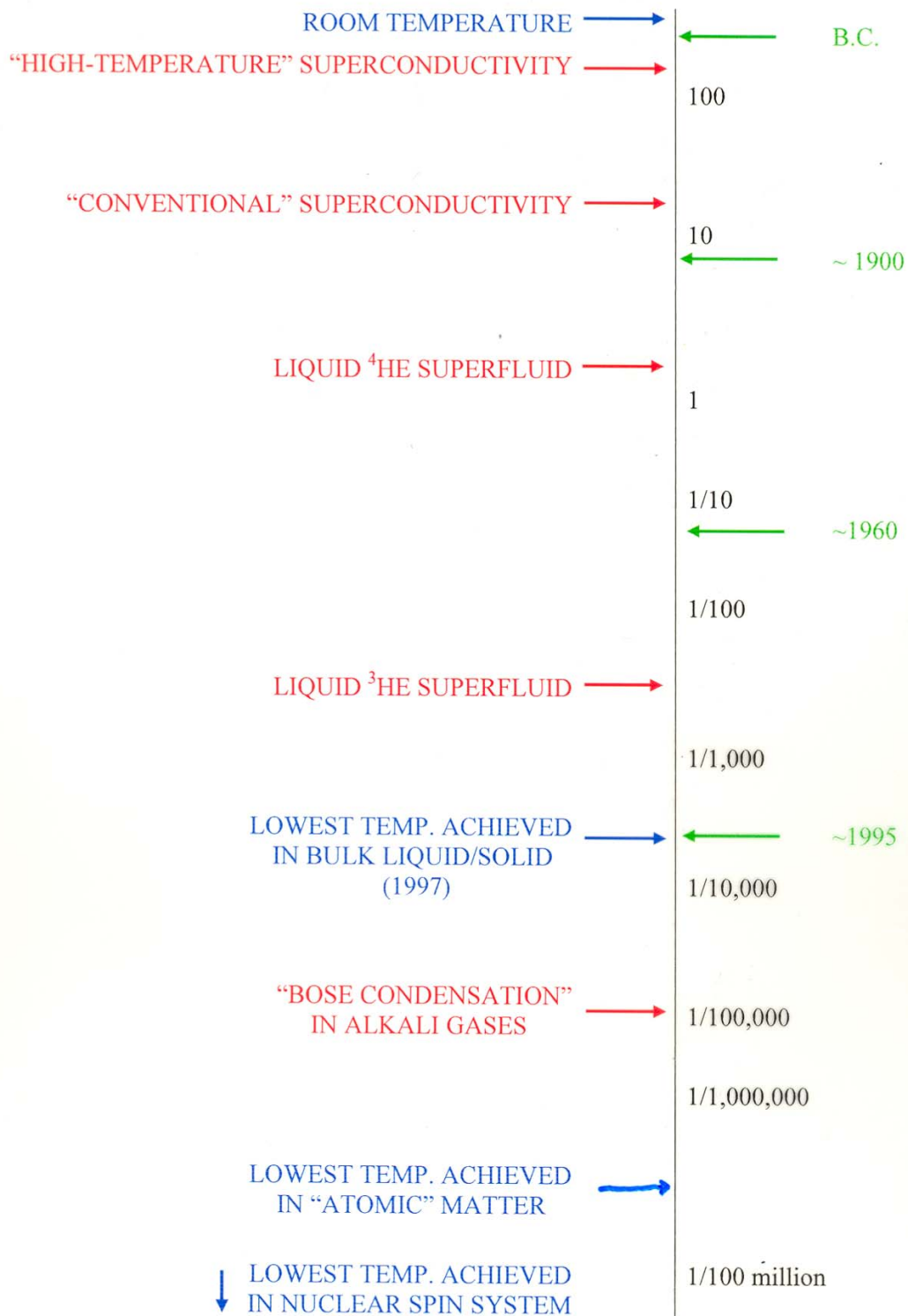
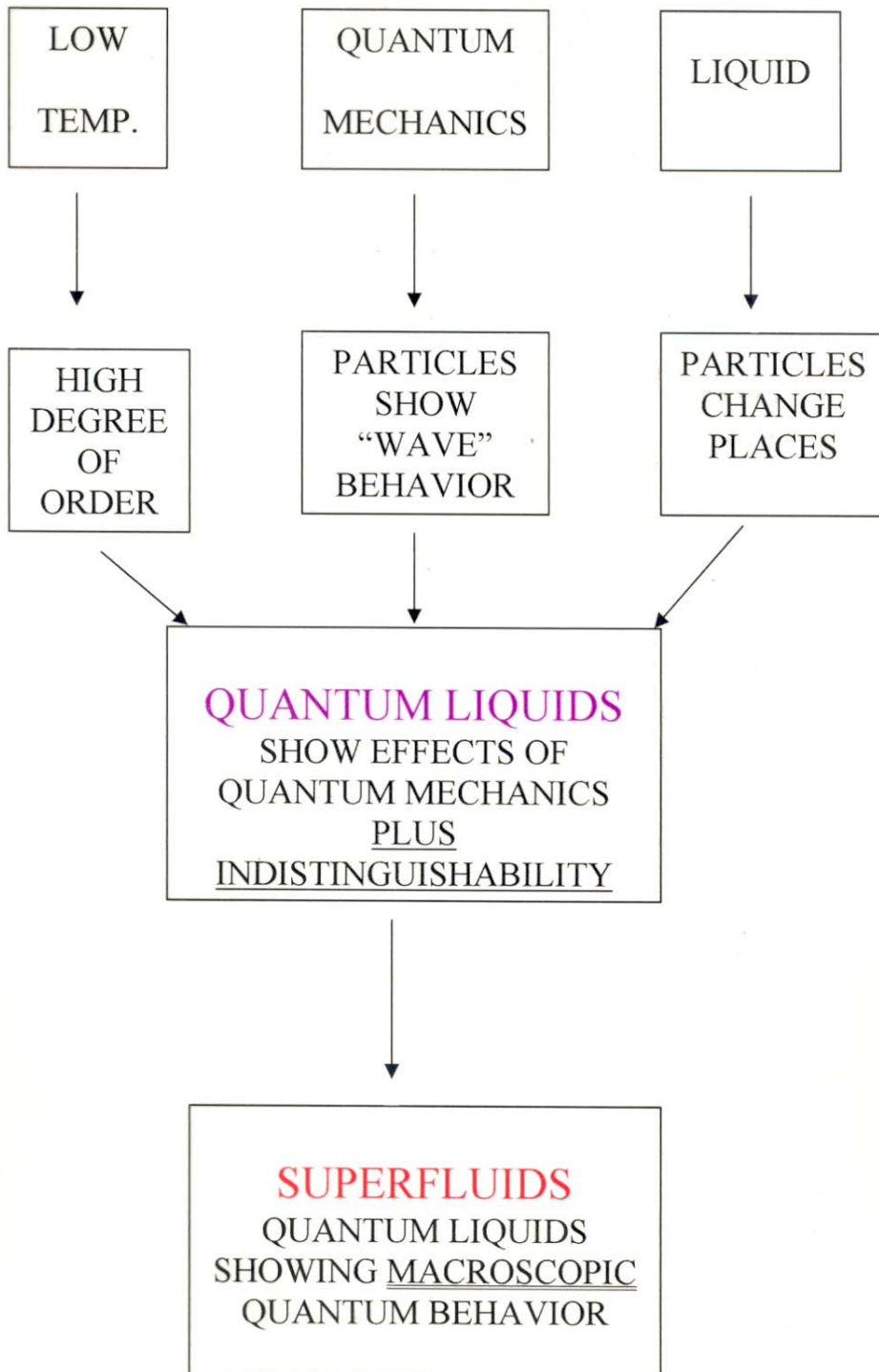


SUPERCONDUCTIVITY
(1997)

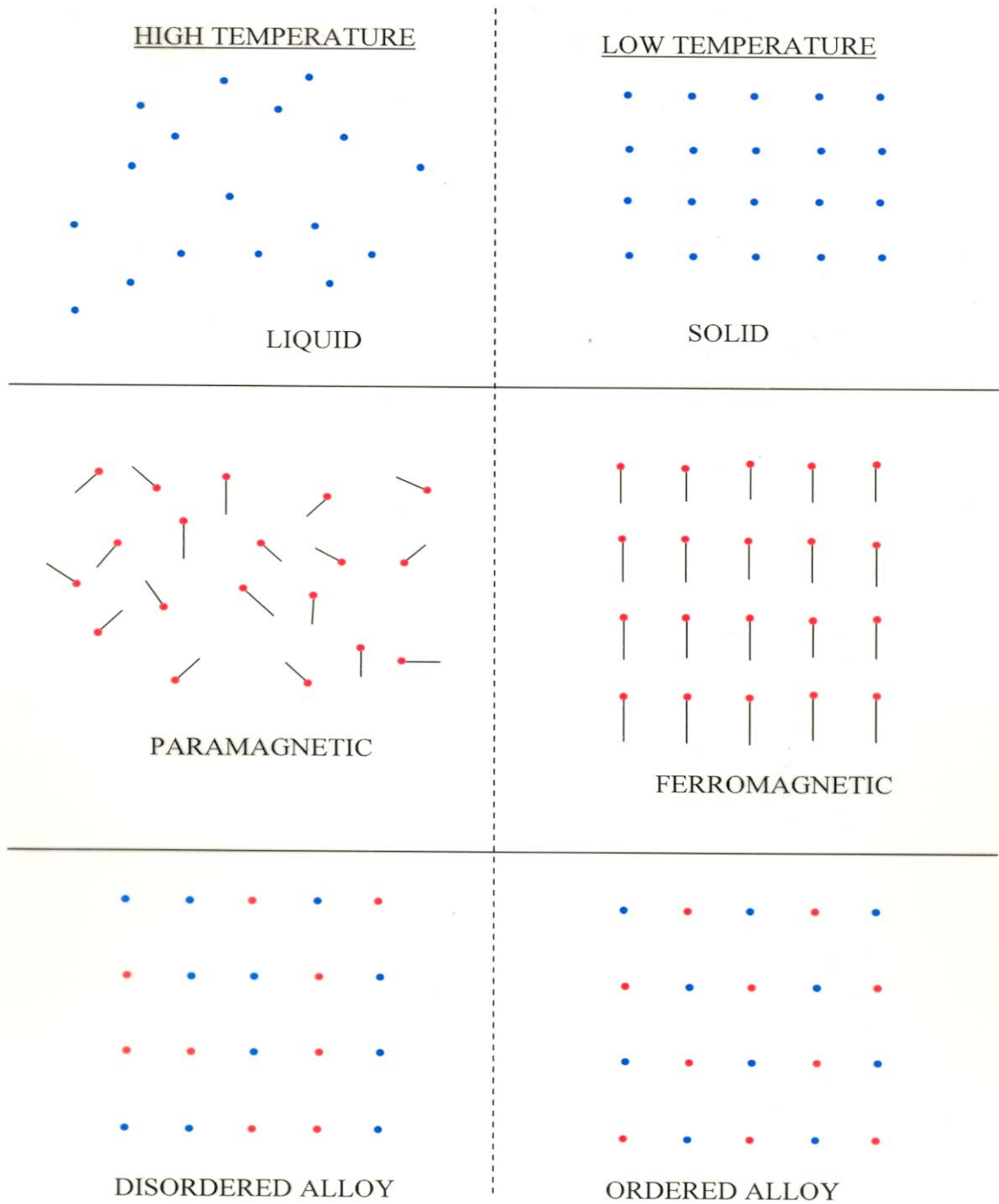


“LOGARITHMIC” TEMPERATURE SCALE
(EACH INTERVAL CORRESPONDS TO A FACTOR OF 10)

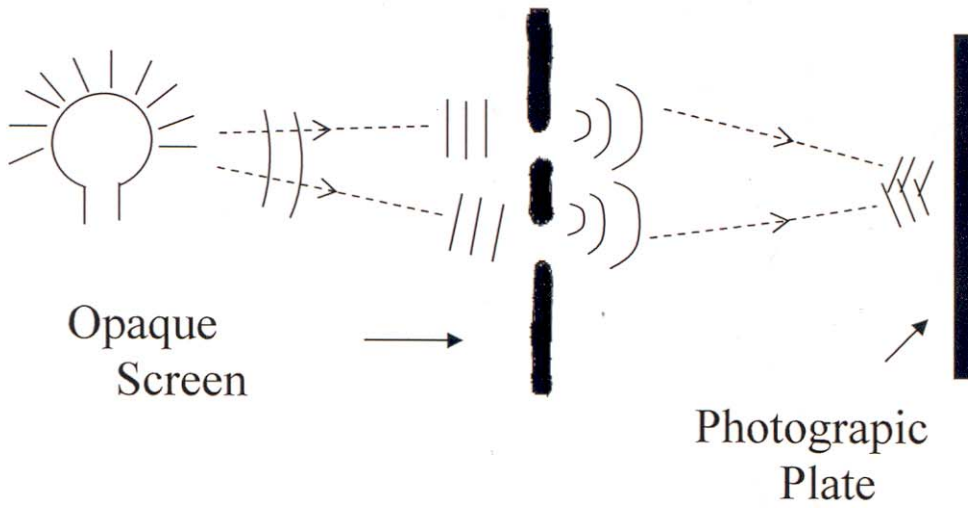




TEMPERATURE, ORDER and DISORDER



PARTICLES AS WAVES



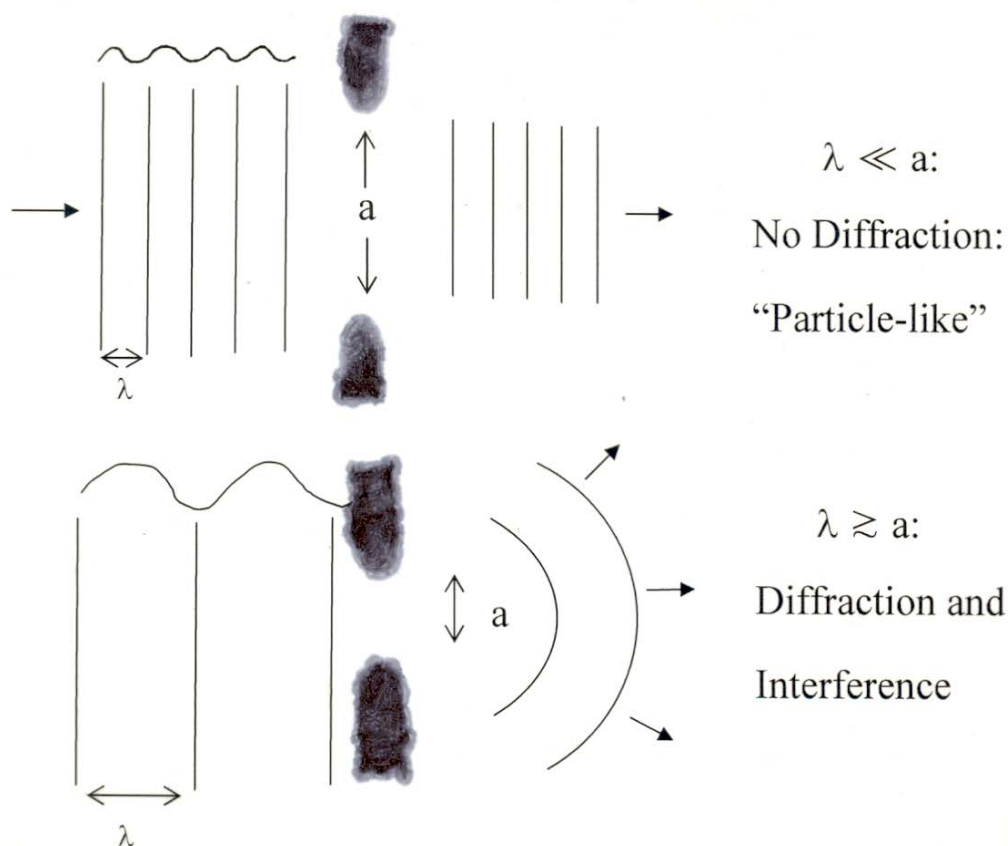
For Particles:

$$\lambda = h/mv$$

Wavelength

“DE BROGLIE RELATION”

When does a “wave” behave like a “particle”?



since $\lambda = h/mv$ (De Broglie) need

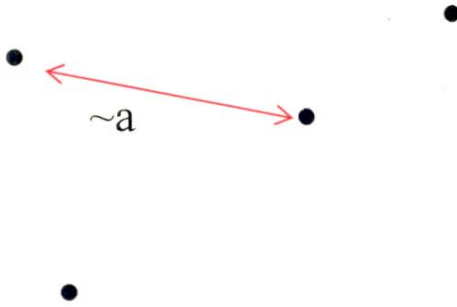
$$v \lesssim h/ma: \text{ but } \frac{1}{2}mv^2 \sim k_B T$$

so to see “wave” effects need

$$T \lesssim h^2/2mk_B a^2$$

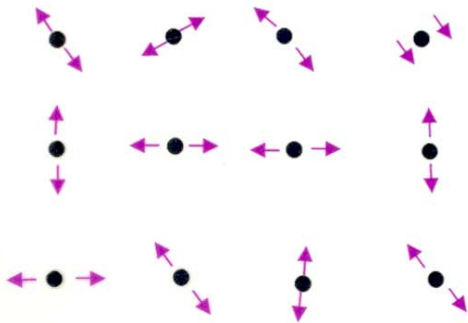
Boltzmann's
constant

Why “Quantum Liquids”?



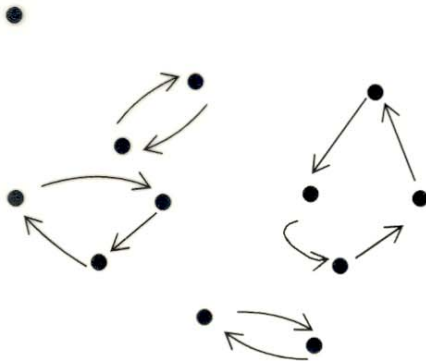
Gas: (usually)

$\lambda \ll a$
so no “wave”
(quantum) effects



Solid at low T:

$\lambda \gtrsim a$ but atoms
don't change places

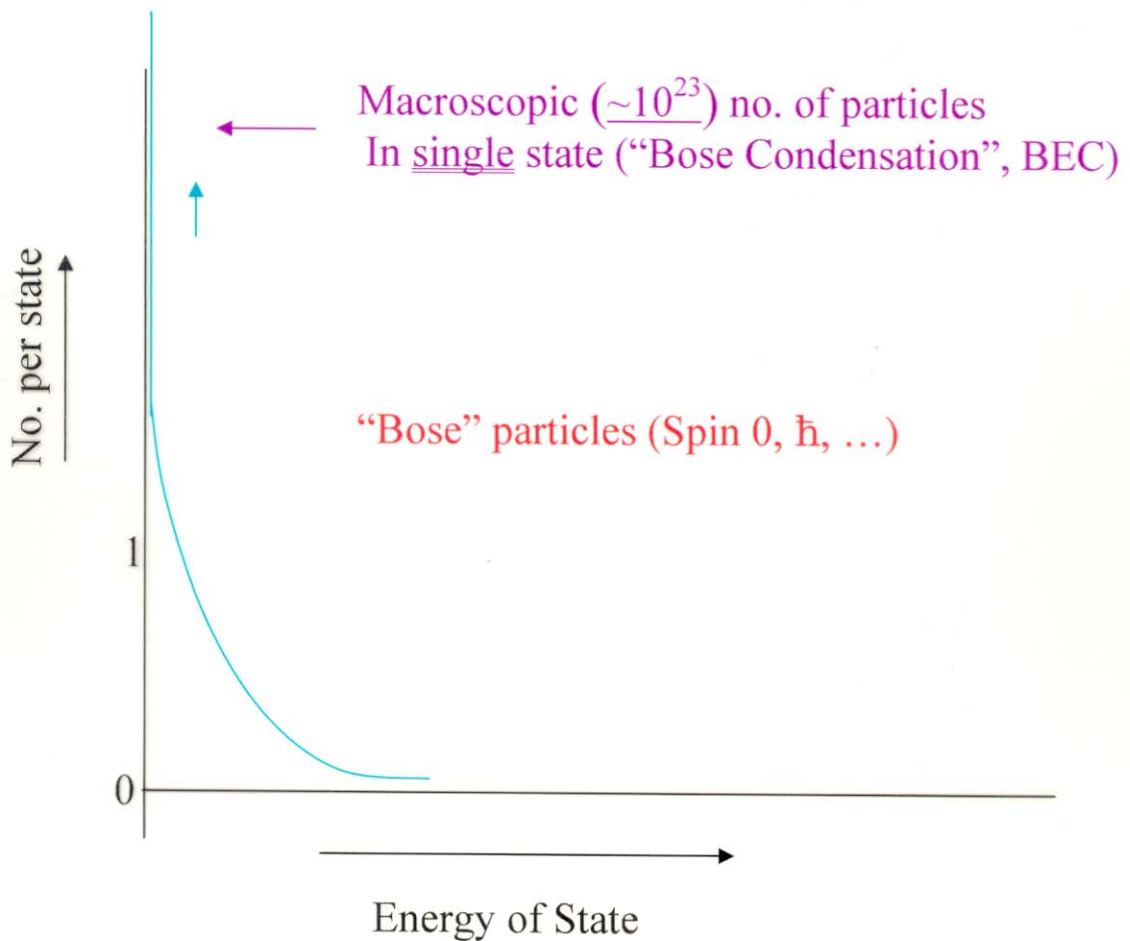
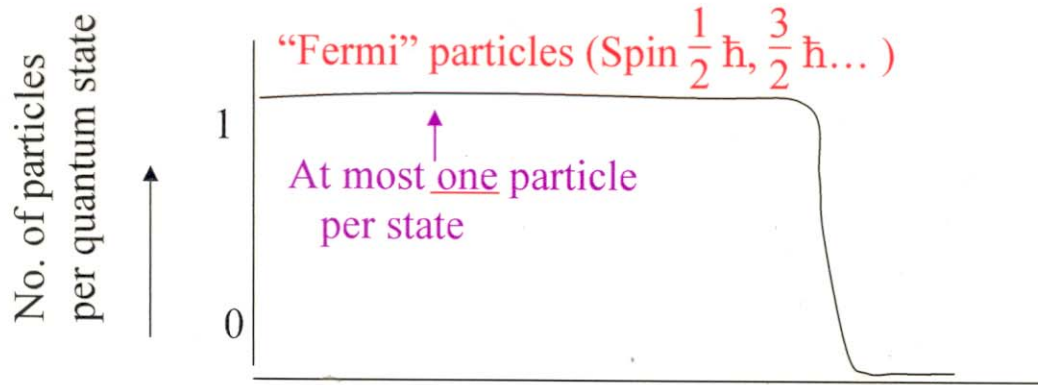


Liquid at low T:

$\lambda \gtrsim a$ and
atoms change places

$T \lesssim 20^\circ \text{K}/(\text{Atomic No.})$

“QUANTUM STATISTICS”





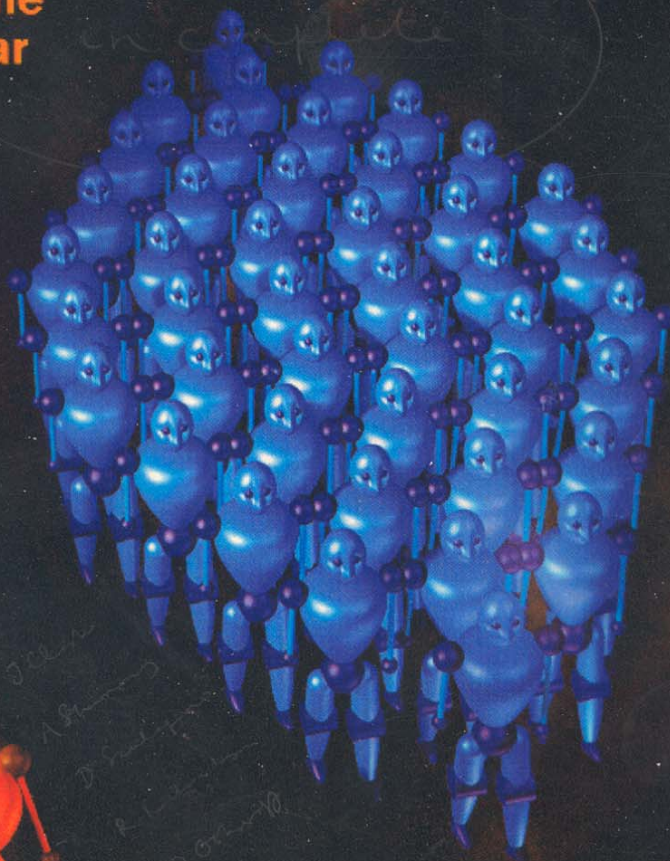
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SCIENCE

SCIENCE

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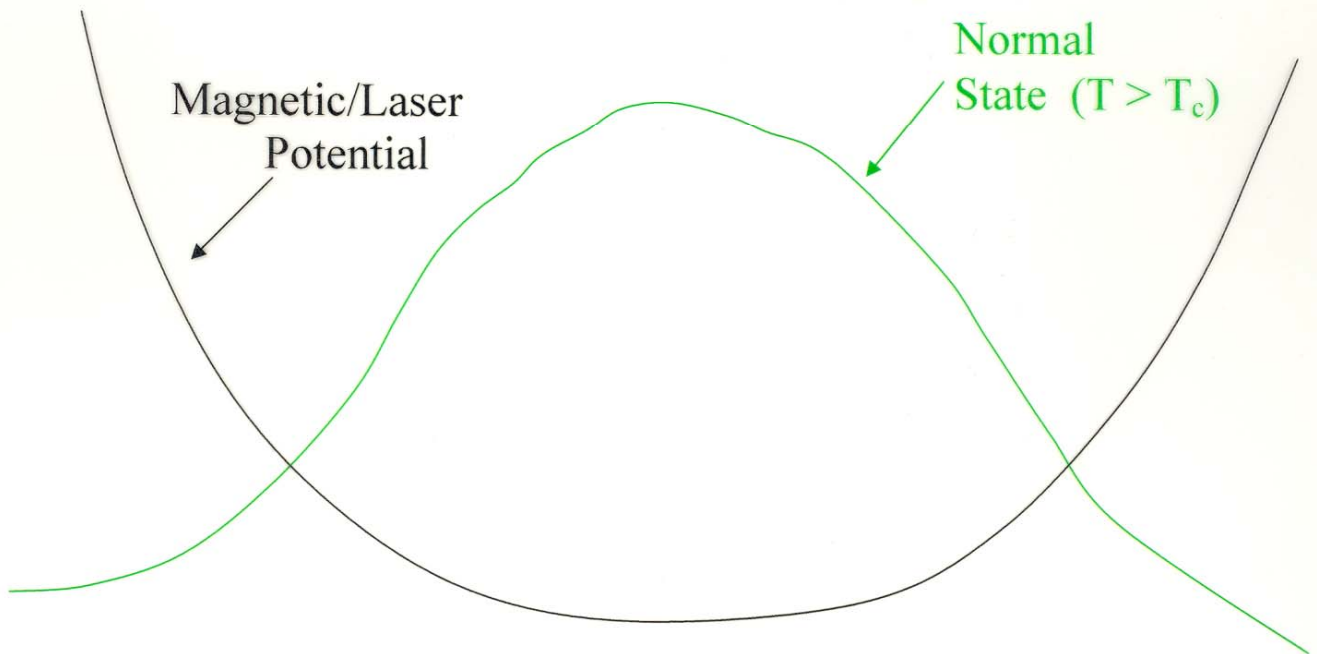
**Molecule
of the
Year**



**the
Bose-Einstein
Condensate**

HOW TO SEE BEC OCCURRING?

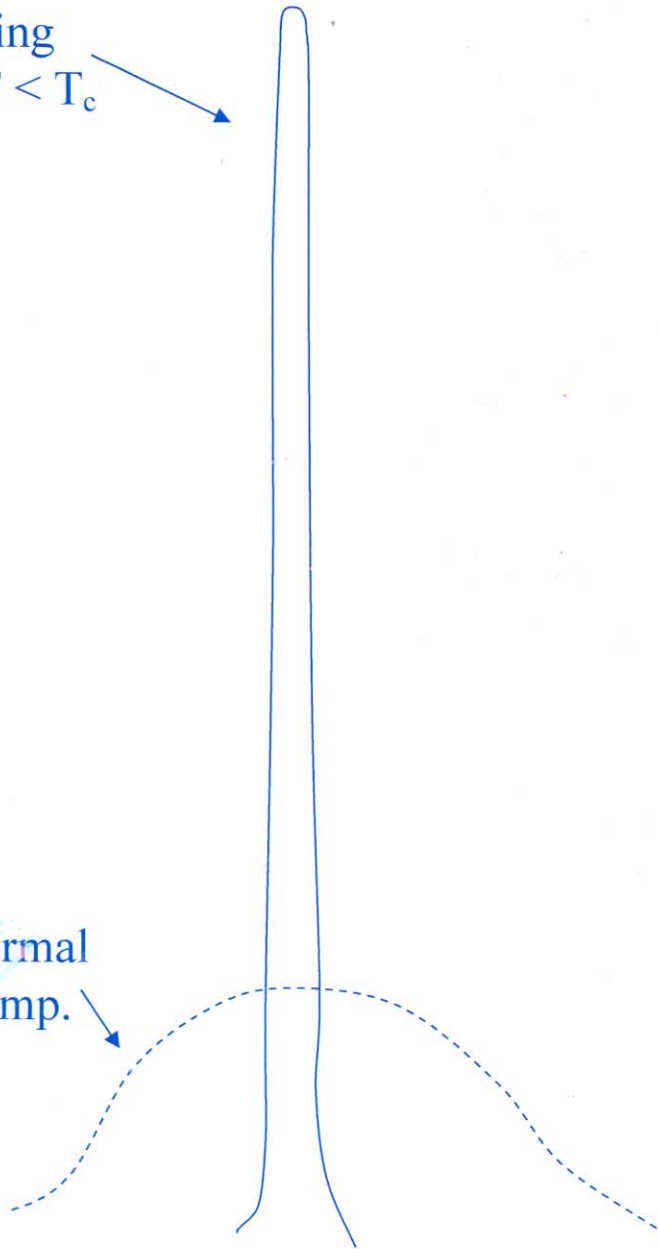
LITERALLY



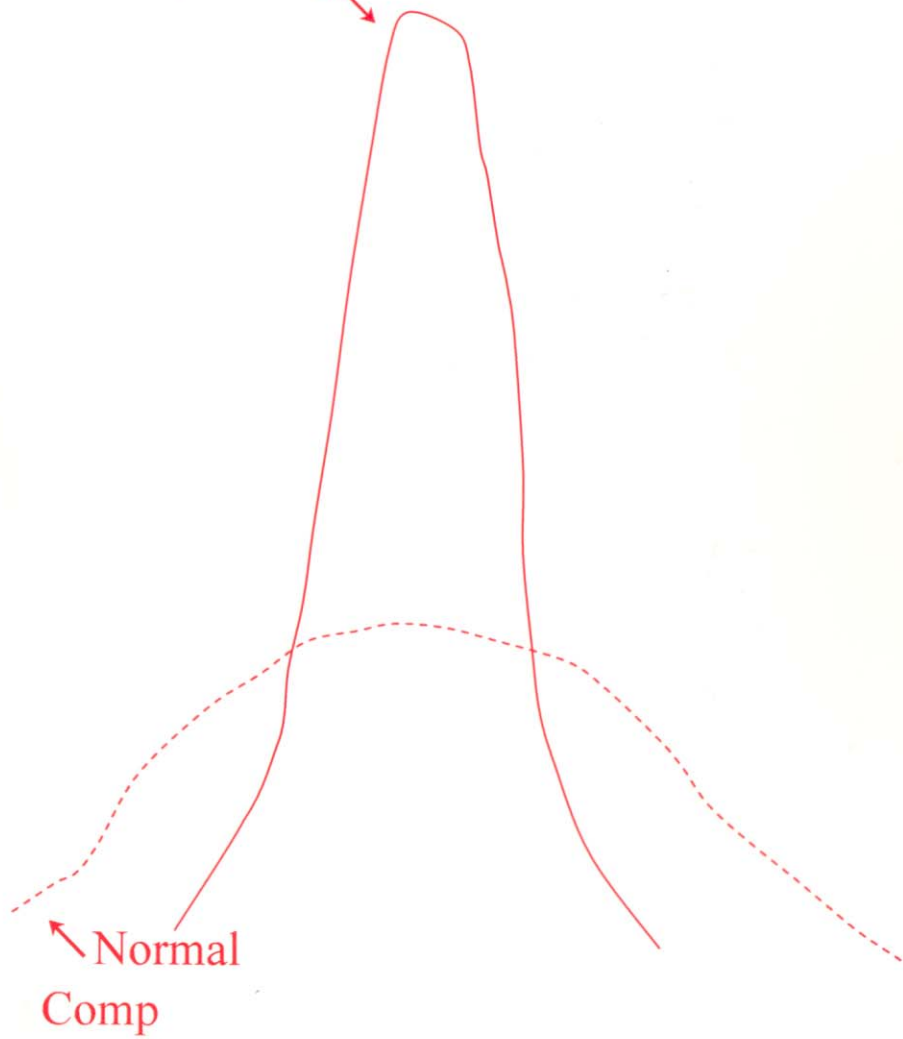
Noninteracting
Bose Gas, $T < T_c$

Normal
Comp.

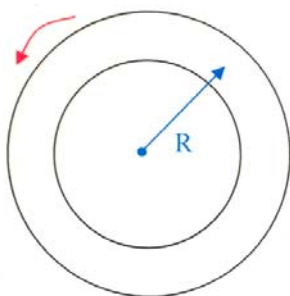
$\sim X_{ZP}$



Interacting
Bose Gas, $T < T_c$



“NO-ROTATION” EFFECT IN LIQUID ^4He



Walls rotating with ang. velocity

$$\omega \lesssim \omega_c \Leftrightarrow \equiv \hbar/m R^2$$

What does liquid do?

General principle: Average ang. velocity of atoms ($\bar{\omega}$) as close as possible to ω

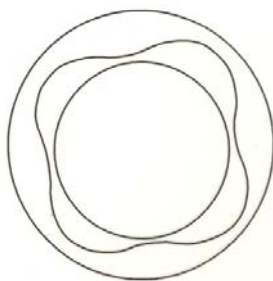
↑ : Single-atom states must obey
quantization condition: $\omega = n\omega_c$ ($l = n \hbar$)

$$n\lambda = \underbrace{2\pi R}_{\text{circumference}} + \text{d.B. } \lambda = h/p$$

$$\Rightarrow L = pR = n \hbar$$

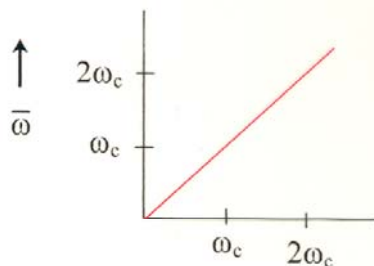
↙ ang. mom^m.

$$\Rightarrow L/mR^2 \equiv \omega = n\omega_c$$

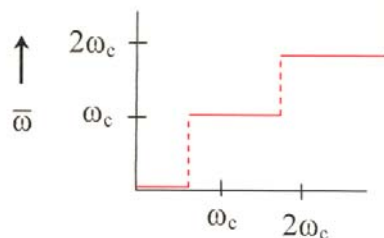


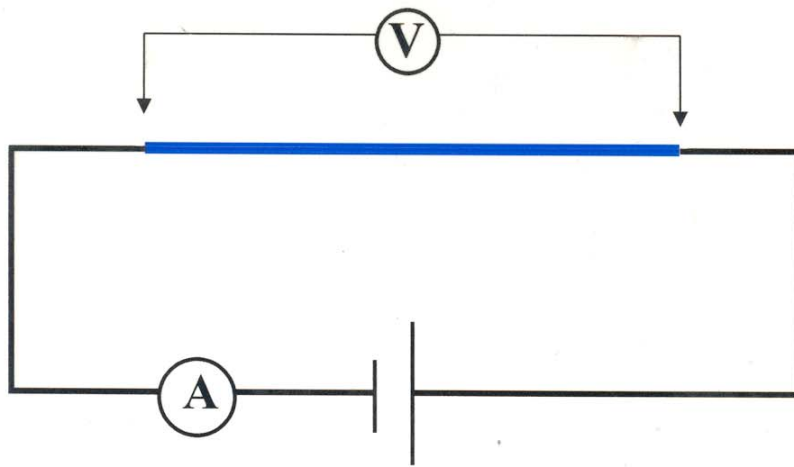
- A. “Normal” (non-BEC) system:
many different single-particle
states occupied (typical value of
 $n \sim (kT/\hbar\omega_c)^{1/2} \sim 10^7$)

\Rightarrow to get $\bar{\omega} = \omega$, just shift atoms
slightly between states.

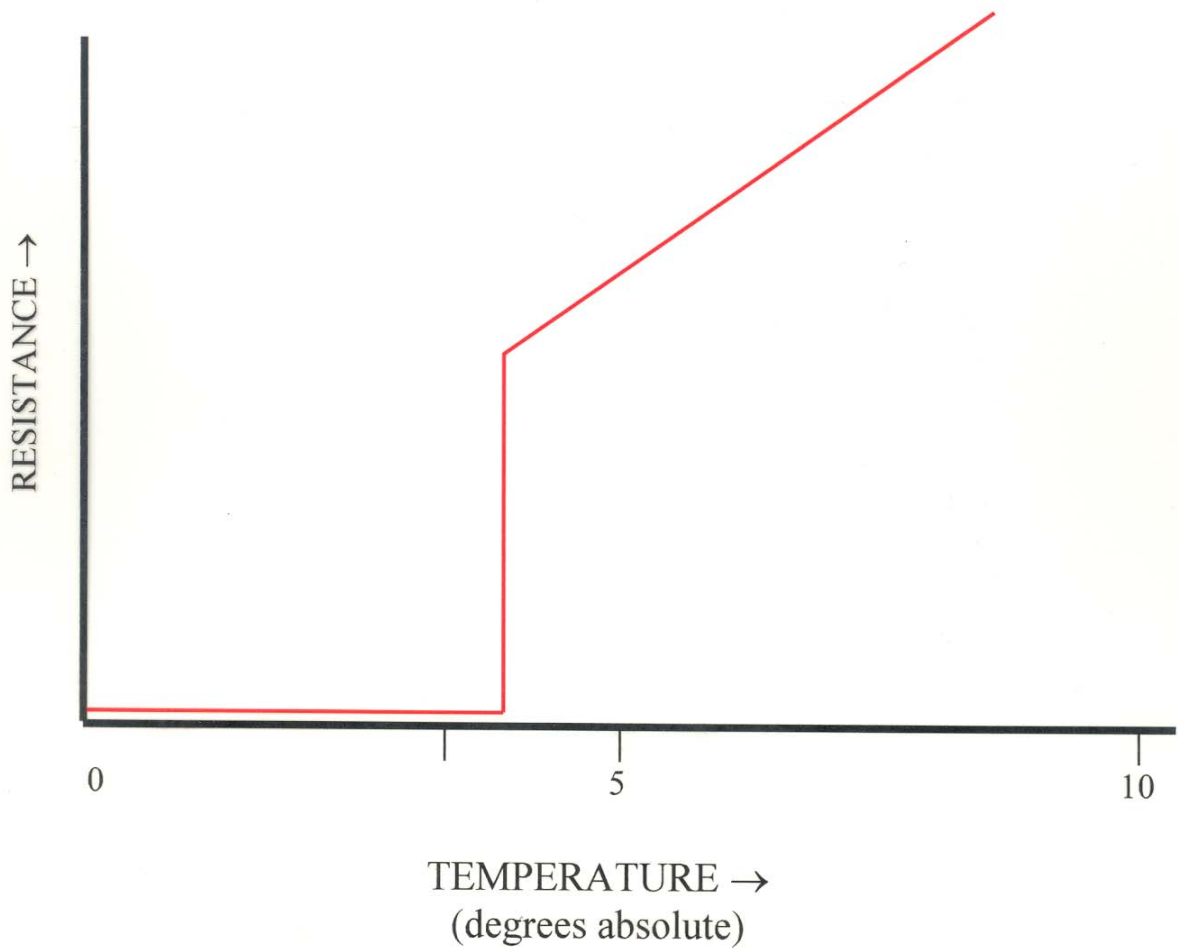


- B. BEC system ($T \ll T_c$):
(almost) all atoms in
condensate must have **same**
value of n . (n_0) $\Rightarrow \bar{\omega} \cong n_0 \omega_c$

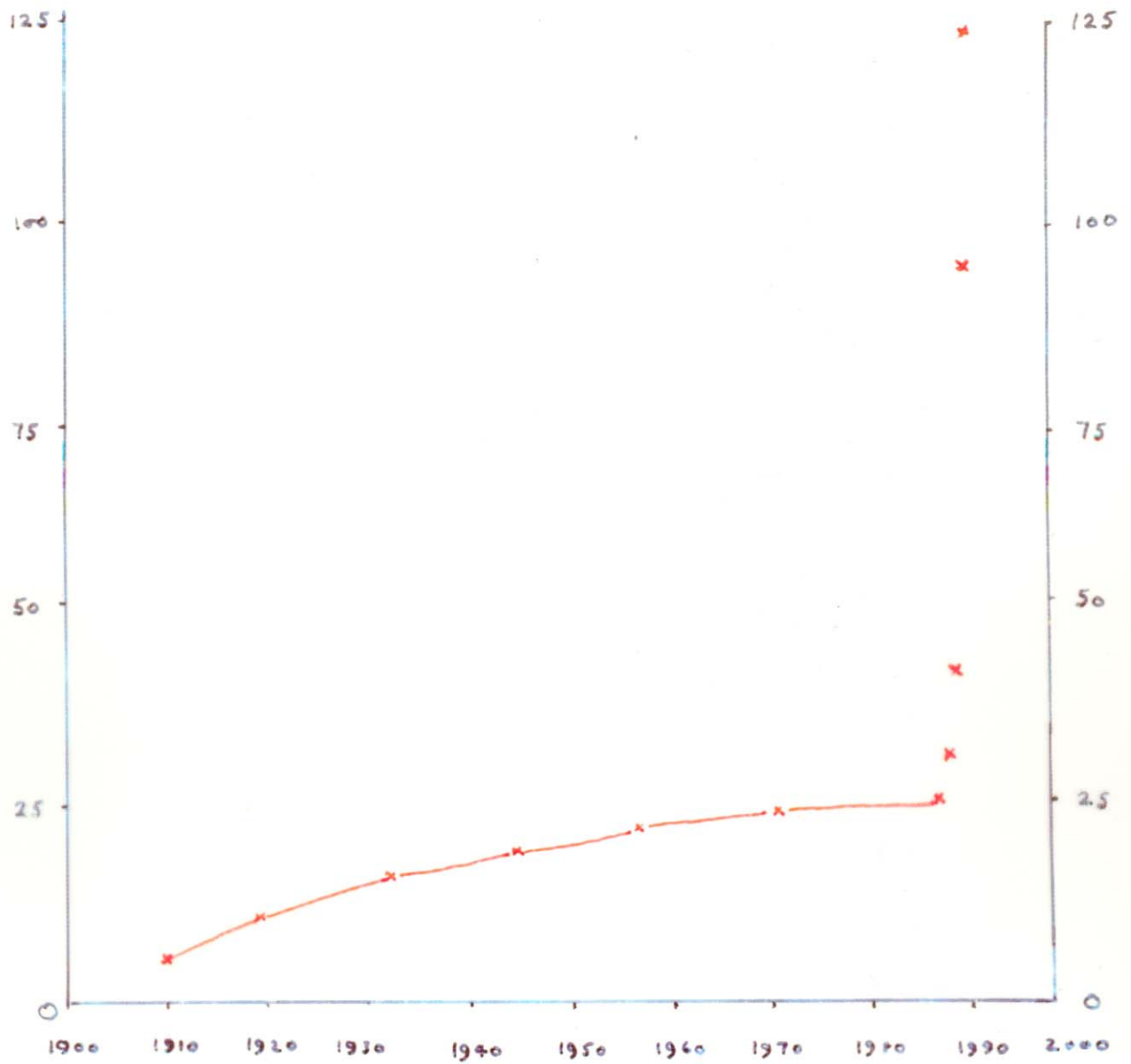




resistance of **—** = V/A = voltage/current



HISTORY OF THE HIGHEST TEMPERATURE
("T_c") AT WHICH SUPERCONDUCTIVITY KNOWN

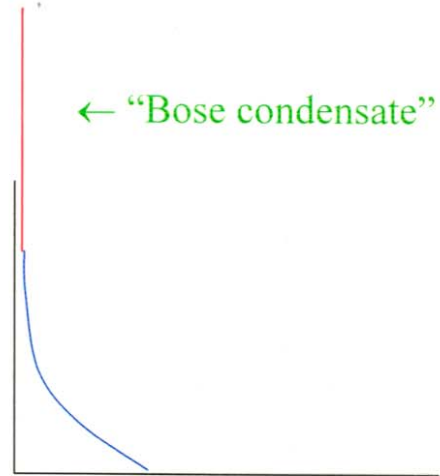
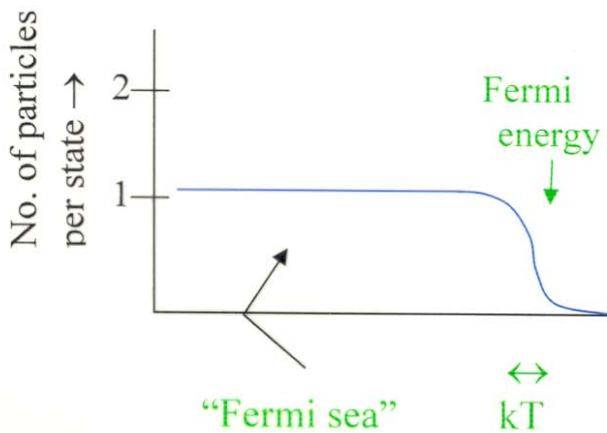


PHYSICS OF SUPERCONDUCTIVITY

“Spin” of elementary particles = $\frac{n}{2} \hbar$

0, 1, 2, ... bosons
 $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$ fermions

At low temperatures:



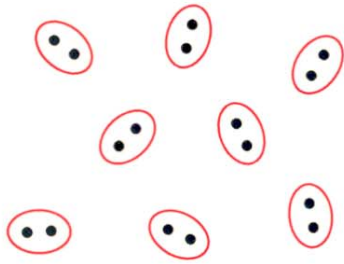
Electrons in metals: spin $\frac{1}{2} \Rightarrow$ fermions

But a compound object consisting of an **even** no. of fermions has spin 0, 1, 2 ... \Rightarrow boson.

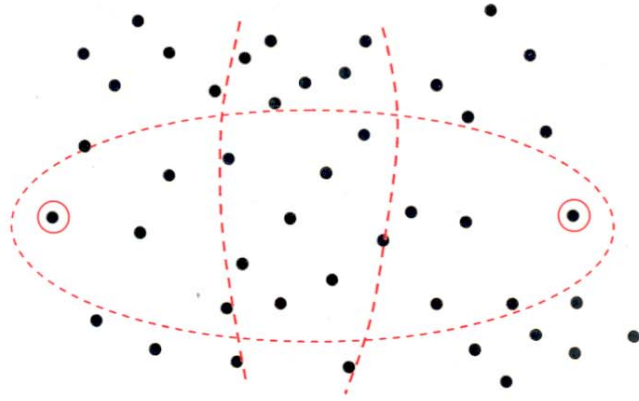
(Ex: $2p + 2n + 2e = {}^4\text{He}$ atom)

\Rightarrow can undergo Bose condensation

Pairing of electrons:



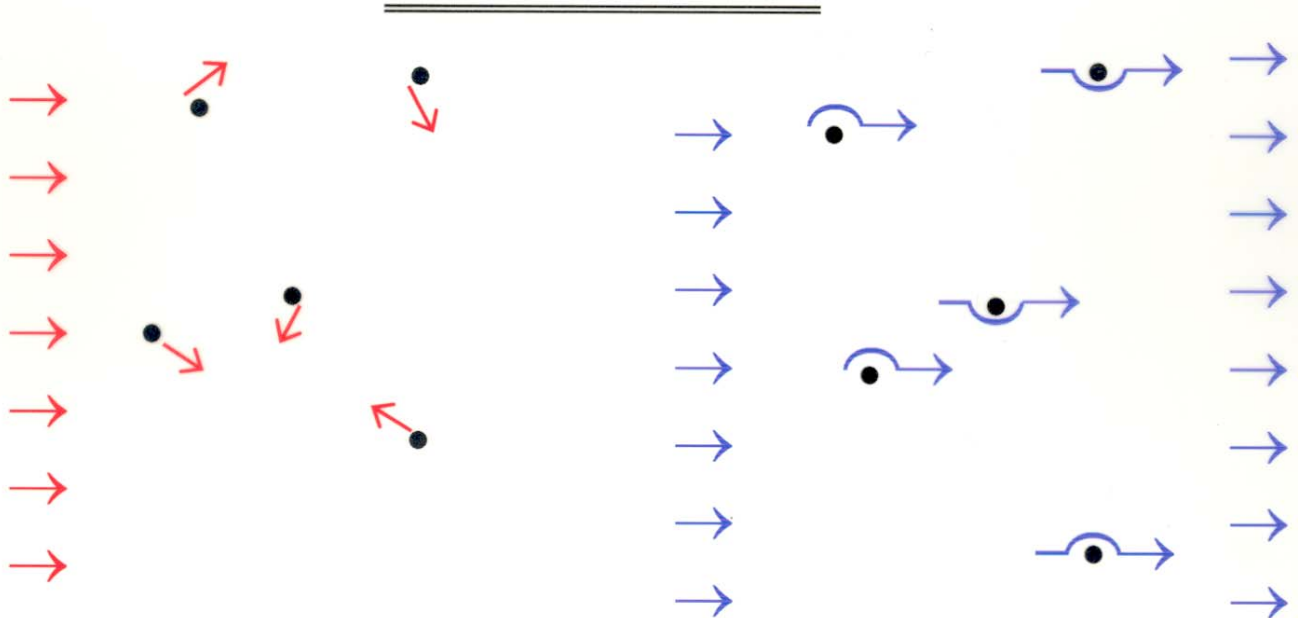
“di-electronic molecules”



Cooper Pairs

In simplest (“BCS”) theory, Cooper pairs, once formed, must automatically undergo Bose condensation!

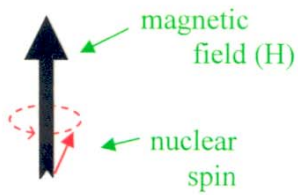
⇒ must all do exactly the same thing at the same time (also in nonequilibrium situation)



Single electrons in
normal metal

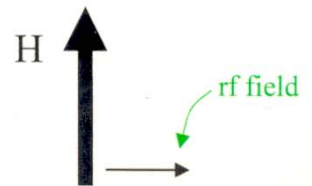
Cooper pairs in
superconductor

NUCLEAR MAGNETIC RESONANCE

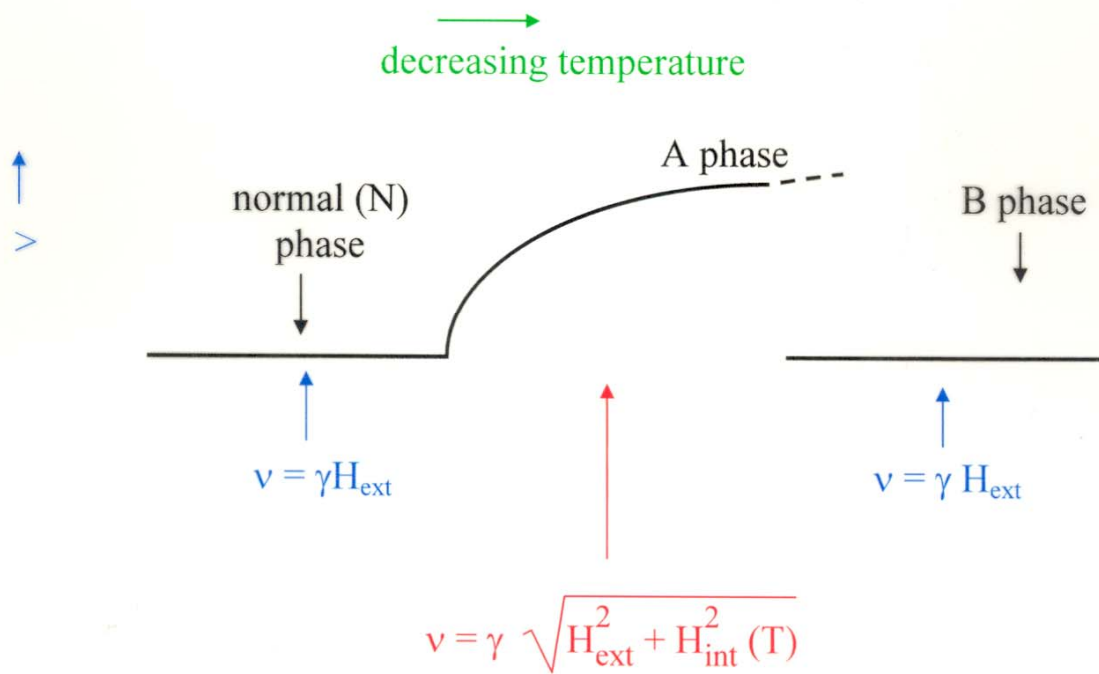


Rate of "precession"
 $\nu = \gamma H$
 "gyromagnetic ratio"

γ is known, (in ^3He , $\sim 3 \text{ kHz/gauss}$)
 so, rate of precession (ν) measures magn. field (H)
 To measure ν , apply
 oscillating (r.f.) field $\perp H$:
 field is strongly absorbed when its frequency is ν .



NMR IN LIQUID ^3He BELOW 3mK:

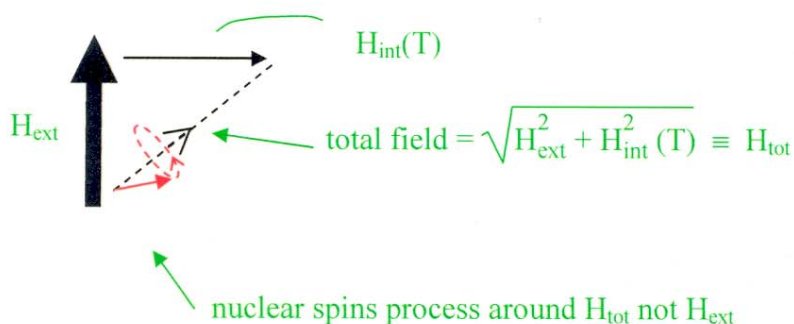


THE ^3He NMR PUZZLE (cont.)

In A phase, precession freq. ν is larger than value (γH_{ext}) in N phase, and given by expression of form

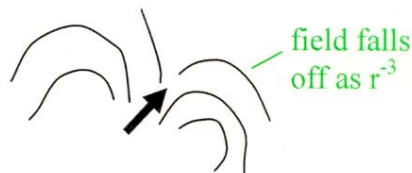
$$\nu = \gamma \sqrt{H_{\text{ext}}^2 + H_{\text{int}}^2} \text{ (T)}$$

Simplest interpretation:



Problem:

Only possible origin of $H_{\text{int}}(\text{T})$ is other nuclear spins



Max. value of field of one nuclear spin on another
(at distance of closest approach of atoms) < 1 gauss.

But, experimental value of $H_{\text{int}}(\text{T})$ is ~ 30 gauss!

**FIRST EVIDENCE FOR BREAKDOWN
OF QUANTUM MECHANICS?**

RESULT OF MORE SOPHISTICATED APPROACH:

A. Simple classical argument too naive.
(no “transverse” internal field)

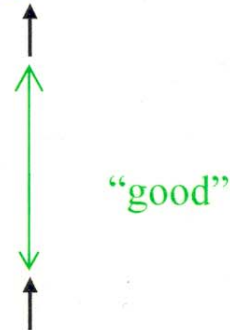
B. Nevertheless, indeed predict formula

$$\nu = \gamma \sqrt{H_{\text{ext}}^2 + H_o^2(T)}$$

where $H_o^2(T)$ is proportional to average value of nuclear dipole interaction energy $E_{\text{dip}}(T)$.

Experimental value of $H_o(T) \rightarrow E_{\text{dip}}(T) \sim 10^{-3}$ ergs/cm³

Why is this a problem?



- energy difference (ΔE) between “good” and “bad” orientations $< 10^{-7}$ K per pair.
- thermal energy (E_{th}) ($= k_B T$) $\sim 10^{-3}$ K.
 \Rightarrow preference for “good” orientation over “bad”
only $\sim \Delta E/E_{\text{th}} < 10^{-4}$
 \Rightarrow resulting value of $E_{\text{dip}}(T)$ **much too small to fit experiment.**
Need preference for “good” ^{over} ~~and~~ “bad” ~ 1 not $\sim \Delta E/E_{\text{th}}$!

SPONTANEOUSLY BROKEN SPIN-ORBIT SYMMETRY:

the analogy with ferromagnetism

FERROMAGNET



difference in energy per spin = ΔE (small)

Above Curie temp.

(“paramagnetic” phase), spins behave independently \Rightarrow thermal energy E_{th} competes with $\Delta E \Rightarrow$ polarization only $\sim \Delta E/E_{\text{th}} \ll 1$

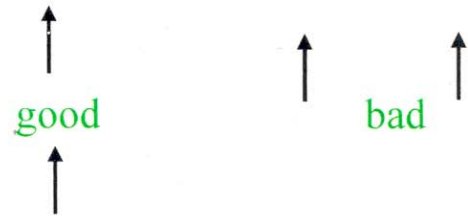
Below T_c (“ferromagnetic” phase): strong (exchange) forces constrain all spins to lie parallel:

↑↑↑↑↑... or ↓↓↓↓↓...
“good” “bad”

$$E_{\text{good}} - E_{\text{bad}} \sim N\Delta E \gg E_{\text{th}}$$

\Rightarrow polarization ~ 1

LIQUID ^3He



difference in energy per pair $\equiv \Delta E < 10^{-7} \text{ K}$

In normal phase, pairs behave

independently $\Rightarrow E_{\text{th}}$ competes with $\Delta E \Rightarrow$ “polarization” (pref. for good orientation over bad) only $\sim \Delta E/E_{\text{th}} \ll 1$.

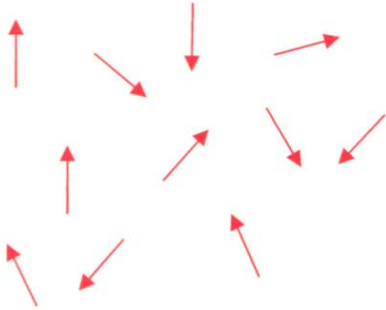
In A phase, **assume**: strong (kinetic-energy, VDW) forces constrain all pairs to behave in same way \Rightarrow either all “good” or all “bad”

$$E_{\text{good}} - E_{\text{bad}} \sim N\Delta E \gg E_{\text{th}} \quad \sim 10^{23}!$$

\Rightarrow polarization can be ~ 1

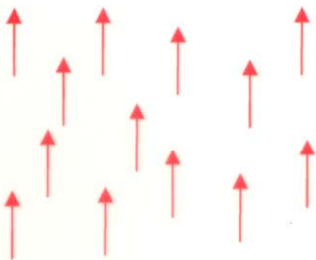
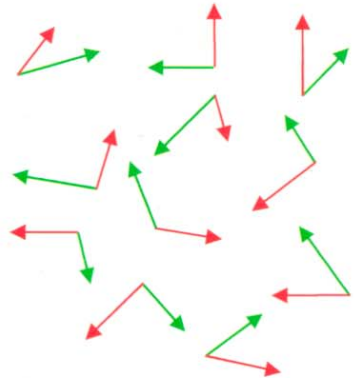
SBSOS: ORDERING MAY BE SUBTLE

FERROMAGNET

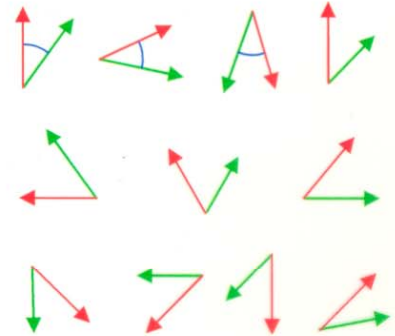



⇐ NORMAL
PHASE ⇒


LIQUID ^3He



⇐ ORDERED
PHASE ⇒



( = total spin of pair

 = relative orbital ang. momentum)

$$\langle \tilde{S} \rangle \neq 0$$

$$\langle \tilde{S} \rangle = \langle \tilde{L} \rangle = 0$$

$$\text{but } \langle \tilde{L} \times \tilde{S} \rangle \neq 0!$$

Amplification of ultra-weak effects by BEC (cf NMR):

Example: P- (but not T-) violating effects of neutral current part of weak interaction:

For single elementary particle, any EDM \underline{d} must be of form

$$\underline{d} = \text{const. } \underline{J} \leftarrow \text{violates T as well as P.}$$

But for ${}^3\text{He} - \text{B}$, can form

$$d \sim \text{const. } \underline{L} \times \underline{S} \sim \text{const. } \hat{\omega}$$



violates P but not T.

Effect is tiny for single pair, but since all pairs have same value of

$\underline{L} \times \underline{S}$, is multiplied by factor of $\sim 10^{23} \Rightarrow$

macroscopic P-violating effect?